

PHYSICS

Grade 11

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Ministry of Education, Science and Technology
Curriculum Development Centre
Sanothimi, Bhaktapur

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Preface

The curriculum is the central guide that decides what is essential for teaching and learning. A textbook is one of the main tools to deliver the intent and the content of the curriculum. Therefore, it should have such an attribute that it presents relevant, practical, and useful knowledge and the process of skills for the overall development of a learner in accordance with the curricular aims and intents. This textbook '**Physics Grade 11**' has been developed to address the main aims of the Secondary Level (Grade 11-12) Physics Curriculum and is aligned with the intent of the National Curriculum Framework for School Education, 2076.

This book was prepared and edited by an academic team that includes Mr. Arjun Sedai, Mr. Bagirath Neupane, Mr. Upendra Acharya and Mr. Lav Dev Bhatta. Several people notably; the Director Generals Mr. Ima Narayan Shrestha and Mr. Baikuntha Prasad Aryal, Professor Dr. Krishna Bakta Maharjan, Dr. Kamal Prasad Achary, Mr. Umanath Lamsal, Mr. Keshar Bahadur Khulal, Mr. Heramba Raj Kandel, MS. Mina Shrestha, MS. Pramila Bakhati, Mr. Purna Shrestha, Mr. Bishnu Paudel, Mr. Rosan Adhikari, Mr. Dipak Paudel and Mr. Yub Raj Adhikari also contributed significantly on the development of this book. The language of this book was edited by Mr. Ramesh Dhakal.

Effort has been made to make the book activity oriented and interesting to the learners. All the contents of each lesson in this textbook are equally important. However, teachers can adapt the contents and tasks to the need of their learners and classroom contexts. This textbook can be used as a major resource for classroom teaching but it is not all in all. The teachers are also encouraged to explore other resources in addition to this book and use them to supplement the curricular contents as provisioned in the curriculum. The Curriculum Development Centre always welcomes constructive feedback for the betterment of its publications.

Curriculum Development Centre
Sanothimi, Bhaktapur

Table of Contents

Chapter	Contents	Page No.
1	PHYSICAL QUANTITIES	1
2	VECTORS	21
3	KINEMATICS	41
4	DYNAMICS	73
5	WORK, ENERGY & POWER	103
6	CIRCULAR MOTION	132
7	GRAVITATION	154
8	ELASTICITY	184
9	HEAT AND TEMPERATURE	208
10	THERMAL EXPANSION	222
11	QUANTITY OF HEAT	246
12	RATE OF HEAT FLOW	270
13	IDEAL GAS	289
14	REFLECTION AT CURVED MIRROR	307
15	REFRACTION AT PLANE SURFACES	319
16	REFRACTION THROUGH PRISMS	336
17	LENS	345
18	DISPERSION	364
19	ELECTRIC CHARGES	378

20	ELECTRIC FIELD	387
21	ELECTRIC POTENTIAL, POTENTIAL DIFFERENCE AND POTENTIAL ENERGY	403
22	CAPACITORS	416
23	D.C. CIRCUITS	431
24	NUCLEAR PHYSICS	459
25	ENERGY LEVELS	487
26	SOLIDS	496

Chapter 1

Physical Quantities

Joseph Fourier (21 March 1768 – 30 May 1830) was a French mathematician and physicist. Fourier was born at Auxerre as a son of a tailor. He lost both his parents at the age of 9. He then went onto study at the Convent of St Mark.

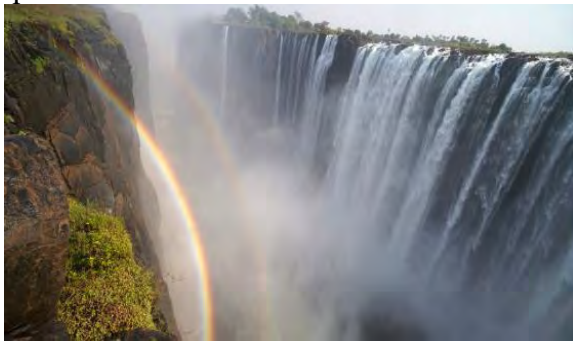
The concept of physical dimension, and of dimensional analysis was introduced by Joseph Fourier in 1822. The modern concept of dimension was introduced in 1863 by Clerk James Maxwell, who synthesized earlier formulations by Fourier.



Joseph Fourier (1768- 1830)

1.1 Introduction

Think of different phenomena occurring in our nature: formation and disappearance of the clouds in the sky, appearance of a rainbow, rainfall, evaporation of pond water, strong wind or storm, cold dry winter, and hot summer. These are examples of physical or natural phenomena. A clear understanding of such phenomena requires some laws of physics, and these laws are expressed in terms of physical quantities. Length, time, mass, velocity, force, momentum, temperature, etc. are examples of physical quantities. The study of physics starts with the measurement of these diverse physical quantities. Hence, in this chapter, we focus on physical quantities and the need for their measurement.



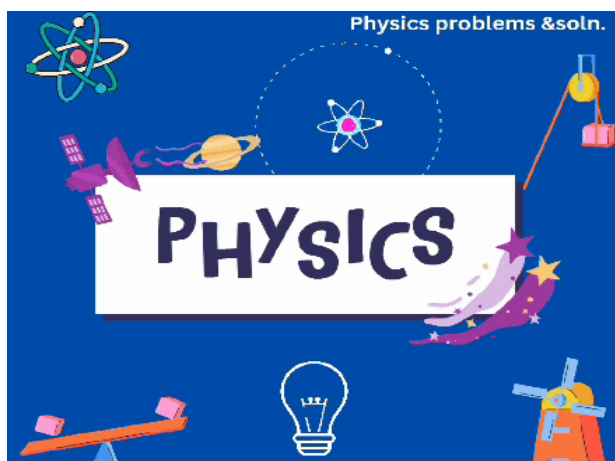


Fig 1.1(a) Showing what physics studies

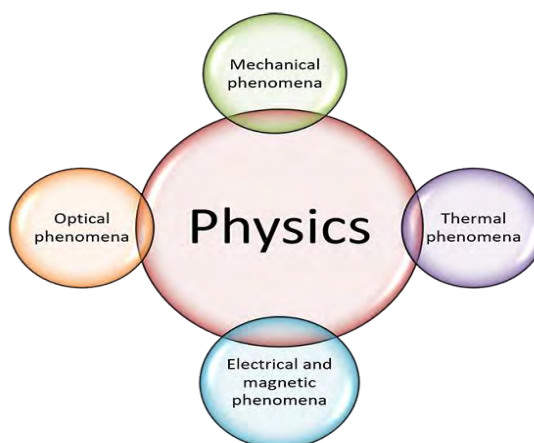


Fig. 1.1(b) Showing physical phenomena

Physics is the branch of science and is one of the most fundamental scientific disciplines. The main goal of physics is to understand how the universe functions. A scientist who specializes in the field of physics is called a **physicist**.

The word *physics* comes from the Latin word *physica* ('study of nature'), which itself is derived from the Greek term φύσις (*phúsis* 'meaning the *origin, nature, property*).

In Sanskrit, physics is called Bhautika, a physical world.

If a student looks around him/her, he/she finds desks, tables, benches, a board, a projector, a duster, and the school building. Beyond it, there are neighboring buildings, trees, vehicles on the road, and many more. All of these represent masses. If they are burnt, energy is released. The greater the mass, the more will be the energy liberated, and vice-versa. Apart from mass and energy, there is nothing around us. Hence, the entire universe is composed of only two things; mass and energy. *Ultimately, the word 'physics' refers to the science of matter and energy and their interactions.*

Physics is a quantitative science as it is based on the measurement that comes with numbers. Mathematics is used in physics to construct models aimed at predicting natural phenomena. Then these predictions are compared to the way the real world works. Physicists continuously strive to refine their models so that they can represent the world better. Physics studies very large things, as well as very small things. For instance, physicists cannot only study stars, planets, and galaxies but also small pieces of matter, such as atoms and electrons. They may also study about sound, light, heat, radioactivity, or even space and time. Physics not only helps people understand how objects move, but also how they change form, how they make sound, how hot or cold they will be, and what they are made of at the microscopic level. Physics is the branch of science that deals with the properties of matter and energy along with their interaction.

1.2 Physical Quantity

A physical quantity is something that can be measured. For example, Mass = 87.5 kg, Length = 1.4 m, etc.

Physical quantities can be measured with instruments or can be derived.

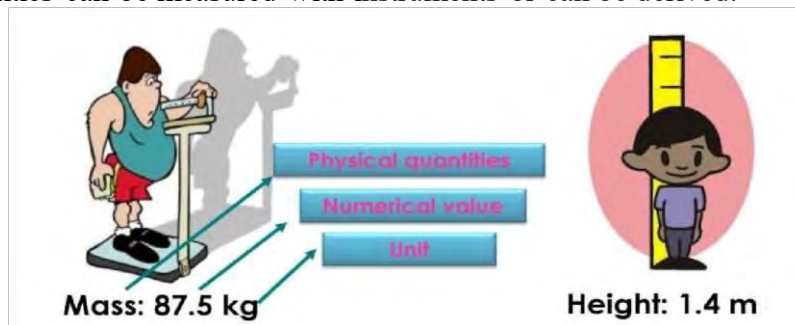


Fig 1.2 Showing measurement of physical quantity

All the measurable quantities from which the laws of physics are governed are the physical quantities. When we speak about something, say 2 hours journey, here '2 hours' refers to the physical quantity of time. Mass, length, time, temperature, velocity, acceleration, pressure, etc. are physical quantities. To understand them, one must measure and express them quantitatively in numerical values. This is done using units, which serve as standards of measurement. Physical quantities are expressed as multiples of these units. For example, the mass of a bag of apples is 5 kg, the distance is 200km, and the time is 2 hours.

In all these, Physical Quantity = nu , where n is the numerical value of the physical quantity, and u is the unit of measurement.

Physical quantities are of two types

1. Fundamental (or Basic) Quantities,
2. Derived Quantities

Fundamental Quantities

These are the physical quantities that are self-defined and independent of others. Examples are mass, length, time, temperature, illuminating power, electric current, and amount of a substance. Moreover, there are two sub-fundamental quantities; a plane angle and a solid angle.



Fig 1.3 Fundamental quantities

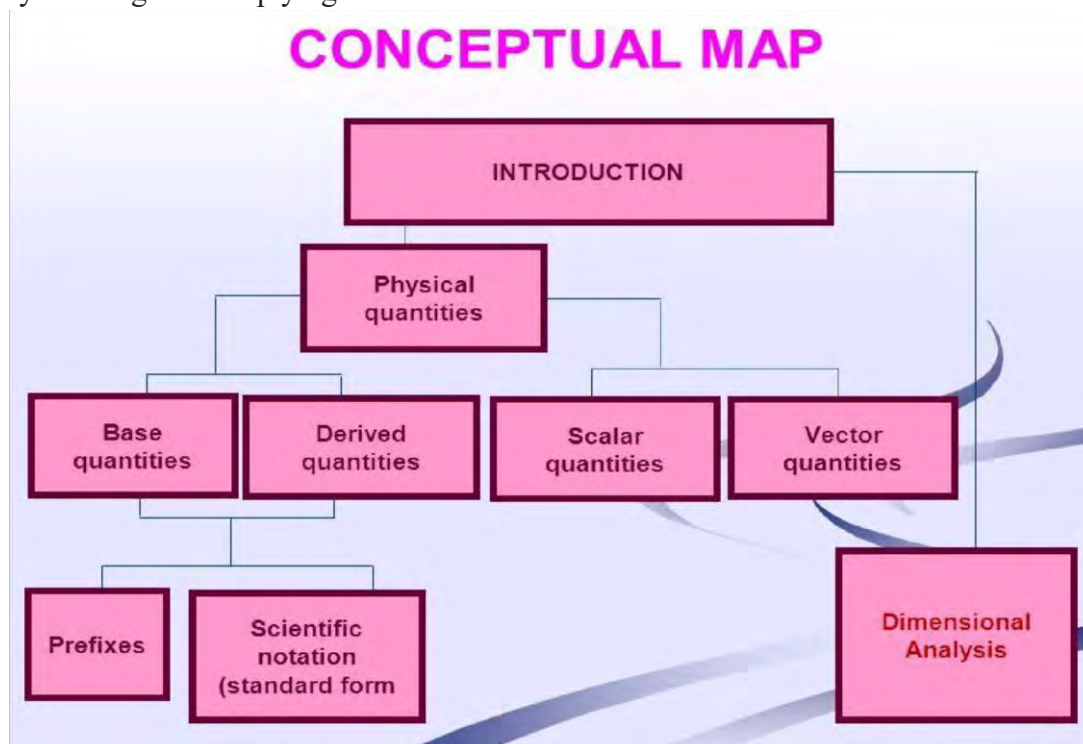
Derived Quantities

These are the quantities that are derived from the fundamental quantities. Examples: velocity, momentum, force, pressure, etc.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{And momentum} = \text{mass} \times \text{velocity} = \text{mass} \times \frac{\text{displacement}}{\text{time}}$$

Here, the two derived quantities were obtained from different fundamental quantities simply by dividing or multiplying them.



1.3 Measurement and its need

In the past, especially at the time of Aristotle, the observations on natural science were subjective and the observations varied from one to another because they depended on one's senses. This made communication among physicists difficult. For the description of a physical quantity, it should have the system of measurement as per **Lord Kelvin (1824 – 1907)**, any measurement should be expressed in numerical value for a complete sense. Hence otherwise it has no place in physics. An unknown quantity if compared to a known reference value is called **measurement in physics**. The known reference value is called the unit.

Why do we need measurement?

- i) To test the concepts in our hypothesis, we need measurement. The measured concepts finally become the variables.

- ii) To understand the phenomenon in physics, one has to perform the experiments. The experiments require measurement and one measures several physical properties like length, mass, time, temperature, etc
- iii) For the experimental verification of laws and theories, one needs the measurement of physical properties.

Concept of measurement:

Assigning the numerical values to the physical quantities by comparing them with their respective units is measurement.

1.4 Unit of a Physical Quantity

As mentioned earlier, a physical quantity should have a system of measurement; it should be measured and expressed in numbers. The standard of measurement of reference is the unit.

Conventions in Writing a Unit:

- i) For the unit that is named after a scientist, write the first letter as small letter instead of a capital letter.

For example, we write newton but not Newton. Similarly, it is written joule but not Joule.

- ii) If the unit named after a person is to be written as a symbol, we use a capital letter whereas the symbols for other units not fromed from a person's name aren't written in capital letters. For example, N for newton (not n) and m for meter (not M).
- iii) The symbols of units are never expressed in the plural form. We use 5 erg (and not 5 ergs), 5 kilo (and not 5 kilos), 10 m (but not 10 ms).
- iv) For the symbols of the derived unit, multiplication is joined with a centre dot or a non-brake space as N.m or Nm
- v) For the units of derived quantities, the symbols obtained after division are joined using solidus or as a negative exponent. For example, metre per second is written as either ms⁻¹ or ms⁻¹. The solidus is used only as one i.e., kg/ms⁻² and not as kg/ms-1⁻².

However, kgm⁻¹s² can also be written in place of kg/ms⁻².
- vi) Symbols must be treated as mathematical entities but not as abbreviations, and they do not have an appended period and full stop.
- vii) Full stops aren't written after the abbreviated units. For example, 50cc 50 ml, but not 50 c.c and 50 m. l.

- viii) Names of units are pluralized i.e., henries and not henry. Lux, hertz, and siemens are the exceptional cases.

Since a prefix is a part of unit, it is prepended to the unit symbol without a separator.

Examples: G in GHz, M in MPa, and k in km.

- ix) Units are of two types; *Fundamental unit and derived unit*.

x) 1.4.1 Fundamental and Derived Units

The units involved in the measurement of fundamental physical quantities are the fundamental units.

The units involved in the measurements of derived physical quantities are the derived units.

The derived units are brought from the fundamental units by simply multiplying or dividing two or more fundamental units.

1.4.2 System of Units

At different times in history, and in different parts of the world, people used different sets of units for measurement. The commonly used systems of units are MKS, CGS and FPS. The internationally accepted system of units used in science and engineering is called the SI unit.

MKS system:

In this system of measurement, length is measured in metre, mass is measured in kilogram, and time is measured in second. It was developed in France.

CGS System:

In it, length is measured in centimeter, mass in gram and the time is measured in second. This system was also developed in France.

FPS System:

In it, length is measured in foot, mass in pound and the time is measured in second. It was developed in Britain and is known as British system of units.

SI Units:

In 1960, to bring uniformity in measurement in the worldwide sense, the convention of scientists held in Paris introduced units based on scientific facts, and is now accepted internationally. This system of units is called **SI unit** which stands for the **Système international d'unités** in the French language.

The seven fundamental quantities and the two supplementary (sub-fundamental) quantities with SI units and symbols have been shown in the table below:

Table 1.1 Fundamental and Supplementary Quantities with Units

Fundamental Quantities	Fundamental Units (SI)	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Current	ampere	A
Luminous Intensity	candela	cd
Amount of Substance	mole	mol
Supplementary (sub-fundamental) Physical Quantity	Supplementary (sub-fundamental) Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

SI Base Units

Metre: The metre (m) has been internationally defined as the length of the path that light travels in a vacuum during a time interval of $1/299\,792\,458$ of a second. It is first introduced as a unit of length in the metric system. And $1\text{ m} = 3.2\text{ feet}$.

Second: The second (s) is the time that elapses during 9,192,631,770 cycles of the radiation produced by the transition between two hyperfine levels of the ground state of cesium-133 atom.

Kilogram: The kilogram(kg) is the mass equal to the mass of a standard Platinum – Iridium alloy cylinder with 90% platinum and 10% iridium) kept at the International Bureau of Weights and Measures in Paris. It is 4cm high and 4cm in diameter.

Concept to know:***Distinction in mN, nm, and Nm:***

mN refers to milli newton ($1\text{ mN} = 10^{-3}\text{N}$), nm represents nano meter ($1\text{ nm} = 10^{-9}\text{m}$), and Nm represents newton metre which is the unit of work or energy.

1.5 Precision and Significant Figures

Activity 1.1

- Stand in front of a dart board and hit the centre of the board with a dart.
- Mark the point the dart hits on the board and repeat the process for several times.

One of the following may be the observation you made.

- The marked points are neither near the centre nor consistent at anywhere.
- The marked points are close to each other (consistent) but are regardless of proximity to the centre.
- The marked points of the darts are all closely near the vicinity (or proximity) of the centre.
- All the marked points are near the centre and are consistent.

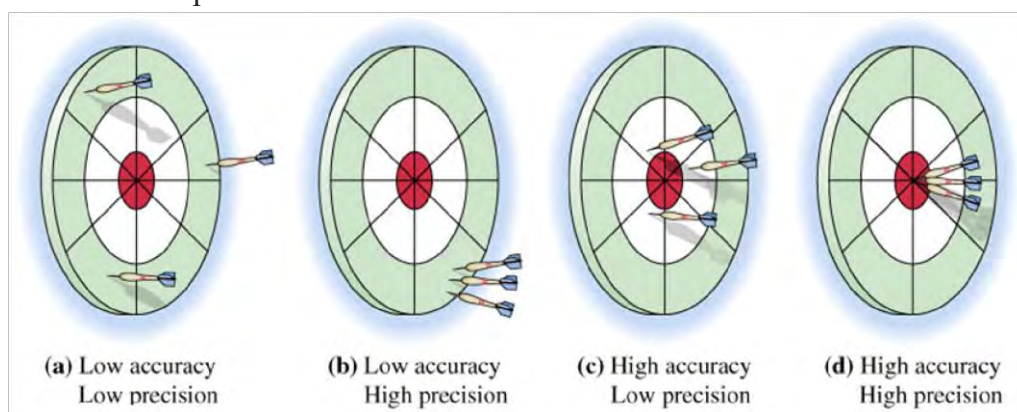


Fig 1.5 (i) Concepts on Accuracy and Precise Measurement

What are the conclusions from the different observations made above?

Accuracy refers to how close the measurements or observations come to lie at the center of the targeted point.

Precision refers to how consistent our results of observations are, regardless of closeness to the centre.

Hence accuracy is the measurement of the observations near the targeted (or true) value while precision is getting consistent observations. In other words, if the observations are made near the true value, it gives accuracy. But, the observations consistent to each other irrespective of the true value give precision. The precision and accuracy can better be understood with the support of the following

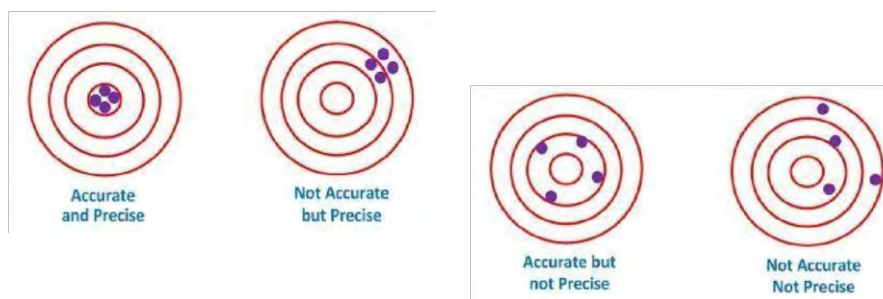


Fig 1.5(ii) Concepts on Accuracy and Precise Measurement

In the first fig in 1.5, the observation points are near the true value and consistent, hence it is the case of *accuracy and precision*.

In the second fig in 1.5, there is a consistent result of the observation regardless of the true target and it is the case of precision.

In the third fig in 1.5, since the measurements are near the true value and inconsistent, it is the case of accuracy but not precision.

In the fourth fig in 1.5, neither the data are around the true value nor they are consistent, it is the case of no accuracy and no precision.

A similar activity can be done on a given load. Suppose the loads are said to be 10 kg backpacks. If several measurements are made near 10kg, such as 9.8kg, 10.2 kg, 10.4 kg; the case is of accuracy. But if the measurements made are consistent (or close) regardless of the true value such as 12.5kg, 12.6 kg, and 12.4kg: then it is the case of precision.

A Better Concept to Know:

To understand the real-life example of accuracy and precision, measure the wt. of a 10 kg mass in the lab and find 3.4 kg, it is not the accurate value of the measurement. If several other measurements were 3.5 kg, 3.3 kg 3.4 kg and so on, these all represent the precise measurement. You precisely measure the data but not accurately. However, if the measured data are at around 10kg as 10.2 kg, 10.1 kg, 9.9 kg, the data all are precise and accurate too.

1.5.1 Significant Figures:

Significant measure is an art of measuring the physical quantity more accurately and precisely and hence is important in scientific and technical measures. The larger the number of significant figures, the greater is the degree of accuracy. The number of significant digits increases if we measure more precisely with smaller range of uncertainty. The measurement will have some digits that are certain and one digit is uncertain (or guessed). In understanding significant figures in measurements, it is important to realize that the precision of a measured value cannot be more precise than the equipment used for measurement.

Let us take the case that we are measuring the diameter of a ball that is found as 2.64cm. Here, the digits 2 and 6 are reliable and certain while the last digit '4' is uncertain. Hence the diameter of the ball has a total 3 significant figures.

Let the measurement for the length of a rod be 3.5 cm. It has a total of 2 significant figures with 3 as reliable (certain) and the last digit 5 as uncertain. If we find the length as 3.56cm with some other device, it will be more precise and of more significant figures than 3.5cm. If the length of the same rod is measured as 3.567 cm using a micrometer screw gauge, it is more accurate than the former two.

Here are some rules to find the number of significant figures:

i) All the non –zero digits in a given measurement are significant.

i.e. 21.48 has total 4 significant figures

ii) All the zeros between two non-zero digits are significant but leading zeros are not significant.

i.e. 15.0041 → 6 significant figures.

2001 → 4 significant figures.

0015 → 2 significant figures

iii) The zeros to the right of the last non-zero digit are not significant.

i.e. 9800 → 2 significant figures.

i.e. 050600 → 3 significant figures.

iv) The zeros to the right of the decimal point and before (to the left of) the first non-zero digits are not significant. They only represent the position of the decimal point.

i.e. 0.006 → 1 significant figure.

0.00025 → 2 significant figures.

0.0080038 → 5 significant figures.

0.25 → 2 significant figures.

v) The zeros to the right of the decimal number are significant only when there is a non-zero digit on its left

5.00 → 3 significant figures

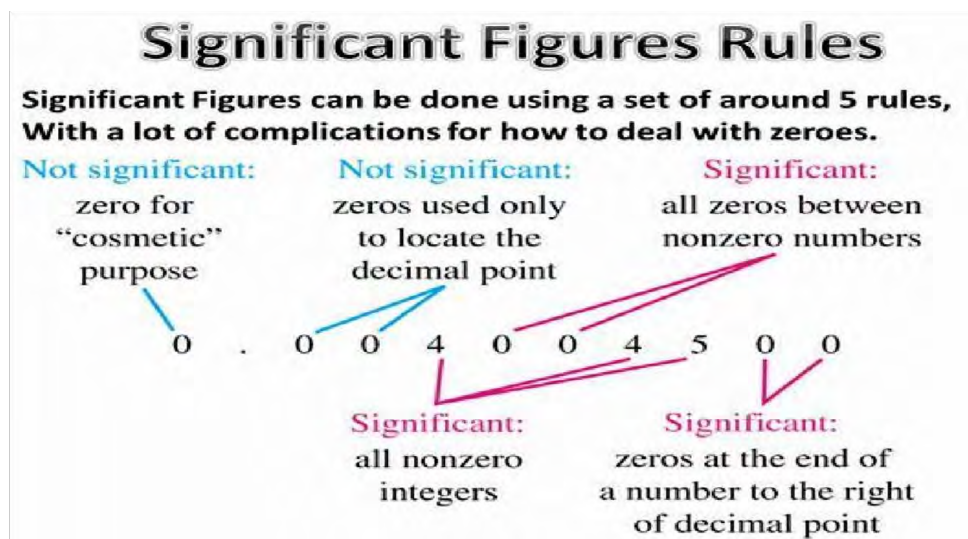
5.0 → 2 significant figures

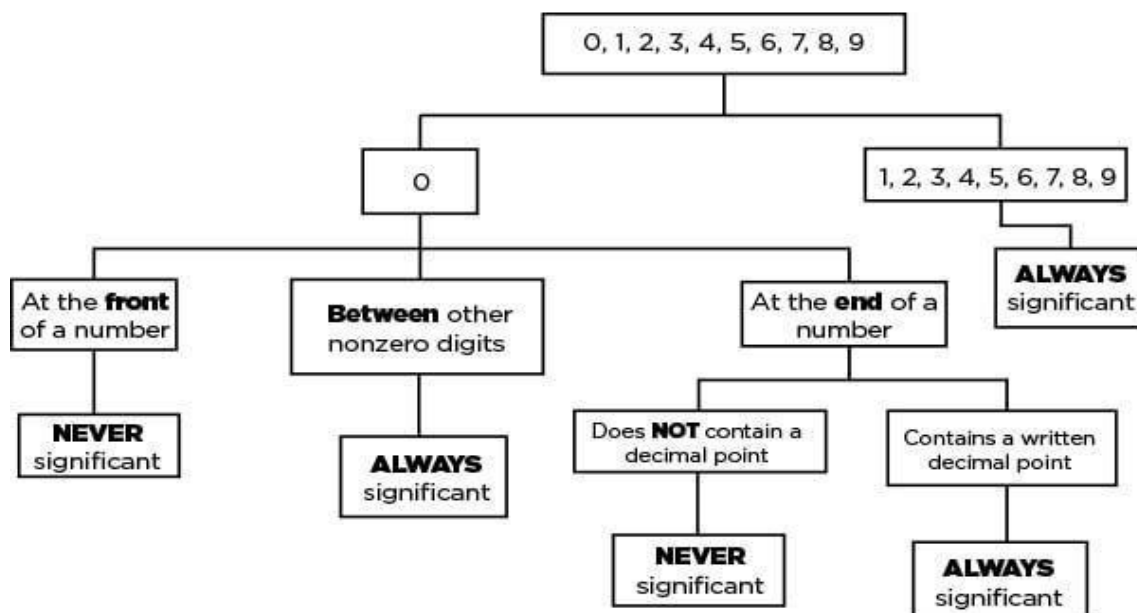
0.200 → 3 significant figures

vi) In a measurement expressed in scientific notation, such as $N \times 10^A$, the 10^A does not follow the rule of significant figures.

$1.6 \times 10^{-19} \text{ C}$ → 2 significant figures

$6.62 \times 10^{-34} \text{ Js}$ → 3 significant figures





1.5.2 Round off Significant Numbers

The rounding off of the significant figures is to omit one or more digits from the right side of the number until we attain the number of significant figures we want to round it off to. For the roundoff, look at the digit on the right end of the number.

The rules for rounding off are as follows

- If the digit to be dropped out is less than 5, the preceding digit remains unchanged on rounding off. e.g. $123.42 \approx 123.4$ (in 4 significant numbers)
 ≈ 123 (in 3 significant number)
- If the digit to be dropped out is greater than 5, the preceding digit is increased by 1 in rounding off. e.g. $123.167 \approx 123.17$ (in 5 significant numbers)
 ≈ 124.2 (in 4 significant numbers)
- If the digit to be dropped out is exactly 5, the preceding digit to be retained is
 - Increased by 1 if it is odd. eg. $12.35 \approx 12.4$ (in 3 significant numbers)
 - kept unchanged if it is even. eg. $12.45 \approx 12.4$ (in 3 significant

1.5.3 Significant Figures in mathematical operation

a) In addition, and subtraction:

When two measurements are added or subtracted one from the other, the answer should have the lowest decimal place as that of the input.

e.g. $3.240 \text{ m} + 7.14 \text{ m} = 10.380 \text{ m}$ (Incorrect)
 $= 10.38 \text{ m}$ (Correct)

And $3.266 \text{ m} - 2.1 \text{ m} = 1.166 \text{ m}$ (Incorrect)
 $= 1.2 \text{ m}$ (Correct)

b) In Multiplication and Division:

When two measurements are multiplied or divided one by other, the answer should have the lowest significant figures as that of the input.

e.g. $118.6 \times 16.2 = 1921.32$ (Incorrect)
 $= 1920$ or 1.92×10^3 (Correct, in 3 significant figures)

$118.6 \div 16.2 = 7.3209876543$ (Incorrect)
 $= 7.32$ (Correct, in 3 significant figures)

$125 \div 3 = 41.6666\dots$ (Incorrect)
 $= 40$ (Correct, in 1 significant figure)

Worked Out Example:

Q: Find the number of a significant figure in 0.005.

Ans: One significant figure

Sample Question:

1. Which of the following measurements is accurate?

- a) 5 kg (b) 5.0 kg (c) 5.00 kg (d) 5.000 kg

Ans: (d)

1.6 Dimensions of a Physical Quantity

The concept of dimension of physical quantities was introduced in 1822 by **Joseph Fourier**.

The dimensions of a physical quantity are the powers that are raised to the fundamental quantities involved in a physical quantity.

In mechanics; mass, length, and time are the three fundamental quantities represented respectively as [M], [L], and [T] in dimensional purpose. Temperature is represented by [K], electric current is denoted by [A] or [I], and the amount of substance by [N] or [mol]. Likewise, the dimensional notation for the luminous intensity is [J] or [cd]. Hence in a dimensional analysis, the basic quantities with their symbols in a square bracketed form for the dimensional analysis are:

Mass = [M]

Length = [L]

Time = [T]

Temperature = [K]

Electric current = [A] or [I]

Amount of substance = [N] or [mol]

And Luminous Intensity = [J] or [cd]

Unless mentioned otherwise, the dimensions of a physical quantity X are written as

$$X = [M^a L^b T^c] \quad \dots\dots\dots (i)$$

Here $[M^a L^b T^c]$ is the **dimensional formula** for X and a, b and c are the dimensions (power) in mass, length, and time respectively.

The equation (i) is the **dimensional equation** for the quantity.

The square brackets here are read as “dimension of”. One can use either small brackets or without the brackets at all, as in MLT^{-2} . However, it is commonly presented within square brackets.

Examples:

Let velocity be a physical quantity and velocity = displacement/ time

$$= \frac{[L]}{[T]} = [LT^{-1}] = [M^0 L T^{-1}]$$

Here 0, 1 and -1 are the dimensions in M, L and T while $[M^0 L T^{-1}]$ is the dimensional formula for the *velocity*, and $v = [M^0 L T^{-1}]$ is the dimensional equation for the velocity.

Hence, to determine the dimensional formula for a physical quantity, we must first establish its mathematical relation or formulation. By assigning dimensions to each quantity involved, the dimensional formula is completed.

Followings are the dimensional formula of some physical quantities:

S.N	Physical Quantity	Mathematical formulation	Dimensional Formula	SI unit
1.	Area	Area= length x breadth	$[L] \times [L] = [L^2]$	m^2
2.	Volume	Length x breadth x height	$[L] \times [L] \times [L] = [L^3]$	m^3
3.	Density	Mass/volume	$[M]/[L^3] = [ML^{-3}]$	kg/m^3
4.	Specific gravity	Density of a substance/Density of water.	$[M^0 L^0 T^0]$	No unit
5.	Frequency	cycles/time	$[T^{-1}]$	Hz
6.	Angle	Arc/radius	$[M^0 L^0 T^0]$	No unit
7.	Speed	distance/time	$[L T^{-1}]$	ms^{-1}
8.	Velocity	displacement/time	$[L T^{-1}]$	ms^{-1}
9.	Acceleration	Velocity/time	$[L T^{-2}]$	ms^{-2}
10.	Linear momentum	Mass×velocity ($p = mv$)	$[M L T^{-1}]$	$kg \ ms^{-1}$
11.	Force	Mass× acceleration ($F = ma$)	$[MLT^{-2}]$	N
12.	Impulse	Force×time	$[MLT^{-1}]$	$kg \ ms^{-1}$ or Ns
13.	Weight	$W = mg$	$[MLT^{-2}]$	N
14.	Work or energy	Force. displacement	$[ML^2T^{-2}]$	Nm or J
15.	Power	Work/time	$[ML^2T^{-3}]$	W or Js^{-1}
16.	Pressure	Force/area	$[ML^{-1}T^{-2}]$	Nm^{-2} or Pa
17.	Angular Velocity	$\omega = \theta/t$	$[T^{-1}]$	$rad \ s^{-1}$
18.	Angular acceleration	$\alpha = \omega/t$	$[T^{-2}]$	$rad \ s^{-2}$
19.	Gravitational Constant	$G = \frac{Fr^2}{Mm}$	$[M^{-1}L^3T^{-2}]$	$kg^{-1}m^3s^{-2}$
20.	Gravitational Field Strength	$\frac{\text{Force}}{\text{Mass}}$	$[L T^{-2}]$	Nkg^{-1}
21.	Gravitational Potential	$\frac{\text{Work}}{\text{Mass}}$	$[L^2 T^{-2}]$	W or Js^{-1}

22.	Stress	$\frac{\text{Force}}{\text{Area}}$	$[M L^{-1} T^{-2}]$	Nm^{-2} or Pa
23.	Strain	$\frac{\text{Change in length}}{\text{Original length}}$	No dimensional formula	No unit
24.	Modulus of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$[M L^{-1} T^{-2}]$	Nm^{-2} or Pa
25.	Surface Tension	$\frac{\text{Force}}{\text{Length}}$	$[ML^0 T^{-2}]$	Nm^{-1}
26	Velocity gradient	$\frac{dv}{dx}$	$[T^{-1}]$	s^{-1}
27.	Coefficient of viscosity (η)	$\frac{\text{Force}}{\text{Area} \times \text{velocity gradient}}$	$[ML^{-1} T^{-1}]$	$kg m^{-1} s^{-1}$ or Nsm^{-2}
28.	Torque or Couple or moment of force	Force. distance	$[ML^2 T^{-2}]$	$Kg m^2 s^{-2}$ or Nm
29.	Coeff. Of linear expansion (α)	$\frac{\text{Change in length}}{\text{Original length} \times \text{change in temperature}}$	$[K^{-1}]$	K^{-1}
30.	Specific heat capacity (s)	$\frac{Q}{m \times \Delta \theta}$	$[L^2 T^{-2} K^{-1}]$	$J kg^{-1} K^{-1}$
31.	Latent heat (L)	$\frac{Q}{m}$	$[L^2 T^{-2}]$	$J kg^{-1}$
32.	Coeff. Of thermal conductivity	$K = \frac{Q \times dx}{t \times A \times d\theta}$	$[M L T^{-3} K^{-1}]$	$Wm^{-1} K^{-1}$
33.	Stefan's constant(σ)	$\frac{Q}{T^4}$	$[M T^{-2} K^{-4}]$	$W m^{-2} K^{-4}$
34.	Gas constant	$R = \frac{PV}{nT}$	$[L^2 T^{-2} K^{-1} N^{-1}]$	$Jmol^{-1} K^{-1}$
35.	Electric charge	$Q = I \times t$	$[T^1 I^1]$	As or C

- i) Sometimes the dimensional formula for a physical quantity is expressed in the actual system of units. Such units are the base units in SI measurement.
- ii) For the concepts of dimensional analysis, the units in the dimensional analysis help in formulating the physical equations with their valid identity. Further, the units help to correct the basic errors in the equation.

Worked Out Examples

Q: Find the dimensional formula of power.

Solution: Power is the work done per unit time. Hence, Power (P) = Work/ time = (Force $\times$ displacement)/time

$$\text{Also, } [F] = [MLT^{-2}]$$

$$[d] = [L]$$

$$[t] = [T]$$

$$\text{Thus, } [P] = [MLT^{-2}] [L] / [T] = [ML^2T^{-3}]$$

Q: Taking force, length, and time as fundamental quantities, find the dimensional formula for energy.

Solution: Here, Energy = Work = Force $\times$ Displacement

$$[\text{Force}] = [F]$$

$$[\text{Displacement}] = [L]$$

$$\text{Then, } [\text{Energy}] = [F] [L] = [F L]$$

$$\therefore [E] = [F L T^0]$$

Sample Question:

1. The power is expressed in a new system of units with fundamental quantities of force, length, and time. Find the dimensional formula for the power. (Ans: $[F L T^{-1}]$)

Principle of Homogeneity:

In a valid physical equation, the dimensions of each term on one side must be consistent and identical to the dimensions of each corresponding term on the other side. This principle is based on the fact that only the physical quantities of the same kind can be added, subtracted, or compared. According to this, mass can be added to or subtracted from mass but not to length. Similarly, force can be added to or subtracted from force but not to energy.

For example, in the equation $v = u + at$, dimensional formula of v , u and at are same, and hence the equation follows the principle of homogeneity.

Types of Physical Quantity

Based on dimension, physical quantities are of the following types:

- Dimensional variables:** These are physical quantities that do not have a fixed value but have dimensions. Examples are work, power, pressure, velocity, etc.
- Dimensionless variables:** These are physical quantities that possess neither a fixed value nor dimensions. Examples are strain, specific gravity, etc. Such quantities have no units.
- Dimensional Constants:** These are the physical quantities that have dimensions and a fixed value. Examples are Gravitational constant, Planck's constant, etc.
- Dimensionless constants:** These are physical quantities that have no dimension but a fixed value. Examples are Coefficient of friction, pi (π), etc.

1.6.1 Application of Dimensional Analysis:

Following are the applications of dimensional analysis:

- To check the correctness of a physical relation,
- To derive a relation between various physical quantities,
- To convert one system of physical quantity to another system of physical quantity
- To find the dimensions of an unknown physical quantity in a given relation.

Let us discuss some of the applications

To check the correctness of a physical relation:

Let $s = ut + \frac{1}{2} at^2$ be a physical relation.

To test the correctness of this equation whether it is correct or not, let us put the dimensional formula of each physical quantity in the relation

$$[L] = [LT^{-1}] [T] + [LT^{-2}] [T^2],$$

Here $\frac{1}{2}$ is the dimensionless constant and it is needless to write.

$$\text{Or, } [L] = [L] + [L] = [L]$$

Hence, by the principle of homogeneity of dimension, both sides have the same dimension, and thus the equation is dimensionally correct.

If the above relation is written as $s = ut + at^2$ and on testing, dimensionally it is found to be correct as above but the relation is physically wrong as the multiplier $\frac{1}{2}$ in the second term is missing.

Also, take the physical equation as $s = ut^2 + \frac{1}{2} at^2$. On testing the equation dimensionally, it is found incorrect, and it is also incorrect physically.

From above it is strongly concluded that a dimensionally correct equation may not be physically correct. But it is sure that a dimensionally wrong equation is physically wrong.

Worked Out Examples:

Q: A student writes the escape velocity as $v = \sqrt{2 \frac{GM}{R}}$. Test the correctness of the equation.

Here, using the dimensional formula for the concerned physical quantities in the relation, $[M^0$

$$L T^{-1}] = \sqrt{[M^1 L^3 T^{-2}][\frac{M}{L}]} = [M^0 L T^{-1}]$$

Hence the student has written the correct formula.

Q: A student writes in a dimensional sense as $2[L] + [L] = [L]$ and $[L] - [L] = [L]$. Justify the expression.

In the first expression, some length is added to the other length so that the result is again some length $[L]$.

In the second expression, some length is subtracted from the other length so the result is again length.

More precisely, two similar quantities of the same dimension can be added to the result of the quantity of the same dimension and taking the difference between two similar quantities of the same dimension also results in the third quantity of the same dimension.

Sample Question:

1. Test the correctness of the physical equation $v^2 = u^2 + as^2$

To derive the relation between various physical quantities:

Let us derive the relation for the period (T) of a simple pendulum. Consider that the period T of pendulum depends on its mass m , length l , and acceleration due to the gravity g .

That is, $T \propto m^a l^b g^c$

Or, $T = k m^a l^b g^c$

Now, using the dimensional formula for the concerned quantities,

$$[M^0 L^0 T^1] = k [M^a L^b [L T^{-2}]^c] = k [M^a L^{b+c} T^{-2c}]$$

Using the principle of homogeneity, we get

$$a = 0, b = 1/2 \text{ and } c = -1/2$$

The value of k is experimentally observed to be 2π .

Recalling these values in the above equation,

$$T = k \sqrt{\frac{l}{g}}$$

The value of k is experimentally observed to be 2π .

$$\therefore T = 2\pi \sqrt{\frac{l}{g}}$$

Hence the physical relation has been established.

Sample Question:

1. The frequency of vibration of a stretched string is a function of tension T on it, its length L and its mass per unit length, μ . Using the knowledge of dimensional analysis establish the relation of the frequency.

To find the dimension of an unknown physical quantity:

The velocity of a particle is written as $v = A + Bt + Ct^2$. If t is time, then find the dimensional formula for A , B , and C .

In the given relation, according to principle of homogeneity, the dimensional formula of A , Bt , and Ct^2 should be consistent with that of v .

Thus, $[v] = [LT^{-1}] = [A] = [Bt] = [Ct^2]$

Then, $[A] = [LT^{-1}]$

$$[B] = \frac{[LT^{-1}]}{[T]} = [LT^{-2}]$$

$$[C] = \frac{[LT^{-1}]}{[T^2]} = [LT^{-3}]$$

Sample Question:

1. In the relation $y = a \cos(\omega t - kx)$, find the dimensional formula for k , where t is time and x is position. (Ans: $[L^{-1}]$)

1.6.2 Limitation of Dimensional Analysis

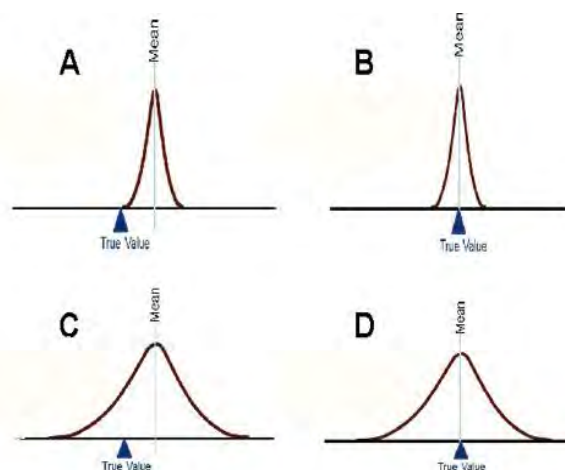
Following are some of the limitations of the dimensional analysis.

- i) It does not provide information on whether a physical quantity is a scalar or a vector.
- ii) It cannot solve the problem related to trigonometric functions, exponential and logarithm functions.
- iii) It does not give the information on dimensionless constant.

Exercises

- Which of the following is dimensionally incorrect?
a. $s = ut + at^2$ b. $s = ut - \frac{1}{2}at^2$ c. $s = ut + 5at^2$ d. $s = ut + \frac{1}{2}at$
- Which of the following is physically correct?
a. $v = u + 2at$ b. $v = u - \frac{1}{2}at$ c. $v = u + \frac{1}{2}at$ d. $v = at + u$
- Which of the following is the best option for a physical quantity in any two systems of units?
a. They have the same unit but different dimensions.
b. They have the same dimensions but different units.
c. They have the same units and the same dimensions
d. They have different units and dimensions.
- Which of the following physical quantities is dimensionless?
a. Gravitational constant b. Strain
c. Universal Gas constant d. Stefan's constant.
- The velocity of a particle is given in the equation $v = \frac{b}{t} + ct^2$. What is the dimensional formula for b in the above equation?
a. [L] b. [M L] c. [T] d. [M L T⁻¹]
- If force [F], length [L], and time [T] are considered as fundamental units, what is the dimensional formula for momentum?
a. [FT⁻¹], b. [FT²] c. [FT] d. [FT⁻²]
- What are the dimensions of a light year?
a. [T] b. [LT⁻¹] c. [L] d. [T⁻¹]
- The sides of a rectangle are (0.5 ± 0.01) m and (0.2 ± 0.01) . Find its perimeter with its uncertainty.
a. (1.4 ± 0.02) m b. (1.4 ± 0.01) m c. (0.1 ± 0.01) m d. (0.1 ± 0.02) m
- Which of the following numbers has the least number of significant figures?
a. 0.70860 b. 0.50300 c. 0.07086 d. 60.5370
- What is the number of significant figures in the measurement 0.0050×10^{-12} ?
a. 1 b. 2 c. 4 d. 5

11. The figure below represents the nature of distribution of data of a measurement. Which one represents both accurate and precise measurement?



12. (a) Define dimensional formula of a physical quantity.

(b) Taking force, length, and time to be fundamental quantities, find the dimensional formula of energy. (Ans: $[F L T^0]$)

(c) The rate of flow of liquid through a horizontal pipe of length l is given by $\frac{kPa^x}{\eta l}$, where k is a

dimensionless constant, P is the pressure, a is the radius of the pipe and η is the coefficient of viscosity ($[ML^{-1} T^{-1}]$). Using the dimensional method, find the value of x . (Ans: 4)

13. (a) The frequency f of vibration of a string depends on the length of the string L , tension produced on it T , and mass per unit length of the string m . Deduce the relation

$$f = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

(b) A 5N force is expressed as $5N = a + bt + ct^2 + dt^3$. Find the dimensional formula for a , b , c and d .

(Ans: $a \rightarrow [MLT^{-2}]$, $b \rightarrow [MLT^{-3}]$, $c \rightarrow [MLT^{-4}]$, $d \rightarrow [MLT^{-4}]$)

(c) Using the relation for the frequency above in 1 (a), find the dimensional formula for the frequency.

14. (a) How do you differentiate between precision and accuracy? How is precision related to a significant figure?

(b) Which of the following measurements represents the most accurate and why?
i. 2 cm, ii. 2.00 cm, iii. 2.000 cm, iv. 2.0000 cm

(Ans: iv)

15. The velocity of a particle is given as $v = a + bt + ct^2$, $t \rightarrow$ time. Find the dimension of a , b and c if v is in ms^{-1} and t is in s . (Ans: LT^{-1} , LT^{-2} , LT^{-3})

16. A body moving in air at a high speed v experiences a retarding force F given by $F = k A \rho v^x$, where $A \rightarrow$ surface area of the body, ρ is density of the body, and k is the dimensionless constant. Deduce the value of x . (Ans: $x = 2$)

17. The velocity of a particle $v = a + bt + c/d + t$. Find the dimensions of a , b , c and d .
(Ans: LT^{-1} , LT^{-2} , $[L]$ and $[T]$)

18. Find the value of 10 joule on a system which has 10 cm 100 g and 30 s as the fundamental units. (Ans: 9×10^6 new units)

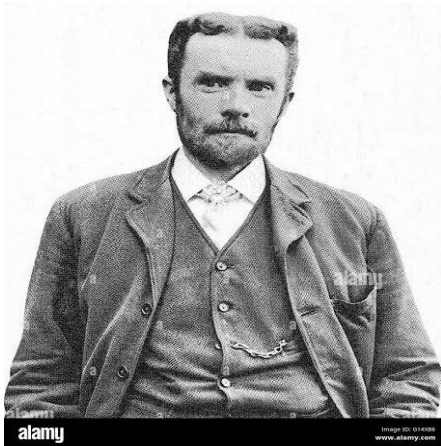
19. The length and the breadth of a field measure $l = (110 \pm 2)$ m and $b = 100 \pm 4$ m respectively. What is the area of the field? (Ans: $(1.1 \pm 0.06) \times 10^4 m^2$)

20. Convert (i) 72 km/h into ms^{-1} and 20 g/cm³ into kg/m³ (Ans: 20 ms^{-1} and $2.0 \times 10^4 kg m^{-3}$)

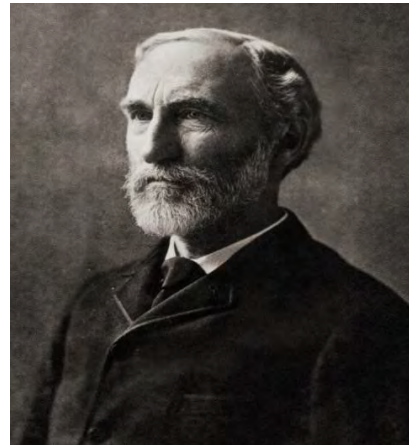
21. The frequency of vibration of a stretched string is a function of its tension T , the length L , and the mass per unit length m . From the knowledge of dimension, prove that $f \propto \frac{1}{L} \sqrt{\frac{T}{m}}$

Chapter 2

Vectors



J. Willard Gibbs (1839-1903)



Oliver Heaviside (1850-19250)

Vector calculus and its sub-objective Vector Fields was invented by two men J. Willard Gibbs and Oliver Heaviside at the end of the 19th century. This allowed scientists and mathematicians to calculate such things as speed and direction from a graph.

J. Willard Gibbs (February 11, 1839 – April 28, 1903) was an American mathematician and physicist who became an inventor of vector analysis. Josiah Willard Gibbs, born in New Haven, Connecticut, USA, graduated from Yale (College) in 1858 and continued his career at Yale, earning his doctorate of philosophy in 1863.

Oliver Heaviside FRS (18 May 1850 – 3 February 1925) was an English self-taught mathematician and physicist who invented a new technique for solving differential equations to the Laplace transform, independently developed **vector calculus**, and rewrote Maxwell's equations.

2.1 Introduction

Vector, in physics, is a quantity that has both magnitude and direction. It is typically represented by an arrow whose direction is the same as that of the quantity, and whose length is proportional to the quantity's magnitude. Although a vector has magnitude and direction, it does not have position.

Consider the Phenomena:

- i) Pokhara lies at a distance of 200 km from Kathmandu. Can a pilot safely land at Pokhara after flying 200km just in any direction from Kathmandu?
- ii) A screw is rotated in a clockwise direction. What will be its possible direction of advancement? Do you anticipate any sense from it?
- iii) Imagine your mother asking you to go to a particular shop to get something, will it be enough for her to say go 200 m from the main gate? Is this information enough for you to locate that shop?

You do realize that it isn't enough, right? You need proper directions, these directions are themselves vectors and that's why you need vectors.

Physical quantities are classified into two types: scalars and vectors. Scalars, also known as scalar quantities, are characterized solely by their magnitude. On the other hand, vectors, or vector quantities, are fully defined by both magnitude and direction.

Mass, length, time, temperature, distance, speed, work, money in the pocket, etc. are scalars. On the other hand, displacement, velocity, force, etc. are all vectors.

A hundred-rupee defines only magnitude, and thus it is a scalar quantity. The distance of a place (such as Pokhara lying 200 km from Kathmandu) requires direction for anyone to reach, making it a vector. The rotation of a screw determines its forward or backward movement, which involves direction, thus making it a vector quantity.

Moreover, although electric current has both magnitude and direction, it isn't regarded as a vector quantity. To qualify as a vector, a physical quantity must possess both magnitude and direction and adhere to the principles of vector algebra. *Hence the electric current, which does not adhere to the principles of vector algebra, is not considered a vector quantity despite possessing both magnitude and direction.*

Vector representation

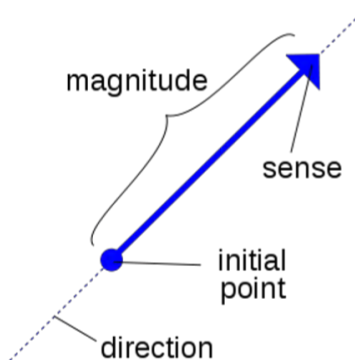


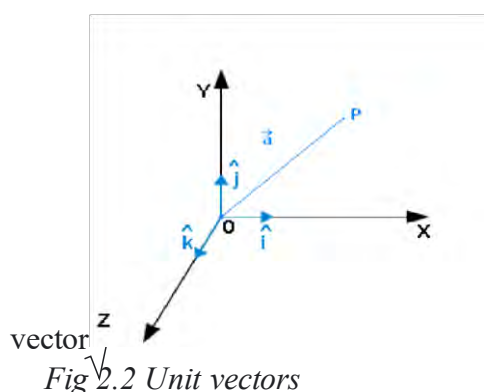
Fig 2.1 Graphical representation of vectors

Graphically, a vector is represented by a straight line with an arrow attached to it. The length of the line indicates the magnitude and the arrow indicates the direction. The starting point of the arrow is the initial point and the ending point is called the terminal point of a vector.

Analytically, a vector is represented as $\vec{A}$ or by a bold-faced letter **A**

2.2 Types of a Vector

i) Unit Vectors: These are the vectors having unity magnitude. For a vector $\vec{A}$, if $A = 1$, it is the case of a unit vector. A unit vector is generally denoted by a symbol $\hat{n}$ and $\hat{n} = \vec{A}/A$. A position vector in the space, $\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \equiv (x, y, z)$, the magnitude of the unit



$$OP = \sqrt{x^2 + y^2 + z^2}$$

Hence, the unit vector along $\vec{OP}$ is $\frac{\vec{OP}}{OP} = \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{\sqrt{x^2 + y^2 + z^2}} \hat{r}$

Similarly; $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the three rectangular coordinates X, Y and Z axes respectively.

Sample Questions:

Q: Find the unit vector for $2\hat{i} + 3\hat{j} - 4\hat{k}$.

(Ans: $2/\sqrt{29}\hat{i} + 3/\sqrt{29}\hat{j} - 4/\sqrt{29}\hat{k}$.)

ii) Parallel Vectors: Two vectors are said to be parallel if they have the same

direction whatever be their magnitudes.

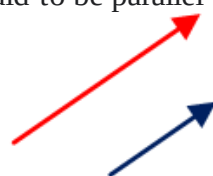


Fig 2.3 (i) parallel vectors

iii) **Equal Vectors:** Two vectors are said to be equal if they have same directions and

magnitude.

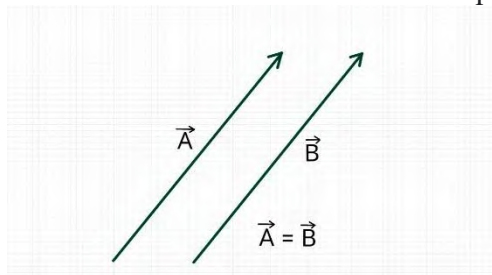
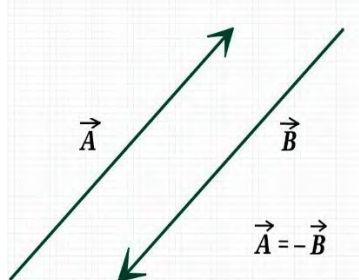


Fig 2.3 (ii) equal vectors.

iv) **Negative Vector:** A vector $\vec{B}$ is said to be the negative of a given vector $\vec{A}$ if it has the

same



magnitude but opposite direction and is written as $-\vec{A}$.

Fig 2.3(iii) Negative vectors

v) **Collinear Vectors:** Vectors lying along the same line or parallel lines are called the



Fig2.3 (iv) Collinear Vectors

collinear vectors. Collinear vectors can be in the same or opposite direction.

Two vectors are said to be collinear if they are parallel to the same line irrespective of their magnitude and direction.

vi) **Coplanar Vectors:** Vectors lying on the same plane are called coplanar vectors.

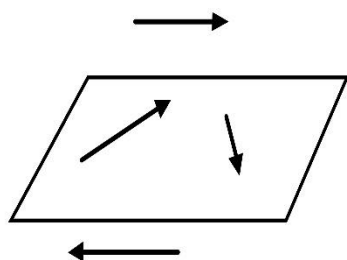


Fig 2.3(v) Coplanar Vectors

vii) **Null (or zero) vector:** A vector is said to be a null vector if its magnitude equals zero i.e, $A = 0$ for a null vector.

1.3 Addition of Vectors

The resultant vector, obtained after performing vector algebra on multiple given vectors, represents the single resultant of those vectors.

The process of finding such a vector is called the composition of a vector. Following are the processes:

2.3.1 Triangle Law of Vector Addition:

If two vectors can be represented by the two sides of a triangle both in magnitude and order, the third side of the triangle taken in opposite order represents the resultant.

Let $\vec{P}$ and $\vec{Q}$ be the two given vectors.

Let $\vec{AB} = \vec{P}$ and $\vec{BC} = \vec{Q}$ be the two given vectors represented by the two sides of a triangle in magnitude and direction.

Then $\vec{AC} = \vec{R}$ be the closing side of the triangle representing the resultant of the two vectors.

And, one writes, $\vec{R} = \vec{P} + \vec{Q}$

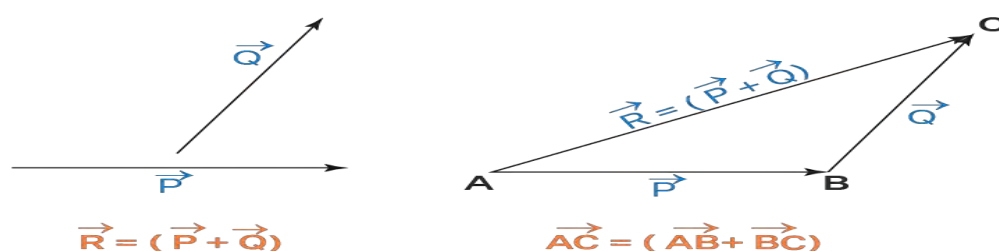


Fig 2.4 Triangle Law of Vectors

To find the magnitude and the direction of the resultant:

Let's draw CD normal on AB produced where θ is the angle between the vectors.

Therefore, $R^2 = AD^2 + CD^2$

$= (AB + BD)^2 + CD^2$

$= AB^2 + 2AB \cdot BD + BD^2 + CD^2$

Or, $R^2 = AB^2 + 2AB \cdot BC \cos \theta + BC^2$

$= P^2 + 2PQ \cos \theta + Q^2$, $\cos \theta = \frac{BD}{BC}$

$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$ is the magnitude of the resultant.

To find the direction of $\vec{R}$:

Let α be the angle of direction that $\vec{R}$ makes with $\vec{AB}$.

Fig 2.5 the resultant of the triangle law of vector addition

Therefore, $\tan \alpha = \frac{CD}{AD} = \frac{CD}{AB + BD}$

Or, $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$

$\therefore \alpha = \tan^{-1} \frac{Q \sin \theta}{P + Q \cos \theta}$

It gives the angle of direction for the resultant.

2.3.2 Parallelogram Law of Vector Addition

Statement: *If any two vectors can be represented by the two adjacent sides of a parallelogram in magnitude and order, then the diagonal of the parallelogram, formed by and passing through the vectors, represents the resultant.*

Proof: Let $\vec{OA} = \vec{P}$, $\vec{OB} = \vec{Q}$ be the two vectors represented by the two adjacent sides of a parallelogram OACB in magnitude and direction.

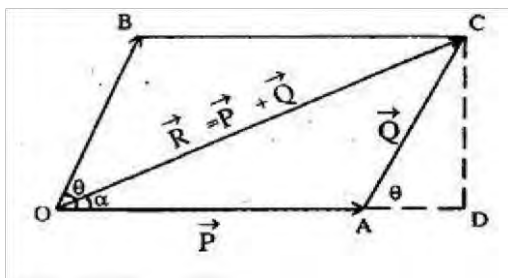


Fig2.6 Parallelogram law of vector addition

Then $\vec{OB} = \vec{R}$ will be the diagonal of the parallelogram representing the resultant of the vectors.

To find the magnitude of $\vec{R}$,

Draw BD normal on OA produced in such a way that OBDO is a rt. $\angle$ d triangle.

Now, we write here,

$$\begin{aligned}
 OC^2 &= OD^2 + CD^2 \\
 &= (OA + AD)^2 + CD^2 \\
 &= OA^2 + 2OA \cdot AD + AD^2 + CD^2 \\
 &= OA^2 + 2OA \cdot AC \cos \theta + AC^2 \quad \because \cos \theta = \frac{AD}{AC} \\
 &= P^2 + 2PQ \cos \theta + Q^2 \\
 \therefore R &= \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \text{ is the magnitude of the resultant.}
 \end{aligned}$$

To find the direction of $\vec{R}$:

Let α be the angle of direction that $\vec{R}$ makes with $\vec{OA}$.

$$\begin{aligned}
 \text{Therefore, } \tan \alpha &= \frac{CD}{OD} = \frac{CD}{OA + AD} \\
 &= \frac{AC \sin \theta}{OA + AC \cos \theta} \\
 &= \frac{Q \sin \theta}{P + Q \cos \theta}
 \end{aligned}$$

$$\text{Or, } \alpha = \tan^{-1} \frac{Q \sin \theta}{P + Q \cos \theta}$$

It gives the direction for the resultant.

Cases:

- i. When $\theta = 0^\circ$, $\vec{P}$ and $\vec{Q}$ act in the same direction.

$$\begin{aligned}
 \text{i.e, } R &= \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \\
 &= P + Q
 \end{aligned}$$

$$\text{And } \alpha = \tan^{-1} \frac{Q \sin \theta}{P + Q \cos \theta} = 0$$

Hence $\vec{R} = \vec{P} + \vec{Q}$ and $\vec{R}$ acts in the same direction as of P & Q.

- ii. When $\theta = 90^\circ$, $\vec{P}$ and $\vec{Q}$ act normally.

$$\begin{aligned}
 \text{Therefore, } R &= \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \\
 \Rightarrow R^2 &= P^2 + Q^2
 \end{aligned}$$

$$\text{And } \alpha = \tan^{-1} \frac{Q \sin \theta}{P + Q \cos \theta} = 45^\circ$$

- iii. When $\theta = 180^\circ$, on following the same way,

$$R = P - Q$$

$$\text{And } \alpha = \tan^{-1} \frac{Q \sin \theta}{P + Q \cos \theta} = 0$$

It gives the direction for the resultant 0.

- i. If two vectors are equal in magnitude and, the angle between them is 120° , then the resultant of the vectors equals the magnitude of either vector i.e. for $P=Q$ and $\theta = 120^\circ$, $R = P = Q$.
- ii. Similarly, if two vectors equal in magnitude have their resultant equal to the magnitude of either vector, then the vectors act at 120° i.e. for $P = Q = R$, we get $\theta = 120^\circ$.

Worked Out Examples:

Q: Two equal vectors in magnitude have their resultant equal to either vector. Find the angle between the vectors.

Solution:

Let $P = Q = R$ be the two equal vectors and again equal to their resultant.

Angle between the vectors, $\theta = ?$

We have,

$$R^2 = P^2 + Q^2 - 2PQ \cos \theta$$

$$\text{Or, } R^2 = R^2 + R^2 - 2R.R \cos \theta$$

$$\text{Or, } R^2 = 2R^2 (1 - \cos \theta)$$

$$\text{Or, } \cos \theta = -1/2 = \cos 120^\circ$$

$$\therefore \theta = 120^\circ$$

Q: The magnitude of two equal vectors are equal and the angle between them is θ . Show that the resultant of the vectors bisects the angle θ equally into two halves

Solution:

Here we have,

$P = Q$ and θ as the angle between them.

Let α be the angle that the resultant makes with one of them say P .

$$\begin{aligned} \therefore \tan \alpha &= \frac{Q \sin \theta}{P + Q \cos \theta} \\ &= \frac{\sin \theta}{1 + \cos \theta} \\ &= \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} \end{aligned}$$

$$\text{Or, } \tan \alpha = \tan \frac{\theta}{2}$$

$$\Rightarrow \alpha = \frac{\theta}{2} \text{ Hence proved.}$$

Sample Question:

Q: Two vectors equal in magnitude act at an angle of 120° . Show that the magnitude of their resultant is equal to either vector.

2.3.3 Polygon Law of Vector Addition:

Activity 1.1

- Take a small piece of stone and displace it first by a displacement OA , then by AB , BC , CD , and finally to E , where E is the final point.
- Join the initial point O with the final point E and determine the net displacement travelled by the stone you took.

Clearly, the net displacement is OE , which is opposite to all the displacements and represents the resultant of all the displacements. This technique is known as the polygon law of vector addition.

Statement: *If the forces (vectors) acting on a particle are represented by the sides of a polygon in magnitude and order, then the closing side of the polygon taken in the opposite order represents the resultant of the vectors.*

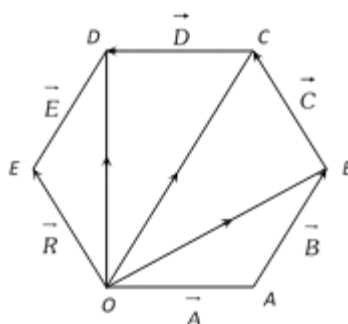


Fig 2.7 Polygon law of vectors

Let $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$ and $\vec{E}$ be the vectors acting on a particle. The vectors are arranged by the sides of a polygon in magnitude and order as $\vec{OA}$, $\vec{AB}$, $\vec{BC}$, $\vec{CD}$ and $\vec{DE}$ respectively. Then, the closing side $\vec{OE}$ taken in the opposite order represents the resultant.

a.Subtraction of a Vector

The subtraction of one vector from another vector is equivalent to adding the negative of the second vector to the first vector.

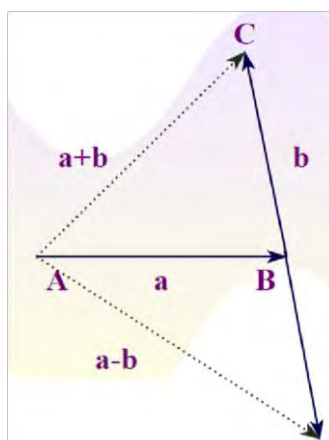


Fig 2.8 Showing subtraction of vectors

Let $\vec{a}$ and $\vec{b}$ be the two vectors and the vector $\vec{b}$ is to be subtracted from the vector $\vec{a}$. For the purpose, we take the negative of the vector $\vec{b}$ by reversing its direction without altering its length as $-\vec{b}$. Then using the triangle law of vector addition on $\vec{a}$ and $-\vec{b}$, we get the resultant $\vec{R}$ as $\vec{R} = \vec{a} - \vec{b}$. The triangle law in the fig. indicates the difference between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

C. Resolution of a Vector:

Resolution is the reverse process of the addition of vectors. In resolution, a vector is split into two or more vectors as components of the original vector. The component of a vector is the effective part of the vector in the direction taken. Generally, it is expressed as the magnitude of the vector times, the cosine of the angle between the vector and the component vector.

Let $\vec{OP} = \vec{P}$ be a vector at an angle α with the direction of OA and β with the direction of OB. The effect or the shadow of vector $\vec{P}$ on OA is $P \cos \alpha$ and the effect or component on OB is

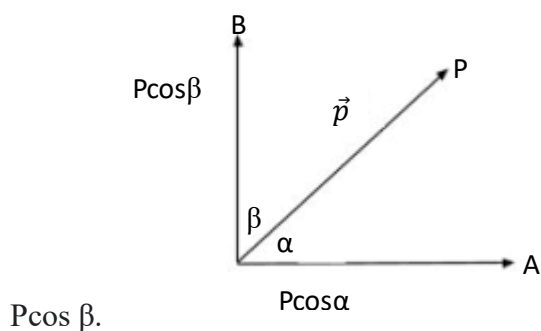


Fig2.8 Resolution of a vector

2.5.1 Rectangular Component of a Vector:

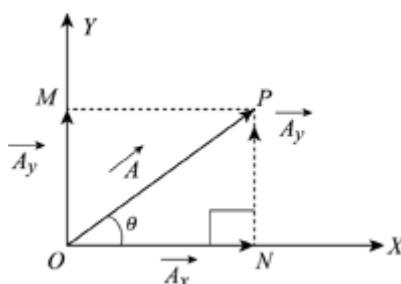


Fig 2.9 Rectangular Components of a vector

Let $\vec{OA} = \vec{A}$ be a vector acting at an angle θ with the X-axis. The shadow of this vector on the X – axis is the X component (or the horizontal component) of the vector, and the shadow on the Y-axis is called the Y- component (or the vertical component) of the vector. Hence to evaluate the components, let's draw $AM \perp OX$ and $AN \perp OY$.

Then, $ON = A_X \rightarrow$ X- component, and $OM = A_Y \rightarrow$ Y- component of the vector.

Now we have, $\cos\theta = \frac{OM}{OA} = A_X/A$

$$\therefore A_X = A\cos\theta$$

And, $\sin\theta = \frac{MA}{OA} = \frac{ON}{OA} = A_Y/A$

$$\therefore A_Y = A\sin\theta$$

Also, $A_X^2 + A_Y^2 = A^2$

Or, $A = \sqrt{A_X^2 + A_Y^2}$

Also, $\vec{A} = A_X\hat{i} + A_Y\hat{j}$

And $\tan\theta = \frac{A_Y}{A_X}$

$$\therefore \theta = \tan^{-1}\frac{A_Y}{A_X}$$

In the 3-D space, the vector $\vec{A}$ is resolved as A_X , A_Y and A_Z and is written as , $\vec{A} = A_X\hat{i} + A_Y\hat{j} + A_Z\hat{k} \equiv (A_X, A_Y, A_Z)$

Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the three mutually perpendicular coordinate axes X , Y and Z respectively.

Worked Out Examples:

Q: A force of 100N acts at an angle of 45° with the X-axis. Find the X – component and the Y – component.

Solution:

Here,

$$F = 100\text{N}, \theta = 45^\circ$$

$$F_x = ?, F_y = ?$$

We have,

$$F_x = F\cos\theta = 100\cos45^\circ = 100.1/\sqrt{2} \\ = 70.42\text{N} = F_y.$$

Sample Question:

1. A vector is defined by $\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$. What is the magnitude of Y component of $\vec{A}$?
(Ans: 4)

a. Scalar and vector Products

Activity 1.3

- i) Take any two natural numbers and multiply them; the result is called a scalar, and the multiplication is referred to as the scalar product of the two vectors.
- ii) Take a screw and rotate it clockwise, then note the direction of the advancement.
- iii) Repeat the same process in the anticlockwise direction, and once again note the direction of advancement.

In the first case of screw rotation, it advances downwards, while in the latter case it advances upwards, indicating a specific direction in each instance. This product is known as the vector product or the cross product of the two vectors.

The physical significance of both these is actually a beautiful concept.

For example, in your airplane case, a dot product would give the combined effect of the coordinates in different dimensions on each other. A cross product would actually help you discover and identify a new path, which would be normal to both the vectors you mentioned.

The cross product could help you identify the path, which would result in the most damage if a bird hits the aeroplane through it. The dot product could give you the interference of sound waves produced by the revving of an engine on the journey.

Now, think that while you're in that town, you're suddenly lost at some weird intersection. You know that there's a path from the place you came from, A and another path to a place, C and all you know is that these roads are at the right angle to B, your destination. Take a cross product and voila! You have the road that you need to take to get to B before the sun sets and the dinner gets cold.

As for why the dot product of two vectors is scalar, you just need to brush up on the definition of the dot product. It is only when you disregard the directions and consider the magnitude (length) of the vectors, multiply them with each other, and then with the cosine of angle between them, you get a dot product. There's no direction here. Now where would I need it in real life? How about if I want to find out how much water is rushing through that little hole in the faucet? I would take the flow of water in different outward directions from the hole as vectors and use the dot product to find how much of it I am losing.

2.6.1 Scalar Products:

For any two vectors $\vec{A}$ and $\vec{B}$, the scalar product

(Or the dot product), denoted by $\vec{A} \cdot \vec{B}$, and read as $\vec{A}$ dot $\vec{B}$ is given as $\vec{A} \cdot \vec{B} = AB \cos\theta$, where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.

Here the product $\vec{A} \cdot \vec{B} = AB \cos\theta$ always yields a scalar quantity and hence the name scalar product (or the dot product).

Cases:

If $\theta = 90^\circ$, $\vec{A} \cdot \vec{B} = AB \cos 90^\circ = 0$

i.e, $\vec{A} \cdot \vec{B} = 0$ with $A \neq 0$, $B \neq 0 \Rightarrow$ property of orthogonality.

Hence two vectors are said to be orthogonal if the dot product vanishes.

The work done by a force $\vec{F}$ through a displacement $\vec{S}$ is a scalar product $\vec{F} \cdot \vec{S}$ and $\vec{F} \cdot \vec{S} = FS \cos\theta$.

Also, for a dot product,

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\vec{A} \cdot \vec{A} = (\vec{A})^2 = A^2$$

For the unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ along the three mutually perpendicular coordinate axes,

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0,$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

Worked Out Examples:

Q: Two vectors are such that $\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, prove the vectors are orthogonal.

Proof:

For the vectors to be orthogonal, their dot product should vanish.

$$\text{i.e. } \vec{A} \cdot \vec{B} = 0$$

$$\text{Therefore, } (3\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})$$

$$= 9 + 16 - 25 = 0$$

Hence the vectors are orthogonal.

Q: Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A} - \vec{B} = \vec{C}$ and $A - B = C$. Find the angle between them.

Let θ be the angle between the vectors.

We have,

$$\vec{A} - \vec{B} = \vec{C}$$

Squaring on both sides-

$$(\vec{A} - \vec{B})^2 = \vec{C}^2 \Rightarrow A^2 + B^2 - 2AB\cos\theta = C^2$$

$$\text{OR, } A^2 + B^2 - 2AB\cos\theta = (A-B)^2 = A^2 + B^2 - 2AB$$

$$\text{OR, } 2AB\cos\theta = 2AB$$

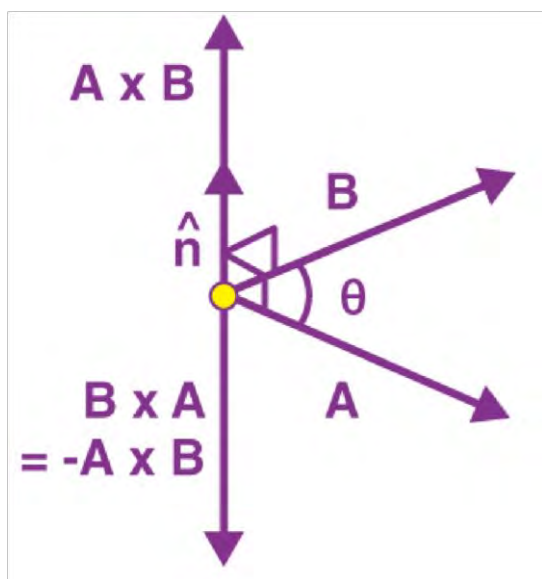
$$\text{OR, } \cos\theta = 1 = \cos 0^\circ \Rightarrow \theta = 0^\circ$$

2.6.2 Vector Product

For any two vectors $\vec{A}$ and $\vec{B}$, the vector product (or the cross product), written as $\vec{A} \times \vec{B}$, and read as $\vec{A}$ cross $\vec{B}$ is given by $\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$, where $\hat{n}$ is the unit vector normal to the plane containing both vectors $\vec{A}$ and $\vec{B}$. Hence the cross product is a vector quantity.

Moreover, $\vec{A} \times \vec{B} = AB\sin\theta\hat{n} = \vec{C}$ is the third vector that is perpendicular to the plane containing both the vectors $\vec{A}$ and $\vec{B}$.

Moreover, $\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = \vec{C}$ is the third vector that is perpendicular to the plane containing both the vectors $\vec{A}$ and $\vec{B}$.



iii) The torque, $\tau = \vec{r} \times \vec{f}$ and the angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ are the examples of a vector product.

Cases:

If $\theta = 0^\circ$, $\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = 0 \Rightarrow$ property of parallelism.

Hence the parallelism of the two vectors is the condition at which the cross product of the two vectors vanishes.

For the cross product,

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

For the unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ along the three mutually perpendicular coordinate axes,

$$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \text{ and } \hat{k} \times \hat{i} = \hat{j}$$

$$\text{But, } \hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i} \text{ and } \hat{i} \times \hat{k} = -\hat{j}$$

It follows the cyclic order $[i \ j \ k \ i \ j]$

And $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ as $\theta = 0^\circ$ and arises the case of parallelism.

In general, $\vec{A} \times \vec{A} = 0$ (The cross product of two identical vectors on themselves equals zero).

The magnitude of the vector product gives the area of a parallelogram.

iv) Dot product for any two vectors, $\vec{A}$ and $\vec{B}$, denoted by $\vec{A} \cdot \vec{B}$ and is given by $\vec{A} \cdot \vec{B} = AB\cos\theta$ and the cross product, denoted by $\vec{A} \times \vec{B}$ and is given by $\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$

v) If the dot product vanishes, the vectors are orthogonal, and if the cross product vanishes, the vectors are the parallel vectors.

vi) For $\hat{i}, \hat{j}, \hat{k}$ as the unit vectors along the three mutually perpendicular coordinate axes,

vii) $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$,

viii) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and

$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}$, and $\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$ and so on

It follows the cyclic order [i j k i j]

Worked Out Examples:

Q: Given the vectors $\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, prove the vectors show the case of parallelism.

Proof: For two vectors to be parallel, their cross product should vanish.

$$\text{i.e., } \vec{A} \times \vec{B} = 0$$

$$\Rightarrow (3\hat{i} + 4\hat{j} - 5\hat{k}) \times (3\hat{i} + 4\hat{j} - 5\hat{k})$$

$$= 12\hat{k} + 15\hat{j} - 12\hat{k} - 20\hat{i} - 15\hat{j} + 20\hat{i} = 0$$

Hence the vectors show the case of parallelism.

Q: If vectors $\vec{A}$ and $\vec{B}$ are non-zero vectors, is it possible for $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$ both to be zero? Explain.

Ans: Here we have, $\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$ and for $\vec{A} \times \vec{B}$ to be zero $\sin\theta = 0$ i.e., the vectors should be parallel or antiparallel and for $\vec{A} \cdot \vec{B}$ to be zero, $\vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos 90^\circ = 0$ i.e., they should act at 90° to each other.

Test yourself:

1. Show that the magnitude of a cross product equals the area of a parallelogram formed by the vectors.

2. What is the angle between $(\vec{A} + \vec{B})$ and $\vec{A} \times \vec{B}$? (Ans: 90°)

Project Work:

Project 2.1 Determining the Shortest Displacement from Home to Shop

A shop lies 200 meters north to the main gate of the house, then 300 meters east, and finally 100 meters southeast.

Hint: *Materials:* Graph paper (optional), Ruler, Calculator, Pen/pencil, Protractor, etc.

- *Procedural steps:* □ Start from the origin (0,0), the main gate, of the house.
- Move 200 meters positive in Y-direction.
- Then move 300 meters positive in X-direction to the east.
- Finally, move 100 meters southeast, which is at a 45-degree angle to the south and east.
- Determine the coordinates after each movement:
 - From (0, 0) to (0,200) (200 meters north).
 - From (0,200) to (300,200) (300 meters east).
 - From (300,200) to the final point.
 - Resolve the southeast movement into x and y components.
- Component of $X = 100\cos 45^\circ =$ Component of $Y = 70.7$

For final coordinates:

- The x-coordinate: $300 + 70.7 = 370.7$ $300 + 70.7 = 370.7 = x$
- The y-coordinate: $200 - 70.7 = 129.3$ $200 - 70.7 = 129.3 = y$
- The displacement vector, $d = \sqrt{x^2 + y^2}$

Note: Select a similar task and execute it for the project work.

Exercise

1. If α be the angle of direction of the resultant of two equal vectors acting at an angle θ , which of the following will be the correct alternative?
 - a. $\alpha = \frac{\theta}{2}$
 - b. $\theta = \frac{\alpha}{2}$
 - c. $\cos\alpha/2 = \cos\theta/2$
 - d. $\cos\alpha/2 = \frac{1}{2}\cos\theta/2$
2. Which one will be the correct alternative for the cross product of two parallel vectors?
 - a. Parallel to one of the vectors
 - b. Normal to one of the vectors
 - c. Zero vector
 - d. A unit vector
3. Two forces of 4 N and 3 N act on a particle at the same time. What is the particle's net force?
 - a. 1N
 - b. 7N
 - c. 5N
 - d. Between 1N and 7N
4. In a rectangular-coordinate system, a particle's position is (3, 2, 5). How is the position vector written?
 - a. $3\hat{i}+5\hat{j}+2\hat{k}$
 - b. $3\hat{i}+2\hat{j}+5\hat{k}$
 - c. $5\hat{i}+3\hat{j}+2\hat{k}$
 - d. $2\hat{i}+3\hat{j}+5\hat{k}$
5. What is the other way of mentioning the vector $12\hat{i}+12\hat{j}$?
 - a. Unit vector
 - b. Null vector
 - c. Vector of magnitude 2
 - d. Scalar

6. Which one of the given quantities is a vector quantity?
 - a. Electrostatic energy
 - b. Electric field intensity
 - c. Electrostatic potential
 - d. Electric current

7. In a hall with dimensions of $10\text{m} \times 12\text{m} \times 14\text{m}$, a fly takes off from one corner and lands in the diametrically opposite corner. What is the size of the fly's flight?
 - a. 17m
 - b. 21m
 - c. 26m
 - d. 36m

8. If the components of a vector in X and Y axes are the same, what is the possible angle made by the vector with the X –axis?
 - a. 30° , b. 45° c 60° d. 0°

9. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the three vectors with magnitude of 3, 4 and 12 respectively. What will be the value of $|\vec{A} - \vec{B} + \vec{C}|$
 - a. 12, b. 13 c. 15 d. undetermined

10. Two equal forces are acting at a point at an angle of 90° . If their resultant is $20\sqrt{2}$ N, what is the magnitude of each force?
 - a. 40N, b. 20N, c. 80N d. 30N

11. Two forces 3N and 2N are at an angle θ such that the resultant is R. The first force is now increased to 6N and the resultant becomes 2R. What is the value of θ ?
 - a. 30° b. 45° c. 60° d. 120°

12. What is the angle between the vectors $(\hat{i} + \hat{j})$ and $(\hat{j} + \hat{k})$?
 - a. 0° b. 60° c. 90° d. 180°

13. A vector is represented by $3\hat{i} + 4\hat{j} + 5\hat{k}$. What is its length in YZ plane?
 - a. 5. b. $\sqrt{41}$ c. $\sqrt{14}$ d. $\sqrt{26}$

14. If the vectors $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} + 2\hat{j} - \lambda\hat{k}$ are the parallel vectors, what is the value of λ ?
 - a. -1 b. -2 c. -3 d. -4

15. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$, what is the projection of $\vec{A}$ on $\vec{B}$?
 a. $3/\sqrt{13}$ b. $3/\sqrt{26}$ c. $\sqrt{3}/26$ d. $\sqrt{3}/13$
16. What is the angle made by the vector $\vec{A} = 2\hat{i} + 2\hat{j}$ with the X- axis?
 a. 0° b. 30° c. 45° d. 60°
17. Three vectors satisfy the relations $\mathbf{A} \cdot \mathbf{B} = 0$ and $\mathbf{A} \cdot \mathbf{C} = 0$. To which vector is vector A parallel?
 a. $\mathbf{B} \cdot \mathbf{C}$ b. $\mathbf{B} \times \mathbf{C}$ c. $\mathbf{B}$ d. $\mathbf{C}$
18. What is the maximum value of $(\mathbf{A} - \mathbf{B})$?
 a. $\mathbf{A} + \mathbf{B}$ b. $\mathbf{A} - \mathbf{B}$ c. $\sqrt{(Ax^2 + By^2)}$ d. $\sqrt{(Ax^2 - By^2)}$
19. (a) Define negative of a vector. State triangle law of vector addition.
 (b) Define component of a vector?
 (c) Define a unit vector. Find a vector in the direction of $(\hat{i} + \hat{j})$ and with magnitude 1. $(0.707\hat{i} + 0.707\hat{j})$
20. (a) State parallelogram law of vector addition.
 (b) Find the angle between two vectors if the magnitude of the cross product of the vectors equals the dot product. (45°)
 (c) A position vector is $\vec{r} = 2\hat{i} + 3\hat{j}$ in a plane. Find the angle it made with the positive direction of X –axis. (56.31°)
21. (a) Define rectangular resolution of a vector.
 (b) Differentiate between $\hat{i} \times \hat{j}$ and $\hat{j} \times \hat{i}$ vector.
 (c) Find the unit vector for the vector $2\hat{i} + \hat{j} - 2\hat{k}$. (Ans: $\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$)
22. (a) Is it possible that $\vec{A} + \vec{B}$ be zero?
 (b) Can a flight of a bird be an example of composition of a vector?
 (c) A particle makes a 2 m displacement in the east, 3 m in the north and 4 m in the upwards direction. Compute the magnitude of its resultant displacement. ($\sqrt{29}$ m)
23. (a) Can a quantity with magnitude and direction be necessarily a vector quantity?
 (b) State and explain the polygon law of vector addition.
 (c) Show that the magnitude of $\vec{A} \times \vec{B}$ gives the area of a parallelogram.

24. (a) How is a vector quantity represented symbolically and graphically?
 (b) State and explain the polygon law of vector addition.
 (c) Two equal vectors are inclined at 120° . Prove their resultant is equal to either vector.
25. (a) What are the methods to multiply two vectors?
 (b) State and explain the parallelogram law of vector addition. What will happen to the resultant if two vectors act at 180° ?
 (c) Illustrate a unit vector. Explain how you would resolve a vector into two rectangular components? Explain why it is $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$. What are the conditions for two vectors to show orthogonality and parallelism?
26. (a) State triangle law of vector addition.
 (b) Derive an expression for the magnitude and direction of the resultant of the two vectors inclined at an angle θ to each other.
 (c) Determine the angle between the two vectors $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ so that their resultant is $\sqrt{(3A^2 + B^2)}$ (Ans: 60°)
27. Two vectors $\vec{P}$ and $\vec{Q}$ have magnitude of 3 units and 4 units respectively. What is the angle between them if the dot product of the vectors equals a. zero, b. 12 units, and c. -12 units?
 (Ans: a. 90° , b. 0° , c. 180°)
28. You are given two vectors as $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$ and $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$, a. find the magnitude of each vector, b. write an expression for the vector difference $\vec{A} - \vec{B}$ using null vectors, c. find the magnitude and direction for the $\vec{A} - \vec{B}$.
 (Ans: a. 5.00, $5\sqrt{29}$, b. $-1.00\hat{i} + 5.00\hat{j}$, c. $\sqrt{26}$ and 101.5°)
29. A man pushes the ground with a force of 100 N at an angle 30° with vertical. Find the reaction of the ground along vertical and the horizontal direction (Ans: 86.6N, 50N)
30. Three vectors **A**, **B** and **C** are such that $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and their magnitudes are 3, 4 and 5 units. Find the angle between the vectors **A** and **C**. (Ans: 53.13°)
31. A disoriented man drives 3.25 km north, then 4.75 km west and finally 1.5 km south. Find the magnitude and direction of the resultant displacement, using the method of components. (Ans: 5.06 km, 69.8° west of north)

32. Two vectors are such that $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. Find the area of the parallelogram formed. (Ans: 27.83)

33. A particle has a displacement of 8 m. If one of its rectangular component is 4 m, what is the other? (Ans: 6.92 m)

34. A rocket fires two engines simultaneously. One produces a thrust of 725N directly forward and the other gives a 513N thrust at 32.4° above the forward direction. Find the magnitude and direction (relative to the forward direction) of the resultant force which these engines exert on the rocket (Ans: 1190N, 13.4°)

35. Given two vectors, $\vec{A} = -2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}$ and $\vec{B} = 3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k}$, do the followings. (a) Write an expression for the vector difference $\vec{A} - \vec{B}$, using unit vectors. (b) Find the magnitude of each vector. (c) Find the magnitude of the vector difference $\vec{A} - \vec{B}$. Is this the same as the magnitude of $\vec{B} - \vec{A}$? Explain.

(Ans: a. $-5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}$. b. 5.38, 4.36 c. 8.83)

36. A spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south and then 280 m at 30° east of north. After a fourth unmeasured displacement, she finds herself back where she started. Determine the magnitude and direction of the fourth displacement. (Ans: 143.55 m, 40.90° south of west)

Answer to multiple choice questions:

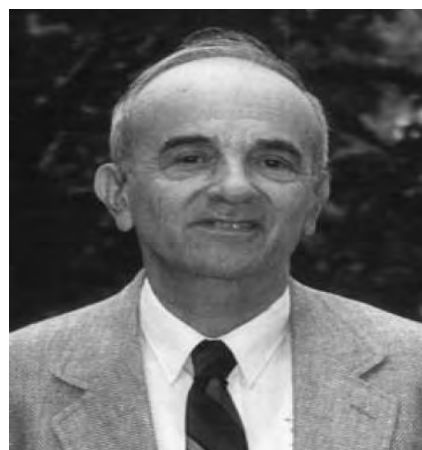
Q.N	1.	2.	3.	4.	5.	6.	7.	8.	9.
Ans	a	c	d	b	a	b	b	b	b
Q.N	10.	11.	12.	13.	14.	15.	16.	17.	18.
Ans	b	d	b	b	b	b	c	b	a

Chapter 3

Kinematics



Franz Reuleaux



Ferdinand Freudenstein

Various physicists and mathematicians including Isaac Newton, Galileo Galilei, and Franz Reuleaux, contributed to the development of kinematics throughout history. The term 'kinematics' was coined in 1834 by the physicist J.C. Ampère. Since the nature of motion was not very precisely defined at the time, he felt it necessary to further study it.

German engineering scientist Franz Reuleaux (1829-1905), known as the “father of kinematics,” conducted extensive studies in the field. He served as a lecturer at the Berlin Royal Technical Academy and later became its president. Reuleaux was a leader in his profession, making significant contributions to various domains of science and knowledge.

American physicist and engineer Ferdinand Freudenstein (12 May 1926 – 30 March 2006) is known as the 'Father of Modern Kinematics.' Freudenstein applied digital computation to the kinematic synthesis of mechanisms.

3.1 Introduction

The word “kinematics” comes from the Greek word “kinesis” meaning motion, and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). Kinematic analysis is the process of measuring the kinematic quantities used to describe motion.

Kinematics deals with the study of motion without considering its causes. At any given instant, a body may be at rest or in motion. If no force acts upon a body, it remains at rest. Similarly, if a body is subject to the influence of numerous forces, and their resultant force is zero, the body remains at rest.

Kinematics is used in astrophysics to describe the motion of celestial bodies and systems; and in mechanical engineering, robotics and biomechanics to describe the motion of systems

composed of joined parts (such as an engine, a robotic arm, or the skeleton of the human body).

If a body changes its position with respect to its surroundings over time, it is said to be in motion. Conversely, if a body does not change its position with respect to its surroundings, it is said to be at rest. Two persons in a moving bus are at rest with respect to each other but seem to be in motion when viewed from outside. Similarly, a person or a tree on the surface of the Earth appears to be at rest when viewed from outside the Earth due to the Earth's motion. Hence, rest and motion are relative terms.

Some terms on Kinematics

Distance and Displacement:

Distance is the total length a particle has traversed during its journey between two points. It depends on the shape of the path that a particle follows.

Displacement, on the other hand, is the shortest distance between the two points in a particular direction, and hence it is a vector quantity.

- If an ant completes a circle during its journey, the displacement is the length of the circumference it traveled, and it returns to the same point, resulting in a zero displacement.
- A ball thrown upwards falls back to the thrower. The net displacement between the initial point (point of throw) and the final point (point of catch) is therefore zero, while distance represents the total length covered during the upward and downward motions.
- If an object at point P reaches point Q along one zig-zag path and returns to point Q following a separate zig-zag path, then the net distance is the sum of the lengths of the two zig-zag paths, while displacement is the shortest distance between P and Q.
- The magnitude of displacement can be less than distance, as it represents the shortest distance in a specific direction.
- In one-dimensional motion, distance and displacement can have the same value.

Speed and Velocity

Speed is the rate of change of distance, and it is a scalar quantity that can be solved using simple ordinary algebra.

$$\text{Therefore, speed (v)} = \frac{\text{distance (d)}}{\text{time (t)}}$$
$$\text{i.e, } v = \frac{d}{t}$$

Velocity is the rate of change of displacement; hence, it is a vector quantity and is solved using vector algebra.

Hence, velocity = speed + direction.

And therefore, velocity, $v = \frac{\text{distance}}{\text{time}}$.

When a particular direction is specified, the term 'speed' is referred to as 'velocity.' However, if the direction is not explicitly stated but the meaning is clear, it is simply referred to as speed.

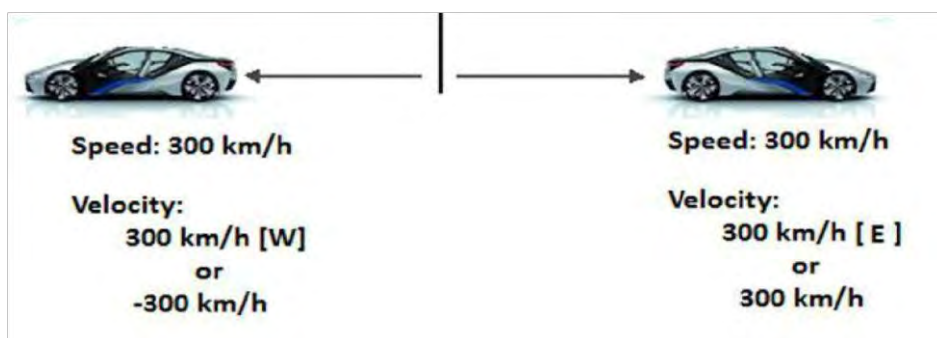


Fig 3.1: Showing the concept of speed and velocity

Uniform Velocity:

If a body undergoes equal displacements in equal time intervals, it exhibits uniform velocity.

Variable Velocity:

If a body changes either the magnitude or the direction of its motion, or both, with time, it is said to have variable velocity.

Average velocity:

It is the ratio of the total displacement to the total time interval taken i.e., average velocity, $v_{av} = \text{total displacement} / \text{time taken}$.

Let s_1 & s_2 be the displacements travelled by a body in time t_1 and t_2 , respectively, then

i.e., average velocity, $v_{av} = \frac{\text{total displacement}}{\text{time taken}}$

Let s_1 & s_2 be the displacements travelled by a body in time t_1 and t_2 , respectively, then

$$V_{av} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

Worked Out Example:

Q: A man travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next equal distance. Find the average velocity.

Solution: Let x be the distance, t_1 and t_2 be the time for the respective velocities v_1 and v_2 .

$$\text{Then average velocity, } v = \frac{x+x}{t_2+t_2} = \frac{2x}{\left(\frac{x}{v_1} + \frac{x}{v_2}\right)} \Rightarrow v = \frac{2}{\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}$$

Instantaneous Velocity

The velocity at any point and at any instant of time is the instantaneous velocity. It is written as

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

The instantaneous velocity indicates the velocity at every possible point at any instant of time.

Distance –Time Graph

The distance-time plot is depicted in the figure. It illustrates how the position of a particle changes over time. The tangent drawn at any point on the path forms a right triangle with sides $\Delta s = S_2 - S_1$ and $\Delta t = t_2 - t_1$. Hence, the average velocity, $V_{av} = \frac{\Delta s}{\Delta t}$, and the instantaneous velocity, $V_{ins} = \frac{ds}{ds}$, are given by the tangent drawn at any point.

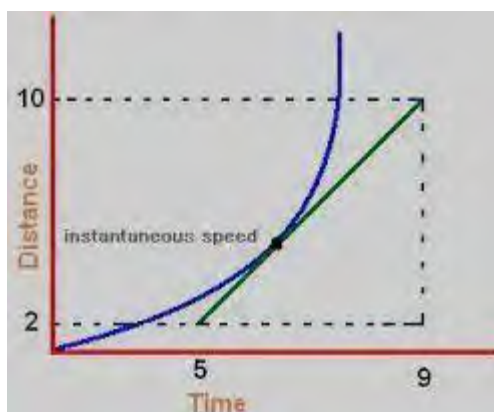


Fig 3.2a

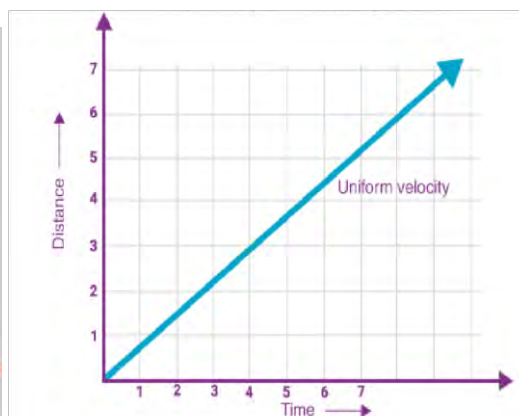


Fig 3.2b

Fig 3.2a Any point on the distance-time plot in this figure gives the value of instantaneous speed, which is represented by the derivative of distance with respect to time, dv/dt . This figure likely shows a curve that represents varying speed over time, where the slope of the curve at any given point gives the instantaneous speed at that moment. Fig 3.2b describes a scenario where there's the same change in distance over the same time interval. This indicates uniform velocity, where the object covers the same distance in equal intervals of time, suggesting constant speed and direction.

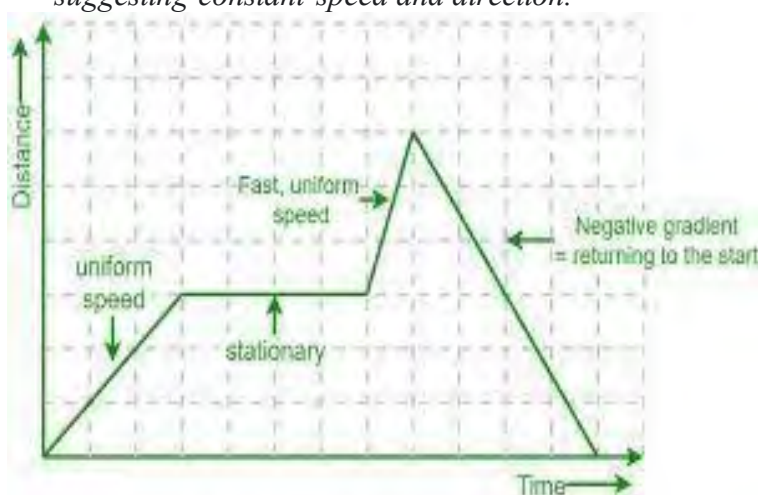


Fig 3.3 The first section, indicated in green, represents uniform speed because there is a consistent increment in distance over equal intervals of time. The second horizontal portion signifies no change in distance over time, representing a stationary or motionless state. The third section, with fast changes in the distance over small intervals of time, depicts fast uniform speed. The final section, showing a decrease in the distance over time, indicates a negative gradient or motion in the opposite direction, ultimately returning to the starting point.

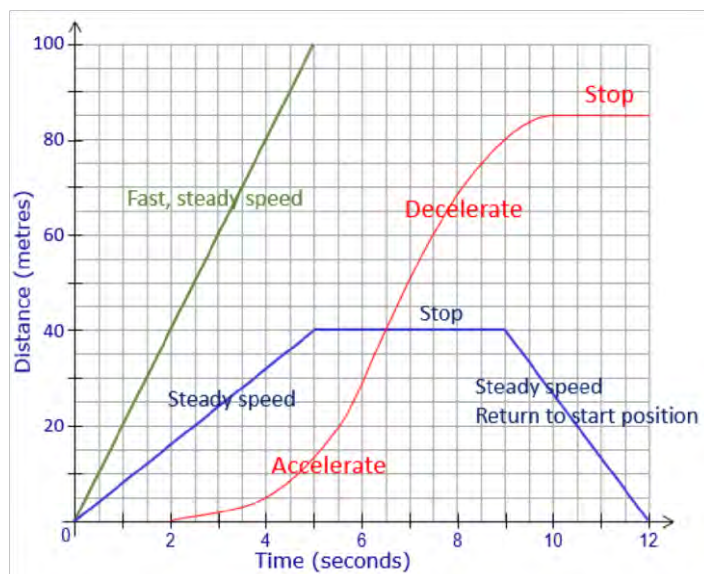


Fig 3.4: Distance – Time graph (The green line indicates a fast, steady speed, moving from 0 to 100 m in 5 seconds. The blue line shows a journey with a stop and a return to the starting position. The red line shows a journey **starting 2 seconds later** than the other two, with an initial acceleration, then a deceleration, and finally a stop.

Note: Any instant of time after the origin of time (or zero time) is called positive time and that before the origin of time is called negative time.

Acceleration:

Think of the following Facts:

- A ball is thrown vertically upwards with an initial velocity of 50 ms^{-1} . After 1 second the velocity decreases to 40 ms^{-1} . Another second later it decreases further to 30 ms^{-1} . By the end of the third second, the velocity is 20 ms^{-1} , and by the fourth second, it decreases to 10 ms^{-1} . Finally, after the fifth second, the velocity becomes zero, and the ball begins to fall back down, after spending a total of 5 seconds in the air. As the ball falls, its velocity increases by 10 ms^{-1} each second, reaching a striking velocity of 50 ms^{-1} when it hits the ground after 5 seconds.
- Consider why there is a consistent change in velocity by an equal amount after each second elapses. What are your thoughts on this uniform change in velocity occurring every second?

The change in velocity in each second is known as acceleration, which can either be an increase or a decrease in velocity. When there is a decrease in velocity in each second, it is referred to as deceleration or negative acceleration.

Therefore, acceleration is defined as the rate of change of velocity.

$$\text{i.e, } a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

$$\Rightarrow \frac{dv}{dt} = \frac{v-u}{t}$$

If the velocity change is equal in equal time intervals, it is considered uniform acceleration.

- If the velocity change varies in each equal time interval, it is considered variable acceleration.
- The total change in velocity over the total time interval represents the average acceleration.

Let v_1 and v_2 be the velocities at a time t_1 and t_2 respectively, then the average acceleration, $a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$ = total change in velocity/total time taken

$$\text{Or, } a = \frac{v_2 - v_1}{t_2 - t_1}$$

- ix) The velocity change at any instant at any point of the path is called instantaneous velocity. It is determined by the limit of the rate of velocity change as time approaches zero.

$$\begin{aligned} \text{x) i.e, } a &= \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} \\ &= \frac{dv}{dt} \end{aligned}$$

- In the case of vertical upward motion, acceleration is always directed downwards and remains constant, while the direction of velocity changes during both ascent and descent.

(Q: Can it be possible that the direction of the velocity is changing, but acceleration remains constant?)

- In the case of pendulum motion, when the pendulum swings eastward from the center, a westward acceleration acts on the bob of the pendulum, seeking to bring it back to the center.
- *(Q: Can an object have an eastward velocity experiencing a westwards acceleration? Justify your answer.)*

In circular motion, velocity changes at every point along the path, but the speed remains constant throughout the motion. (While varying velocity with constant speed is possible, the scenario of varying speed with constant acceleration is not achievable.)

Velocity –Time Graph

A velocity-time graph is a visual representation that illustrates a journey.

For the graph, we take velocity in Y- axis and the time in X – axis. A velocity-time graph tells us how someone/something's velocity has changed over a period of time.

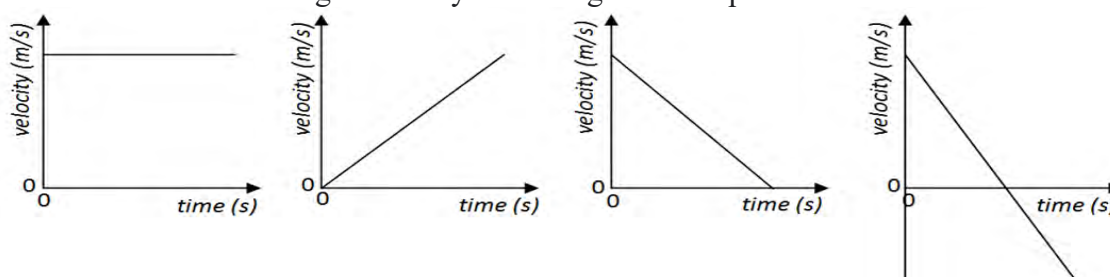


Fig 3.5a: Constant velocity, Fig 3.5b: Constant acceleration, Fig 3.5c: Constant deceleration, Fig3.5d: Body initially thrown upwards.

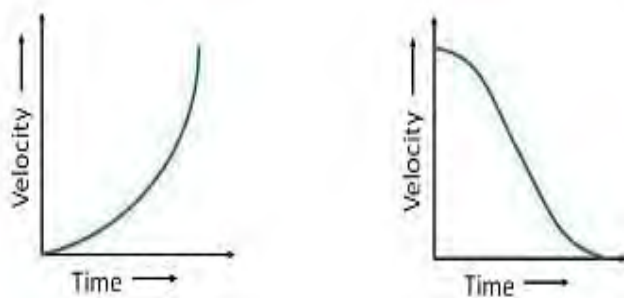


Fig 3.5e: Non-uniform acceleration Fig 3.5f: Non-uniform retardation

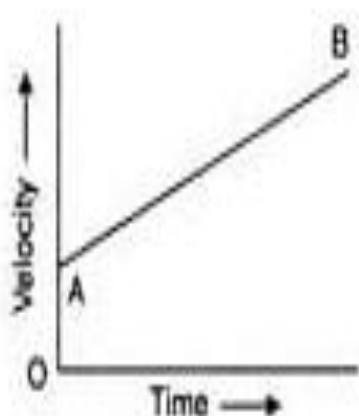


Fig3.5g: Body with some initial velocity $OA = u$

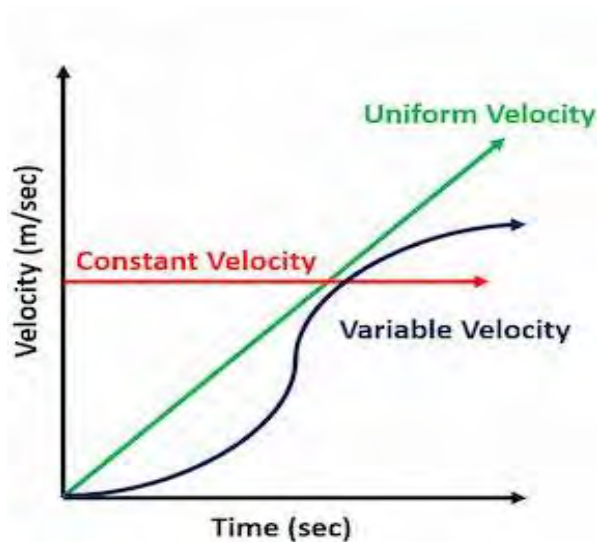


Fig 3.5g: Showing graphically the nature of the velocity. The v-t plot in a red line indicates the constant velocity. The velocity remains the same as time changes. The green-colored plot shows a proportional increase in velocity with time, indicating uniform velocity. The plot in black color depicts varying changes in velocity, signifying variable velocity.

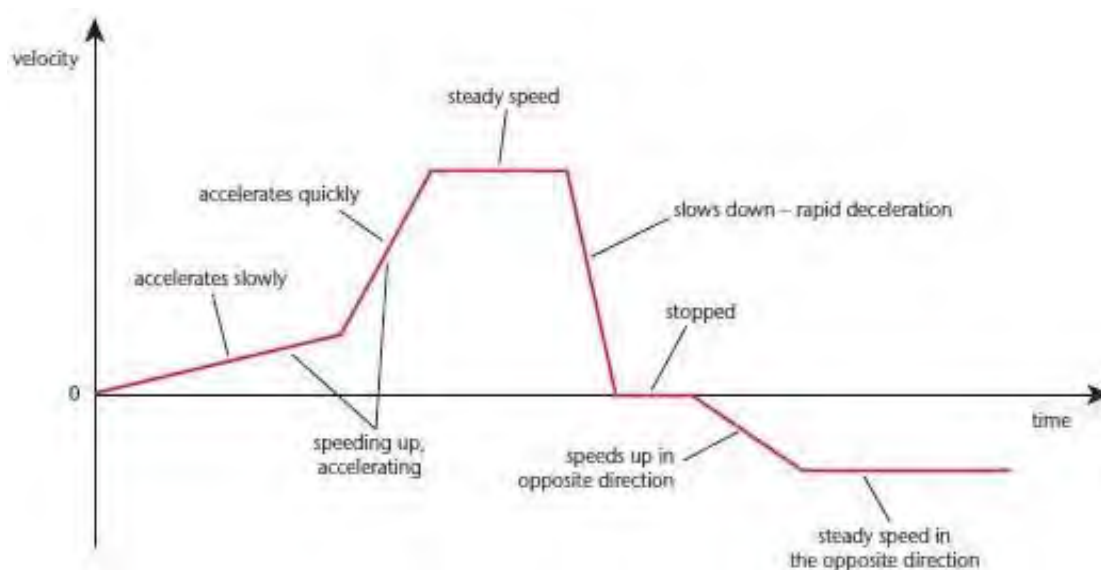


Fig3.3h: the v-t plot for different velocities. In the first section of the plot, there is a slow change in velocity even for a large value of time, hence the body is said to be accelerating slowly. In the second part, there is a rapid increase in velocity over time, signifying quick acceleration. In the third section, there is no change in velocity over time, signifying a steady speed. In the final section, there is a rapid decrease in velocity, representing rapid deceleration that finally returns to its initial state and remains stopped over some interval of time. The body finally speeds up in the opposite direction for a time and remains in a state of steady speed then.

Salient Features:

- (1) Since the slope of the velocity vs. time (v-t) graph at every point represents acceleration, the graph must be continuous and single-valued to be differentiable.
- (2) Time always increases, so the graph should never trend in the opposite direction of the time axis.
- (3) The displacement in a given time interval can be found by calculating the area under the curve between two specified time values. It's crucial that the graph is numerically integrable, meaning it should not have singularities.
- (4) The graph should never become parallel to v -axis.
- (5) The velocity-time (v-t) graph for motion with uniformly increasing velocity is a straight line with a positive slope, while for motion with uniformly decreasing velocity, it is a straight line with a negative slope.
- (6) The velocity-time (v-t) graph of a body with increasing/decreasing acceleration is a curve that exhibits a continuously increasing/decreasing slope as time progresses.

Worked Out Examples:

1. The displacement of a particle is written as $x = 5 + 4t + 2t^2$. Find the value of the instantaneous acceleration.

Solution: Here, $x = 5 + 4t + 2t^2$

Instantaneous velocity, $v = \frac{dx}{dt} = 4 + 4t$

And instantaneous acceleration, $a = \frac{dv}{dt} = 4$ units.

2. As soon as a cyclist starts from rest, an athlete moving with a uniform speed overtakes the cyclist as shown in the v- t diagram in the fig below. Determine
3. (i) The difference between the distances made by the cyclist and the athlete in 6 seconds.
- (ii) The time when the cyclist will catch up the athlete, and
- (iii) The distance of the cyclist and the athlete from the starting point at that instant

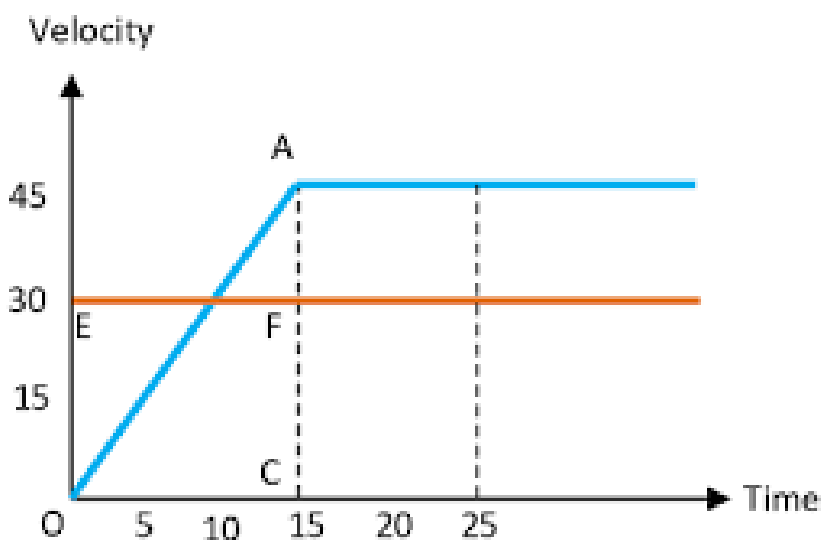


Fig3.6

Solution: Here, OP represents the velocity of the cyclist starting with 0 velocity at O that reaches 30 ms^{-1} at P, and then remains constant.

- (i) Therefore, the distance travelled by the cyclist in 6 s = area of the triangle **OPQ** = $\frac{1}{2} OQ \times PQ = \frac{1}{2} 6 \times 30 = 90 \text{ m}$.

And, the distance travelled by the athlete in 6 seconds = $20 \times 6 = 120 \text{ m}$

Hence, the difference in distance = $120 - 90 = 30 \text{ m}$

- (i) The relative velocity of the cyclist w.r.to the athlete after 6 s = $30 - 20 = 10 \text{ ms}^{-1}$

Therefore, time taken to cover the distance 30 m by the cyclist = $30/10 = 3 \text{ s}$

- (ii) Hence the time taken by the cyclist to catch the athlete after the cyclist starts to travel = $6 + 3 = 9 \text{ s}$. Now the distance travelled by the athlete in 9 s = $9 \times 20 = 180 \text{ m}$

Hence the distance between the cyclist and the athlete from the point of start = 180 m where the cyclist catches the athlete.

3.4 Relative Velocity:

Activity 3.1

- You are following a child running at 20 ms^{-1} while you are running at 50 ms^{-1} . Initially, the child is 300 meters ahead of you. Estimate the time it will take for you to catch up with the child.

Certainly, here the child covers 20m in a second and the man following it covers 50m in the same one second i.e., a total of $50 - 20 = 30 \text{ m}$ closer in 1 second. Hence every second elapse, the man gets by 30 m closer to the child. In doing so 300m distance will be covered in $\frac{300}{30} = 10$ second.

Hence you meet the child after 10 seconds.

To solve the problem mathematically, one must have the knowledge of relative velocity as $50 - 20 = 30 \text{ ms}^{-1}$. The value $50 - 20 = 30 \text{ ms}^{-1}$. Hence is the relative velocity of your with respect to the child. The 20 ms^{-1} in your velocity as $50 \text{ ms}^{-1} = 30 \text{ ms}^{-1} + 20 \text{ ms}^{-1}$ and the child's velocity 20 ms^{-1} remains vain and the child completely comes at rest on subtracting 20 ms^{-1} from your 50 ms^{-1} .

Hence the time taken to meet the child, $t = \frac{v}{t} = \frac{300}{30} = 10 \text{ s}$.

Examples:

- If you run in rainy weather, the rain appears as if it is coming towards you.
- If two vehicles pass each other along a straight road, the passengers in one vehicle perceive the other vehicle as moving with double the speed compared to what is observed by a stationary observer.
- If we observe a flying airplane from a moving bus, we perceive its motion differently compared to what is observed from the stationary ground. The disparity in perception, as studied so far, is known as relative motion.
- To determine the relative motion of a body with respect to a second body, the second body must be brought to rest. Therefore, the motion of one body with respect to another is considered as fixed to compare their relative velocities.



Fig 3.7: Concept for relative velocity

In the figure outlined above, the girl finds herself moving with a velocity of 1 ms^{-1} with respect to the man, and the man sees the girl approaching him with the same velocity. This is called the relative velocity of the girl with respect to the man.

Some basic rules to find relative velocity:

- Let two bodies A and B be moving with velocities V_A and V_B in a direction. Then the velocity of A with respect to B is $V_{AB} = V_A - V_B$, and the velocity of B with respect to A is $V_{BA} = V_B - V_A$.
- If the bodies are moving in opposite directions, then the relative velocity of A with respect to B, V_{AB} , is calculated as $V_A - (-V_B)$, and the relative velocity of B with respect to A, V_{BA} , is calculated as $V_B - (-V_A)$.

In case of two bodies moving in the opposite directions, the relative velocity gets added and that results a greater value than its absolute value.

- For two bodies acting at an acute angle ($0 < \theta < 90^\circ$), to determine the relative velocity of an object with respect to an observer, the velocity of the observer is reversed, and a parallelogram is constructed. The diagonal of this parallelogram represents the relative velocity. The same technique can also be applied using the triangle law of velocity.

Example:

Let $\vec{a}$ and $\vec{b}$ be the two velocities for the bodies A and B acting at an acute angle θ such that ($0 < \theta < 90^\circ$).

Now, for the relative velocity of A with respect to B, we reverse $\vec{b}$ as shown in the

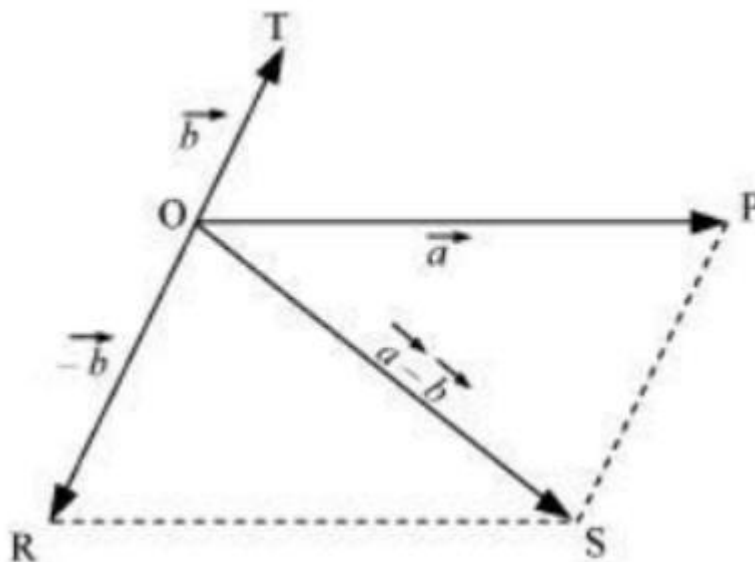


figure.

Fig 3.8

Then taking $\vec{a}$ and $\vec{b}$ as the two sides of a parallelogram, we apply the parallelogram law for the velocity addition.

$$\text{Now, } v = [\vec{a}^2 + \vec{b}^2 + 2ab\cos(180-\theta)]^{1/2}$$

$$= [a^2 + b^2 - 2ab\cos\theta]^{1/2}$$

Cases:

- i) If $\theta = 0$, both $\vec{a}$ and $\vec{b}$ act in the same direction and hence $v = \vec{a} - \vec{b}$
- ii) If $\theta = 180^\circ$, then $\vec{a}$ and $\vec{b}$ act in the opposite direction and hence $v = \vec{a} + \vec{b}$

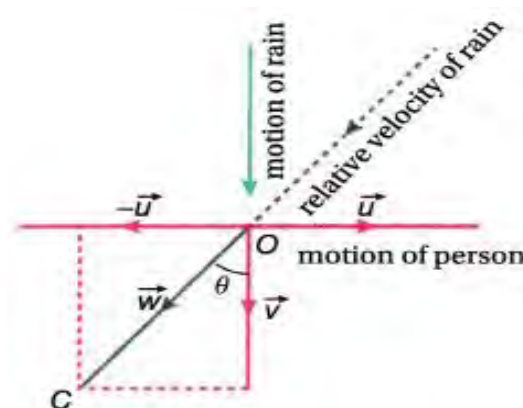
Swimming Case:

- i) For a person swimming in water, let V_{pw} be the velocity of the swimmer with respect to the water, and V_{wg} be the velocity of the water with respect to the ground. Then, $V_{pg} = V_{pw} + V_{wg}$, which implies $V_{pw} = V_{pg} - V_{wg}$.
Therefore, when swimming in the direction of the flow of water, $V_{pg} = V_{pw} + V_{wg}$, and when swimming in the opposite direction to the flow of water, $V_{pg} = V_{pw} - V_{wg}$.

Falling Rain Case:

- For vertical rain, it appears to fall vertically to an observer at rest. We say the relative velocity of the rain with respect to the observer is in the vertical direction. However, to an observer in motion, the situation is different. Let V_r be the velocity of the rain falling vertically, and V_p be the velocity of the person moving horizontally.

To find the relative velocity of the rain with respect to the person, we reverse the direction of V_p and find the resultant.



direction as indicated in the fig 3.9

Fig 3.9

Then the resultant or the relative velocity of the rain with respect to the person V is found using the parallelogram law of vector addition, i.e., the diagonal of the parallelogram of sides V_r and $-V_p$.

$$\begin{aligned}\text{Also, } \tan \theta &= \frac{v_p}{v_r} \\ \Rightarrow \theta &= \tan^{-1} \frac{v_p}{v_r}\end{aligned}$$

It is the angle that the resultant makes with the vertical.

Similarly, if water is falling to the south east direction and a man is marching onwards to the east, the water seems falling vertically as illustrated in the fig.

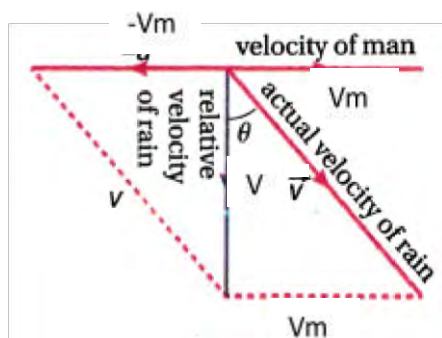


Fig 3.10

Worked Out Examples

- Two ships A and B are 4 km apart. A is due west of B. If A moves with a uniform velocity of 8 km h^{-1} due East and B moves with a uniform velocity of 6 km h^{-1} due South, calculate (i) the magnitude of the velocity of A relative to B and (ii) the closest distance apart A and B.

Solution:

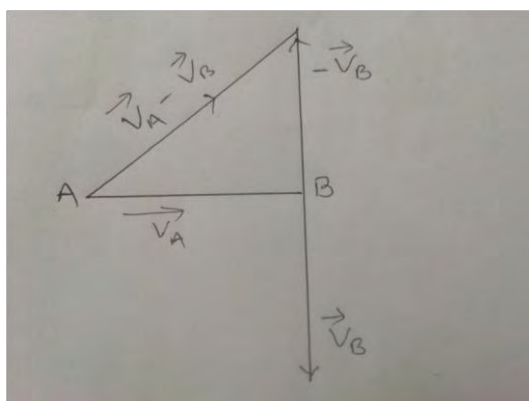


Fig 3.11

Here,

- For the relative velocity of A w. r to B, we reverse V_B due north as $-V_B$.
Therefore, $|V_A - V_B| = \sqrt{V_A^2 + V_B^2} = \sqrt{8^2 + 6^2} = 10 \text{ km h}^{-1}$
- To find the closest distance between A and B, let us assume that B is at rest and the ship A is approaching B. The velocity with which A appears to B is $V_A - V_B$. Therefore, time taken by A to reach B is $t = \frac{d}{|V_A - V_B|} = \frac{4}{10} \text{ hr.}$
- Actually, to say, B is not at rest and the distance it travelled during this time = $V_B \times \frac{4}{10} = 6 \times \frac{4}{10} = 2.4 \text{ km} \Rightarrow$ the closest distance between A and B.

3.5 Equations of Motion:

For the study of the motion of a particle here we have the following variables:

$s \rightarrow$ distance travelled

$u \rightarrow$ initial
velocity

$v \rightarrow$ final
velocity

$a \rightarrow$ acceleration

And, $t \rightarrow$ time taken

(We can use the word 'suvat' to memorise.)

xi) By the definition of acceleration,

$$a = \frac{v-u}{t}$$

$$\Rightarrow v = u + at \dots\dots\dots (3.1)$$

xii) The average distance travelled in time t ,

$$S = \frac{u+v}{2} \times t$$

$$\text{But, } a = \frac{v-u}{t} \Rightarrow t = \frac{v-u}{a}$$

$$\therefore s = (u + v) / 2 \times \frac{v-u}{a}$$

$$= \frac{v^2 - u^2}{2a}$$

$$\therefore v^2 = u^2 + 2as \dots\dots\dots (3.2)$$

Also in $s = \frac{u+v}{2} \times t$, replace v by $v = u + at$

$$s = \left(\frac{u + u + at}{2} \right) \times t$$

$$\therefore s = ut + \frac{1}{2}at^2 \dots\dots\dots (3.3)$$

These are the equations of motion.

Graphical Treatment

Let us take the motion of a particle with uniform acceleration in a straight line.

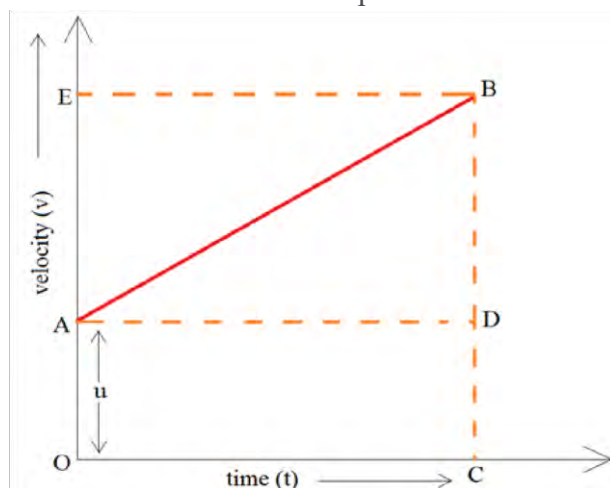


Fig3.12

i) Let $t = 0$, the velocity is u that attains the value v in a time t . Therefore, in the fig,

$$OA = CD = u \quad BD = BC - CD = v - u.$$

The slope of the line AB in a v-t graph,

$$\text{Slope} = \frac{BD}{AD}$$

But, the slope = acceleration

$$\therefore a = \frac{BD}{AD} = \frac{v-u}{t}$$

$$\Rightarrow v = u + at \dots\dots\dots (3.4)$$

i) The distance travelled in time t is the area enclosed by the area of the trapezium OABC.

$$\text{i.e, } S = \frac{1}{2}(OA + BC) \times OC$$

$$\text{But, slope} = a = \frac{BD}{AD} = \frac{BC - CD}{AD} = \frac{BC - CD}{OC}$$

$$\text{OR, } OC = \frac{BC - CD}{a} = \frac{v - u}{a}$$

$$\therefore S = \frac{1}{2}(u + v) \times \frac{v - u}{a}$$

$$\text{OR, } S = \frac{v^2 - u^2}{2a}$$

$$\Rightarrow v^2 = u^2 + 2as \dots\dots\dots (3.5)$$

1. Since we have,

Distance travelled = area of the trapezium OABC

= area of the rectangle OADC + area of the triangle BDA

$$= (OA \times OC) + \frac{1}{2}(v - u) \times t = ut + \frac{1}{2}at^2$$

$$\therefore S = ut + \frac{1}{2}at^2 \dots\dots\dots (3.6)$$

Worked Out Example:

Q. 1 A car is stopped at a traffic light. It then travels along a straight road so that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40 \text{ ms}^{-1^2}$ and $c = 0.120 \text{ ms}^{-1^3}$. (a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0 \text{ s}$, (b) Calculate the instantaneous velocity of the car at (i) $t = 0$, (ii) $t = 5.0 \text{ s}$, (iii) $t = 10.0 \text{ s}$, (c) How long after starting from rest is the car again at rest?

Solution:

Here, distance, $x(t) = bt^2 - ct^3$, where $b = 2.40 \text{ ms}^{-1^2}$, $c = 0.120 \text{ ms}^{-1^3}$

(a) Average velocity of the car (v_{av}) = ? For $t = 0 \text{ s}$ and $t = 10.0 \text{ s}$ (b)

Instantaneous velocity, $v_{inst} = ?$ at $t = 0 \text{ s}$, $t = 5.0 \text{ s}$ and $t = 10.0 \text{ s}$ (c)

Time taken to come to rest, $t = ?$

a) Distance travelled at $t = 0$.

$$x(0) = 0 \text{ and the distance in } t = 10.0 \text{ s, } x(10.0) = 2.40 \times (10.0)^2 - 0.120 \times (10.0)^3 = 120 \text{ m}$$

$$\therefore v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{120}{10.0} = 12 \text{ ms}^{-1}$$

$$\text{b) We have, } x(t) = bt^2 - ct^3 \Rightarrow v_{inst} = \frac{dx(t)}{dt} = 2bt - 3ct^2$$

- ii) Now,
- c)
- i) At $t = 0$, $v_{\text{ins}} = 0 \text{ ms}^{-1}$
- ii) At $t = 5.0 \text{ s}$, $v_{\text{inst}} = 2 \times 2.40 \times 5.0 - 3 \times 0.120 \times (5.0)^2 = 15.0 \text{ ms}^{-1}$
- ii) At $t = 10.0 \text{ s}$, $v_{\text{inst}} = 2 \times 2.40 \times 10.0 - 3 \times 0.120 \times (10.0)^2 = 12.0 \text{ ms}^{-1}$
- (d) We have, $v = \frac{dx(t)}{dt} = 2bt - 3ct^2$

For $v = 0$, $2bt - 3ct^2 = 0 \Rightarrow t = 2b/3c = \frac{2 \times 2.40}{3 \times 0.120} = 13.3 \text{ s}$

Distance travelled in an n^{th} second

Activity 3.2

Consider the activity you have.

xiii) Suppose you are going outside the home for a 2-minutes (120s) walk to reach the destination you considered.

xiv) Guess the distance you traveled in only the 25th second or only the 35th second.

xv) Think and find the technique to deduce the answer.

To find the distance travelled in the 25th second, we first calculate the distance covered in 25 seconds and then in 24 seconds. Subtracting the distance covered in 24 seconds from the total distance in 25 seconds yields the answer.

Similarly, to find the distance in the 35th second, we subtract the distance traveled in 34 seconds from the distance in 35 seconds.

In the same way, here, we develop a general formula to find the distance travelled in an n^{th} second, finding first the distance travelled in n seconds and then in $(n-1)$ seconds respectively.

Therefore, for $t = n$, $S_n = un + \frac{1}{2}an^2$ (3.7)

And, for $t = n-1$, $S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$ (3.8)

$\therefore$ Distance travelled in an n^{th} second $S_{n^{\text{th}}} = S_n - S_{n-1}$

$= un + \frac{1}{2}an^2 - \{u(n-1) + \frac{1}{2}a(n-1)^2\}$

$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{1}{2}a$

$\therefore S_{n^{\text{th}}} = u + \frac{a}{2}(2n-1)$ (3.9)

Note:

For the deceleration or retardation,

$a \rightarrow -a$,

Therefore, $v = u - at$,

$S = ut - \frac{1}{2}at^2$,

$V^2 = u^2 - 2as$

And, $S_{n^{\text{th}}} = u - \frac{a}{2}(2n-1)$

Motion under Gravity

For this,

$a \rightarrow g$ and $s \rightarrow h$

For falling motion, g is positive (+ve).

For upward motion, g is negative (-ve).

Therefore, $v = u + gt$.

$$h = ut \pm \frac{1}{2}gt^2$$

$$v^2 = u^2 \pm 2gh, \quad h_{nth} = u \pm \frac{a}{2}(2n-1)$$

Worked Out Examples:

1. A body begins its motion to move and attains a velocity of 25 ms^{-1} in 5 s and 34 ms^{-1} in 8 s . Calculate the distance it travelled in 12^{th} second.

Solution: here, $a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{34 - 25}{8 - 5} = 3 \text{ ms}^{-2}$

Now distance travelled in the 12^{th} second, $S_{12^{\text{th}}} = u + \frac{a}{2}(2n-1) = 0 + \frac{a}{2}(2 \times 12 - 1)$

$$= 3 \times \frac{23}{2}$$
$$= 34.5 \text{ m}$$

3.6 Projectile Motion

Activity 3.3

Do the activities, observe the phenomena and answer the questions that follow:

- Take a small piece of a stone you feel easy to throw.
 - xvi) Go to the top of a tower and throw the stone horizontally with as much force as you can.
- Observe the motion, the path it follows, and the time it takes to come to a stop.
- Note the horizontal distance it covered on reaching the ground.
- Take two similar bodies again and throw one horizontally forward, while releasing the other to fall vertically downward.
- Note the time taken by the bodies to reach the ground.
 - Q. 1 Sketch the path followed by the particle and find the nature of the path.
 - Q. 2 What types of motion did you see in the journey?
 - Q. 3 Do the two bodies in step 5 fall off together or there is some time lag?

Any object thrown into the region of space that moves under the effect of gravity alone is called a projectile. The path followed by the projectile is called its trajectory. A stone thrown into the air, a bullet fired from a rifle, or a bomb dropped from a flying aero plane are some examples of the projectile. It is hence mainly used in the military field.

In a projectile, there are two motions: horizontal motion free from gravity and the vertical motion under the effect of gravity.

Motion where u and a are constant but mutually perpendicular

The projectile motion is categorized into two parts:

Projectile fired horizontally

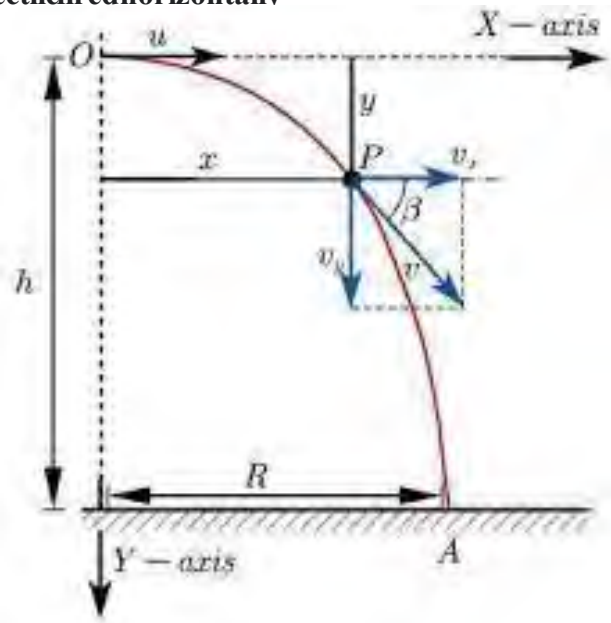


Fig 3.13 Projectile fired horizontally.

Let us consider a particle fired horizontally with a velocity $u_x = u$ from a height h above the Earth's surface. The particle follows the path as indicated and finally reaches the ground at point A. At any instant of time t , let it be at point $P(x, y)$, such that x and y be the horizontal and vertical displacements, respectively.

Therefore, $x = u_x t = ut$

$$\Rightarrow t = \frac{x}{u} \dots \dots \dots (3.10)$$

$$\text{And } y = u_y t + \frac{1}{2} g t^2$$

$$\text{OR, } y = \frac{1}{2} g (x/u)^2$$

$$\therefore y = \frac{g}{2u^2} x^2 \dots \dots \dots (3.11)$$

Equation (3) is similar to the equation of a parabola ($y = kx^2$). Hence, the path followed by the projectile is parabolic in nature. Type equation here.

Time of Flight (T)

It is the time taken by a projectile to remain in the space.

$$\text{We have, } h = u_y t + \frac{1}{2} g t^2$$

$$\text{But, } u_y = 0, t = T$$

$$\therefore h = \frac{1}{2} g T^2$$

$$\Rightarrow T = \sqrt{\frac{2h}{g}}$$

Equation (3.11) is similar to the equation of a parabola ($y = kx^2$). Hence, the path followed by the projectile is parabolic in nature.

Q: Two similar bodies are thrown from the same height above the Earth's surface, one horizontally and another dropped vertically downwards. Which one will hit the ground earlier?

Horizontal Range (R)

It is the total horizontal distance travelled by the projectile on reaching the ground.

From $x = u_x t$,

For $t = T$, $x = R$

$$\therefore R = u_x T = u \sqrt{\frac{2h}{g}}$$

$$\text{Hence } \boxed{R = u \sqrt{\frac{2h}{g}}}$$

Resultant Velocity at any Instant, t

Let V be the resultant velocity.

$$\begin{aligned} \text{Therefore, } v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{u^2 + u_y + gt} \\ &= \sqrt{u^2 + g^2 t^2} \end{aligned}$$

And, the direction at any point,

$$\tan \alpha = \frac{v_y}{v_x} = \frac{gt}{u}$$

$$\therefore \boxed{\alpha = \tan^{-1}\left(\frac{gt}{u}\right)}$$

Worked Out Examples:

1. A student throws a water balloon vertically downwards from the top of a building. The balloon leaves the thrower's hand at a speed of 6.0 ms^{-1} . If air resistance is neglected, the water balloon is in free fall after it leaves the thrower's hand. (a) What is its speed after falling for 2.00 s ? (b) How far does it fall in 2.00 s ? (c) What is the magnitude of its velocity after falling 10.0 m ?

Solution: Here, $u = 6.0 \text{ ms}^{-1}$.

- (a) Time, $t = 2 \text{ s}$, $v = ?$

$$\text{We have, } v = u + gt = 6 + 10 \times 2 = 26 \text{ ms}^{-1}.$$

- (b) $h = ?$ in $t = 2 \text{ s}$

$$\text{We have, } h = ut + \frac{1}{2} gt^2$$

$$= 6 \times 2 + \frac{1}{2} \times 10 \times 2^2 = 32 \text{ m}$$

- (c) $v = ?$ for $h = 10 \text{ m}$

$$\text{We have, } v^2 = u^2 + 2gh = 6^2 + 2 \times 10 \times 10$$

$$\therefore v = 15.36 \text{ ms}^{-1}$$

Projectile Fired Obliquely Upwards

Let a particle be fired obliquely upwards with an initial velocity u and at an angle θ with the horizontal. The velocity u is then resolved into two parts as $u_x = u \cos \theta$ and $u_y = u \sin \theta$ as horizontal and vertical components, respectively.

The particle reaches to a point of maximum height at A and finally on the ground at point B along the path as shown in fig.3.14

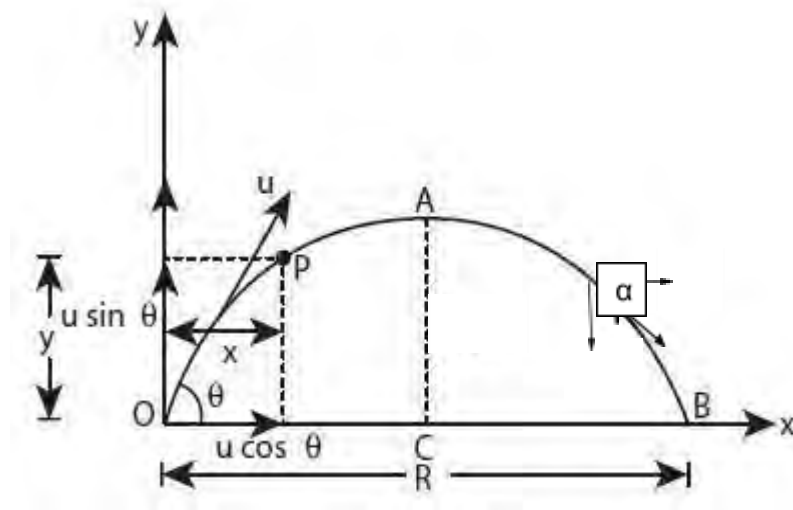


Fig 3.14 Projectile fired obliquely upwards

Let at any instant 't' the particle lies at P(x,y) such that x and y be the horizontal and the vertical displacements respectively.

Here, we have, $x = u_x \cdot t = u \cos \theta \cdot t \Rightarrow t = \frac{x}{u \cos \theta}$(3.12)

$$\begin{aligned} \text{And, } y &= u_y t - \frac{1}{2} g t^2 \\ &= u \sin \theta \cdot t - \frac{1}{2} g t^2 \\ &= u \sin \theta \cdot \frac{x}{u \cos \theta} - \frac{1}{2} g \left(\frac{x}{u \cos \theta} \right)^2 \\ &= \tan \theta \cdot x - \frac{1}{2} u^2 \cos^2 \theta \cdot x^2 \dots \dots \dots (3.13) \end{aligned}$$

Equation (3) is similar to the equation of a parabola ($y = kx^2$). Hence, the path followed by the projectile is parabolic in nature.

Q: Show that the path followed by the projectile fired obliquely upwards is parabolic in nature.

Time of Flight (T)

Since it is the total time taken by the projectile to remain in the space, we have

$$\begin{aligned} \text{here } h &= u_y t - \frac{1}{2} g t^2 \\ &= u \sin \theta t - \frac{1}{2} g t^2 \end{aligned}$$

$$\text{Therefore, } 0 = u \sin \theta T - \frac{1}{2} g T^2$$

Aliter: One may use $v_y = u_y - gt$

$$\therefore T = \frac{2u \sin \theta}{g}$$

Aliter: At a point of maximum height (point A here), final velocity, $V_y = 0$ and the time taken to reach there $= \frac{T}{2}$

$$\text{Therefore, } v_y = u_y - gt \Rightarrow 0 = u \sin \theta - g \frac{T}{2} \Rightarrow \therefore T = \frac{2u \sin \theta}{g}$$

Maximum Height ($H_{\max}$)

It is the greatest height that a particle attains during its flight.

For this, we take

$$v_y^2 = u_y^2 - 2gh$$

For the maximum height, $h \rightarrow H_{\max}$, and $v_y = 0$ (Velocity at the point of maximum height becomes zero.)

$$\Rightarrow 0 = (u \sin \theta)^2 - 2gH_{\max}$$

$$\therefore H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

xvii) If we allow two bodies, one lighter and the other heavier vertically upwards ($\theta = 90^\circ$) with the same velocity, they will reach the same height as per the relation $H_{\max} = \frac{u^2}{2g}$.

xviii) For the same speed, $R_{\max} = \frac{u^2}{g}$ be the maximum range, then $R_{\max} = 2 H_{\max}$.

Hence if h is the greatest height that a man can throw a stone, then $2h$ is the greatest distance that he can throw the stone.

xix) The angle of projection and the initial velocities are the two factors that determine the longer jumping.

Horizontal range(R)

It is the total horizontal distance that a projectile makes on reaching the ground after elapsing the time of flight.

$$\begin{aligned} \text{Therefore, } R &= u_x \cdot T = \frac{u \cos \theta \cdot 2 \sin \theta}{g} \\ &= \frac{u^2 2 \sin \theta \cdot \cos \theta}{g} \\ \therefore R &= \frac{u^2 \sin 2\theta}{g} \end{aligned}$$

Maximum Horizontal Range ($R_{\max}$)

Here we have,

$$R = \frac{u^2 \sin 2\theta}{g}$$

For maximum range, $R \rightarrow R_{\max}$,

Hence for $R \rightarrow R_{\max}$, $\sin 2\theta = 1$

$$\text{Or, } \sin 2\theta = \sin 90^\circ$$

$$\text{OR, } 2\theta = 90^\circ$$

$$\therefore \theta = 45^\circ$$

$$\text{And } R_{\max} = \frac{u^2}{g}$$

Hence, in order to achieve the maximum horizontal range even on projecting with the least speed, one should project the particle at 45° with the horizontal.

Q: 1. Relate the ranges when fired at 15° and 45° (Ans: $R_{15} = \frac{R_{45}}{2}$)

Q: 2. Relate the maximum horizontal range with maximum height (Ans: $R_{\max} = 4H_{\max}$)

An Alternative Possible Angle for the Same Range:

Let θ and $90 - \theta$ be the angles of projection.

$$\text{Therefore, } \theta \Rightarrow R = \frac{u^2 \sin 2\theta}{g}$$

$$\begin{aligned} \text{And } (90 - \theta) \Rightarrow R &= \frac{u^2 \sin 2(90 - \theta)}{g} \\ &= \frac{u^2 \sin (180 - 2\theta)}{g} \end{aligned}$$

$$\therefore R_1 = \frac{u^2 \sin 2\theta}{g} = R.$$

Hence, on projecting a projectile either at θ or at $90 - \theta$, there is the same range though the trajectories are different.

Resultant Velocity at Any Instant:

Let V be the resultant velocity.

$$\begin{aligned} \text{Then } v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{u^2 \cos^2 \theta + (u \sin \theta - gt)^2} \end{aligned}$$

$$\text{And, the direction, } \tan \alpha = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$$

$$\therefore \alpha = \tan^{-1} \frac{u \sin \theta - gt}{u \cos \theta}$$

Q 1. A projectile is fired at an angle 18° with the horizontal. Find the possible angle in order to have the same range. (Ans: 72°)

Q 2. Since a projectile has the same range for two angles of projections, show that the product of the time of flights T_1 and T_2 is related as $T_1 T_2 \propto \frac{1}{R}$.

Worked Out Examples:

1. A projectile is fired from the ground level with a velocity of 500 ms^{-1} at 30° to the horizontal. Find the horizontal range, the greatest vertical height to which it rises, and the time to reach the greatest height. What is the least speed with which it could be projected to achieve the same horizontal range? (The resistance of the air to the motion of the projectile may be neglected.)

Solution:

Here, velocity of projection, $u = 500 \text{ ms}^{-1}$

Angle of projection, $\theta = 30^\circ$

Horizontal range, $R = ?$

Maximum height, $H_{\max} = ?$

Time to reach the greatest height, $T/2 = ?$

$$\begin{aligned} \text{We have, } R &= u^2 \sin 2\theta / g = 500^2 \sin 2 \cdot 30 / 10 \\ &= 21651 \text{ m.} \end{aligned}$$

$$\begin{aligned} H_{\max} &= u^2 \sin^2 \theta / 2g = 500^2 \sin^2 30 / 2 \times 10 \\ &= 3125 \text{ m.} \end{aligned}$$

$$T/2 = u \sin \theta / g = 500 \sin 30^\circ / 10 = 25 \text{ s}$$

Let u' be the least speed with which the projectile could be projected to achieve the same (21651 m) range.

For the maximum range with the least speed, one should project a particle at 45° .
Therefore, $R = \frac{u'^2 \sin 2\theta}{g}$

$$\text{Or, } 21651 = \frac{u'^2}{10} \quad u' = \sqrt{216510} = 465.30 \text{ ms}^{-1}.$$

1. An airplane is flying with a velocity of 90.0 ms^{-1} at an angle of 23.0° above the horizontal. When the plane is at the height of 114 m directly above a dog that is standing on level ground, a suitcase drops out of the luggage compartment. How far from the dog will the suitcase land? You can ignore the air resistance.

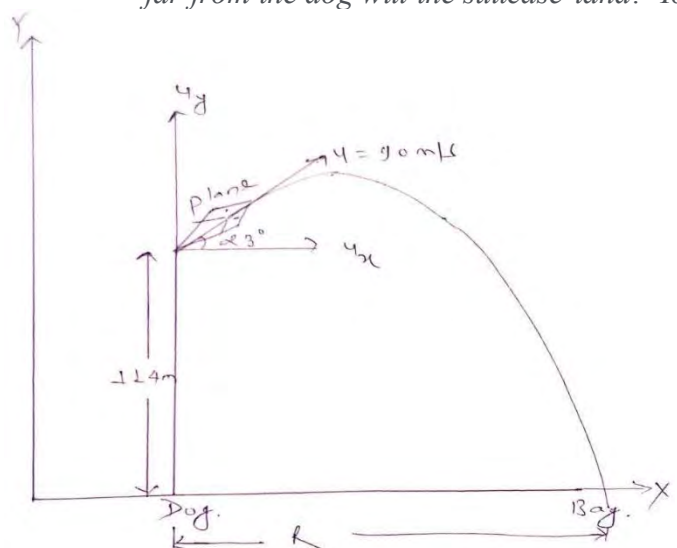


Fig 3.15

Solution: Here, velocity, $u = 90.0 \text{ ms}^{-1}$

Angle of projection, $\theta = 23.0^\circ$

Height of the projection point, $H = 114 \text{ m}$

Range, $R = ?$

We have, horizontal component, $u_x = u \cos 23^\circ = 90 \times \cos 23^\circ$
 $= 82.85 \text{ ms}^{-1}$ and the vertical component, $u_y = u \sin 23^\circ$
 $= 90.0 \times \sin 23^\circ = 35.17 \text{ ms}^{-1}$

Let T be the time taken by the suitcase to reach the ground.

Hence, we apply $H = u_y T + \frac{1}{2} g T^2$

Or, $114 = -35.17T + \frac{1}{2} g T^2$ ($\because$ The suitcase is dropped downwards from 114 m .)

$$\text{Or, } T^2 - 7T - 22.8 = 0$$

$$\text{OR, } T = \frac{7 \pm 11.84}{2}$$

On considering the positive value, $T = 9.42 \text{ s}$

We have, $R = u_x T = 82.85 \times 9.42 = 780.45 \text{ m}$.

Project Work:

Project 3.1 Comparison of Time of Flight for Horizontally Thrown and Vertically Dropped Bodies

Hint:

Materials:

- ☐ Two identical balls.
- ☐ Measuring tape or ruler
- ☐ Stopwatch or high-speed camera
- ☐ Level platform for height
- xx) Markers or chalk for marking positions

Theory: Time of fall, $T = \sqrt{\frac{2h}{g}}$

Procedural idea:

- xxi) Ensure the area below the platform is clear of obstacles.
- xxii) Hold one of the balls at the edge of the platform.
- xxiii) Release the ball from rest, allowing it to fall freely.
- xxiv) Use a stopwatch or high-speed camera to measure the time it takes for the ball to hit the ground.
- xxv) Repeat this step several times and record the times.
- xxvi) Hold the second ball at the same height but give it a horizontal push so it is thrown horizontally and note the time it takes for the ball to reach the ground and repeat the steps for several times. *Note to tabulate all the data.*

Data Analysis:

- xxvii) Find average time for drop and horizontal throw.
- xxviii) Compare the average times to demonstrate that they are approximately equal.

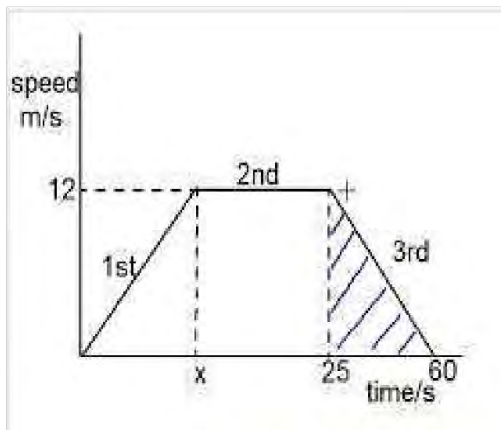
Validation to theory:

- xxix) Compare the experimental times with the theoretical time, $T = \sqrt{\frac{2h}{g}}$

Exercise

1. A bus travels $\frac{1}{3}$ of a distance at 10 km/h, another $\frac{1}{3}$ of the distance at 20 km/h and the remaining $\frac{1}{3}$ rd of the distance at 60 km/h. What is the average speed of the bus?
 - a. 9 km/h,
 - b. 16 km/h,
 - c. 18 km/h and
 - d. 48 km/h
2. The velocity of a particle at an instant is 10 ms^{-1} . After 5 s the velocity of the particle is 20 ms^{-1} . What is the velocity at 3 s before reaching 10 ms^{-1} ?
 - a. 5 ms^{-1} ,
 - b. 4 ms^{-1} ,
 - c. 6 ms^{-1} ,
 - d. 8 ms^{-1}
3. A particle experiences constant acceleration for 20 seconds after starting from rest. If it travels a distance S_1 in the first 10 s and a distance S_2 in the next 10 seconds, which one is the correct alternative?
 - a. $S_1 = S_2$
 - b. $S_2 = 2 S_1$
 - c. $S_2 = 3 S_1$
 - d. $S_1 = 4 S_2$
4. The displacement of a particle is proportional to the first power of time. What is the acceleration of the particle?
 - a. Small finite value
 - b. zero
 - c. infinite
 - d. large finite value
5. What is the motion of the tip of the minute hand of a clock?
 - a. zero displacement
 - b. zero distance
 - c. average speed is zero
 - d. average velocity is non- zero
6. A bullet fired onto a wooden block loses half of its velocity after penetrating 40 cm. What further distance does it cover before coming to rest?
 - a. $4000/3 \text{ m}$
 - b. $2200/3 \text{ m}$
 - c. $2200/5 \text{ m}$
 - d. $2000/3 \text{ m}$
7. A cyclist moving on a circular track of radius 25m completes half a revolution in 25 s. What is its average velocity?
 - a. 4 ms^{-1}

- b. 2ms^{-1}
 c. zero
 d. $\pi \text{ ms}^{-1}$
8. Two railway tracks extend parallel from north to south. If train A moves from north to south at 54 km/h and train B moves at 90 km/h from south to north. The relative velocity of the train A w.r. to the train B is
 a. 20 ms^{-1}
 b. 30 ms^{-1}
 c. 40 ms^{-1}
 d. 50 ms^{-1}
9. _____ is the simplest type of motion and is along a straight line path.
 a) plane motion
 b) rectilinear motion
 c) curvilinear motion
 d) circular motion
10. A train covers 90 miles between 1 p.m. and 4 p.m. How fast was it going at 2 p.m?
 a) 90 mph
 b) 30 mph
 c) 40 mph
 d) 80 mph
11. A police officer living on a planet with no air resistance drops a pair of handcuffs and a handkerchief from the same height at the same time. Which one will reach the ground first?
 a. the handkerchief
 b. handcuffs
 c. The answer depends on how far above the ground they were when he dropped them.
 d. They both hit the ground at the same time.
12. A ball dropped from a tower covers half the total distance in the last second of its motion. The total time of fall for $g = 10 \text{ ms}^{-1}{}^2$ is
 e. a. $(1+\sqrt{2}) \text{ s}$
 f. b. $(2 + \sqrt{2}) \text{ s}$
 g. c. $2\sqrt{2} \text{ s}$
 h. d. $\sqrt{2} \text{ s}$
13. Calculate the distance travelled by a particle in the third stage from the following v-t plot



- a. 220 *Fig 3.16*
- b. 200 m
- c. 105 m
- d. 210 m

14. From the movement of a car in the displacement – time graph in fig 3.17. What is the time for the car to stop?

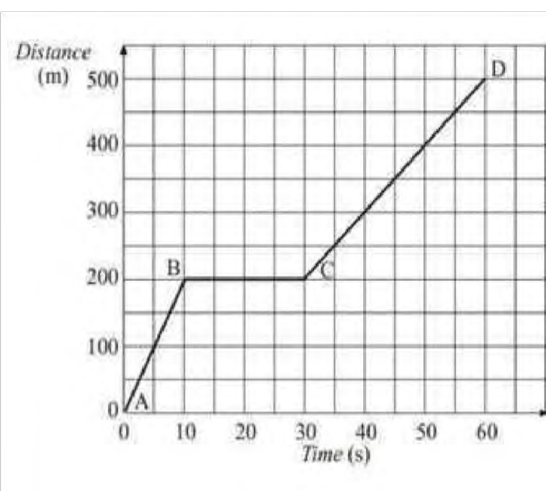
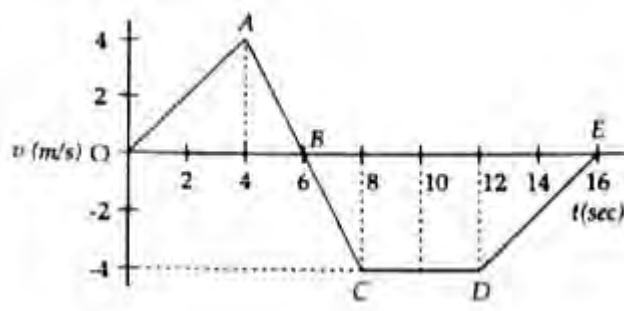


Fig 3.17

- a. 10 s
- b. 20 s
- c. 30 s
- d. 15 s

15. What is the total displacement of a particle from the given $v - t$?



- a. 8 m *Fig 3.18*
 b. -16m
 c. 16m
 d. -8m
16. What is the value of v for a total displacement of 1750m in the following speed–time diagram?

- a. 15 ms^{-1}
 b. 20 ms^{-1}
 c. 25 ms^{-1}
 d. 28 ms^{-1}

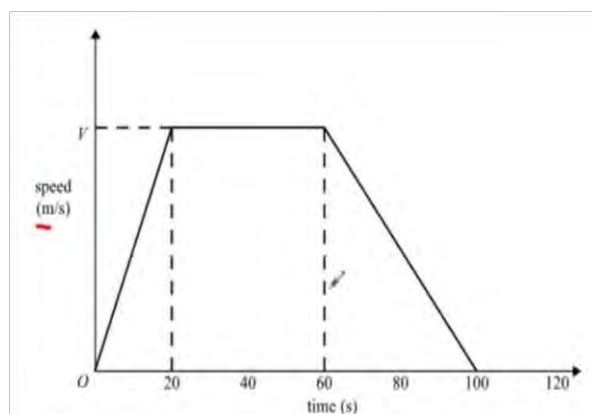
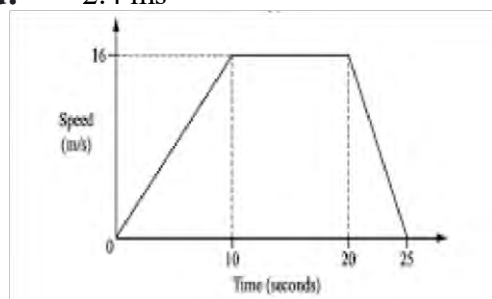


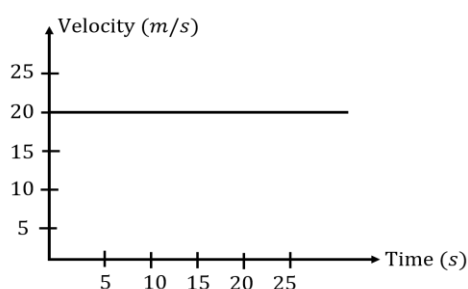
Fig 3.18

17. What is the deceleration of the car for the last 5 s in the speed- time graph given below?

- a. 3 ms^{-2}
 b. 3.2 ms^{-2}
 c. 1.6 ms^{-2}
 d. 2.4 ms^{-2}



18. What is the distance travelled by a body in 20 seconds in the v-t diagram below?



- a. 300 m *Fig 3.19*
 b. 350 m
 c. 400 m
 d. 450 m
19. (a) What are average speed and average velocity?
 (b) The v- t for an object is given as $x = A - Bt^3 + At^4$.
 Calculate the value of instantaneous velocity and instantaneous acceleration (*Ans: $v = 4At^3 - 3Bt^2$, $a = 12At^2 - 6Bt$*)
 (c) The following graph isn't correct, why?

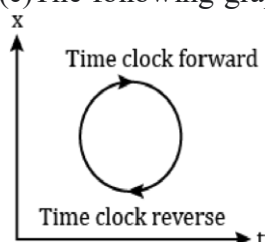


Fig 3.20

20. (a) Can a displacement time graph be negative?
 (b) A person makes 4 complete cycles around a mile track in a total time of 7 minutes. Calculate the (i) average speed and (ii) average velocity in ms^{-1} . (*Ans: (i. 19.16 s, ii. 0)*)
 (c) Why are these two answers in 20 (b) different?
21. (a) Define a projectile.
 (b) Show that the path of the projectile is parabolic when fired horizontally.
 (c) (c) The range of the projectile when fired at an angle of 30° is 2 km. Find its range when it is fired at 45° . (*Ans: $4/\sqrt{3}$ km*)
22. (a) Why is the angle of attack preferred for 32 to 36 degrees for a javelin throw though physics recommends it for 45 degrees for maximum range?
 (b) Show that the range is the same when a particle is projected at θ and $(90 - \theta)$.
 What is the main factor to alter the range for a set of apparatus?
 (c) Write the condition to achieve the same range even with a least speed.
23. (a) A bomb is released earlier from a flying aeroplane before it reaches just above the targeted point. Why?

- (b) Which one will hit the ground earlier when on two similar bodies, one is fired horizontally and another allowed to fall down from the same height at the same time? (c) How would you find the resultant velocity with which a projectile hits the ground?
24. a. Define a projectile. Deduce the expressions for the time of flight, maximum height attained, and horizontal range when a projectile is fired obliquely upwards at an angle with the horizontal.
b. Show that the maximum range is 4 times the maximum height covered by the projectile.
c. If the angle of projection is 18° , find the other possible angle in order to have the same range though the trajectories differ.
25. a. Show that the path followed by a projectile is parabolic in nature.
b. Calculate the time of fall that a projectile takes when fired horizontally.
c. Guess the point on the trajectory of a projectile at which velocity and acceleration are normal to each other.
26. Consider the motion of an object whose velocity– time graph is outlined in the figure aside:
- a. Find the acceleration of an object in the time interval $t = 0$ to $t = 2$ s. If a constant unbalanced force of 10N is applied, what will be the mass of the object?
b. Find the acceleration of the object in the time interval $t = 10$ s to $t = 12$ s.
c. Determine the total displacement of the object in the time interval $t = 0$ to $t = 16$ s.
(Ans: a. 4 ms^{-2} , 2.5 kg, b -2 ms^{-2} , c 100m)

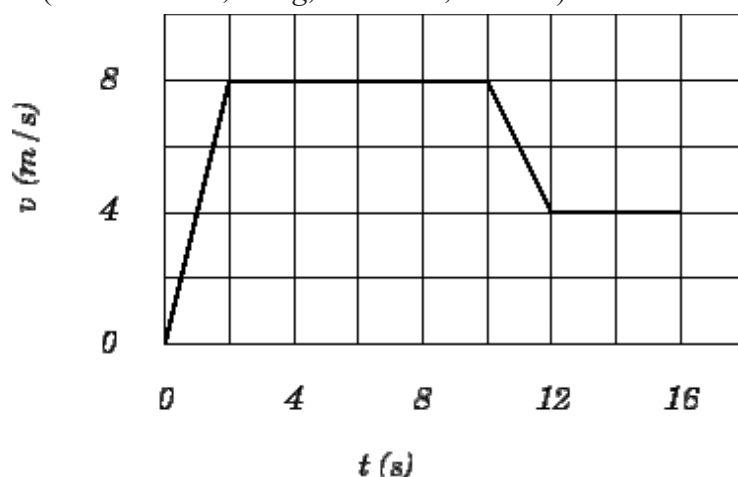


Fig 3.21

27. A child runs from rest on a horizontal level road and moves with a constant acceleration for 8 s reaching a speed of 10 ms^{-1} . The speed then is maintained for T s. The child then decelerates at a constant rate until it stops. The child has run a total of 800 m in 100 s.
- (a) Sketch a v- t graph to illustrate the motion of the child.
(b) Calculate the value of T.
(Ans: 60 s)
28. Consider the motion of an object whose velocity– time graph is outlined in the figure aside. (Fig 3.22)

- (a) Find the acceleration of the object in the time interval $t = 0$ to $t = 4$ s (b) Calculate the value of the deceleration in the time interval $t = 7$ to $t = 10$ s.
- (c) Determine the total displacement of the object in the time interval $t = 0$ to $t = 7$ s. (Ans: a. 0.75 ms^{-2} , b. -3.0 ms^{-2} , c. 16.5 m)

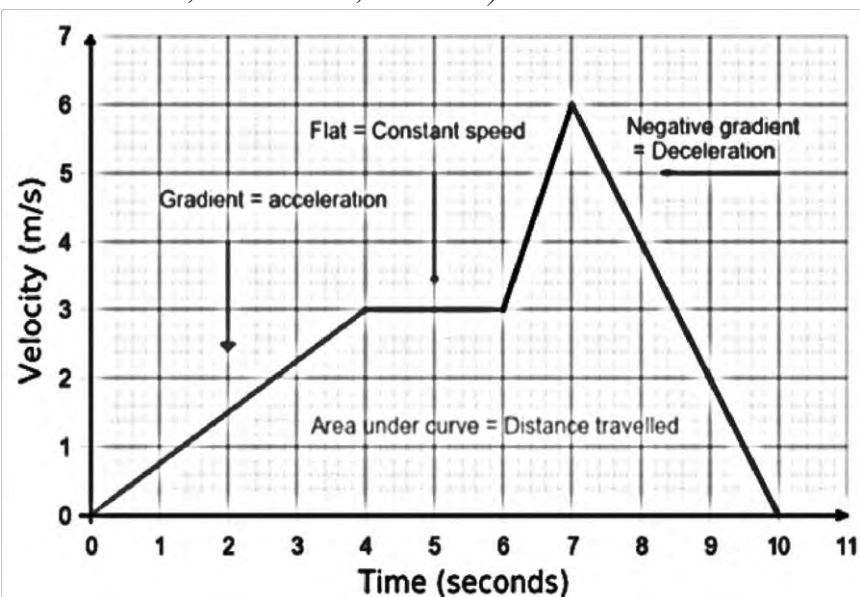


Fig 3.22

29. A car moving with a velocity of 10 ms^{-1} accelerates uniformly at 1 ms^{-2} until it reaches a velocity of 15 ms^{-1} . Calculate (i) time taken, (ii) the distance travelled during the acceleration, and (iii) the velocity reached 100m from the place where the acceleration began. (Ans: i. 5 s, ii. 62.5 m, iii. 17.3 ms^{-1})
30. A ball is thrown vertically upwards with a velocity of 20 ms^{-1} . Calculate (i) the time to return to the thrower, (ii) the maximum height it reached. ($g = 10 \text{ ms}^{-2}$) (4 s, 20 m)
31. A ball is dropped from a height 20 m and rebounds with $\frac{3}{4}$ of the velocity with which it hits the ground. What is the time interval between the first and the second bounces? (Ans: 3 s)
32. A projectile is fired with a velocity of 320 ms^{-1} at an angle of 30° with the horizontal. Find (i) the time to reach the greatest height, (ii) its horizontal range and (iii) with the same velocity, what is the maximum possible range? (16 s, 8868 m, 10240 m)
33. A ball is allowed to fall down from a height of 60 m and at the same time another similar ball is thrown vertically up with an initial velocity of 20 ms^{-1} along the same vertical line. Find when and where they will meet. (take $g = 10 \text{ ms}^{-2}$). (Ans: After 3 s at 15 m above the ground)
34. A body begins its motion to move and attains a velocity of 25 ms^{-1} in 5 s and 34 ms^{-1} in 8 s. Calculate the distance it travelled in the 12th second. (Ans: 34.5 m)
35. Suppose an object is thrown with initial speed 10 m s^{-1} at an angle $\pi/4$ with the horizontal, what is the range covered? Suppose the same object is thrown similarly in

the Moon, will there be any change in the range? If yes, what is the change? (The acceleration due to gravity in the Moon = $1/6 g$)

(Ans: $R_E = 10m$, $R_M = 60 m$)

36. In a game of cricket, a batsman strikes the ball in such a way that it moves with the speed of 30 m s^{-1} at an angle 30° with the horizontal. The boundary line of the cricket ground is located at a distance of 75 m from the batsman. Will the ball go for a six? (Neglect the air resistance and take acceleration due to gravity $g = 10 \text{ m s}^{-2}$).
(Ans: $R = 77.94 \text{ m}$ which is greater than 75 m and hence the ball goes for six.)
37. An athlete executing a long jump leaves the ground at 27° and lands at a point 7.80 m away. (a) What was the take-off speed? (b) If the speed is increased by 5%, how much longer would the jump be? (Ans: a. 10.72 ms^{-1} , b. 8.6 m)
38. A long jumper leaves the ground at 20° above the horizontal at a speed of 11 ms^{-1} . How far does he jump in the horizontal direction? What is the maximum height attained? (Ans: 7.86 m , 0.72 m)
39. A football player kicks the ball from the ground level at 27 ms^{-1} at an angle 30° above the horizontal. Neglecting the air resistance, find;
a. the ball's hang time,
b. the ball's maximum height, and
c. the horizontal distance the ball travels before hitting the ground. (Ans: a. 2.8 s , b. 9.3 m , c. 65 m)
40. A rock is thrown from a 50.0-m high cliff with an initial velocity of 7.0 ms^{-1} at an angle of 53.0° above the horizontal. Find its velocity when it hits the ground below. (Ans: 32 ms^{-1} , 82° from horizontal.)

Answer to the Multiple Choice Questions:

Q.N	1.	2.	3.	4.	5.	6.
Ans	c	b	c	b	a	a
Q.N	7.	8.	9.	10.	11.	12.
Ans	b	c	b	b	d	b
Q.N	13.	14.	15.	16.	17.	18.
Ans	d	b	b	c	b	c

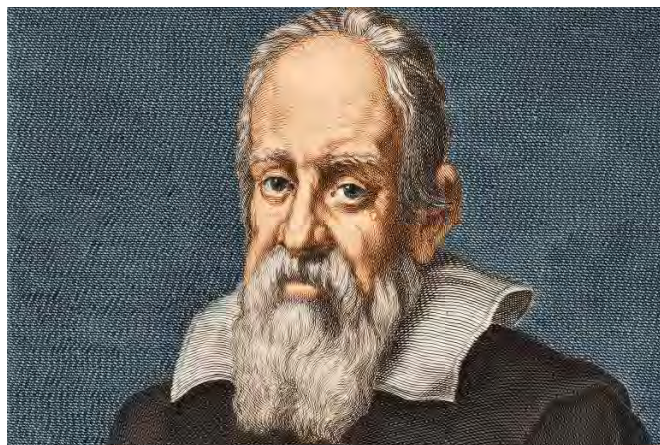
Chapter 4

Dynamics

The foundations of dynamics were laid at the end of the 16th century by Galileo, who, by experimenting with a smooth ball rolling down an inclined plane, derived the law of motion for falling bodies; he was also the first to recognize that force is the cause of changes in the velocity of a body.

Isaac Newton was the first to formulate the fundamental physical laws that govern dynamics in classical non-relativistic physics, especially his second law of motion.

Isaac Newton is responsible for the precise formulation of the laws of motion, describing how speed and position vary over a short period of time.



Galileo (Feb 15, 1564 – Jan 8, 1642)

4.1 Introduction

Dynamics is a branch of mechanics that deals with the study of the motion of an object by taking its cause.

4.1.1 Force:

Activity 4.1

Do the following activities and answer the questions that follow:

- i) As soon as your class ends, arrange the teaching materials such as the duster and markers on the table and leave the class.
- ii) Notice the states of those materials on the following day after you enter the class.
- iii) If you find the states of the body intact, can you guess what's going on?
- iv) If you see the states of the body disturbed, write your guess on what the disturbing agents might be.

Certainly, when one finds something disturbed in the state, one concludes that an external agent must have either pushed or pulled the objects and disturbed them. This external agent is called the force.

Hence, the force is an external agency that causes a change in the state of a body.

4.1.2 Inertia

Every object has a tendency to continue in the state it is in, i.e. objects resist the change in their state. This resistance of a body to change its state is called inertia. Inertia is directly proportional to its mass. Therefore, the greater the mass, the greater the inertia, which is expressed by the equation $I \propto m$ where I represents inertia and m represents mass.

Examples:

- i) Suppose there are two water buckets on the ground, one filled with water and the other empty. If a strong wind blows, the empty bucket gets disturbed, while the water-filled bucket remains in the same state. This is due to the greater inertia of the water-filled bucket because of greater mass. It exhibits greater resistance and remains at rest.
- ii) If a solid iron ball and a hollow rubber ball, both of the same size, are rolling towards you, you will find it easier to stop the hollow one. This is due to its lower mass and less inertia.

Inertia has the following types:

Inertia of rest: When the bus driver suddenly accelerates the bus, the passengers in the bus lean backward. This is because they possess the inertia of rest.

Inertia of motion: When a moving bus suddenly stops, passengers lean forward due to the inertia of motion.

Inertia of direction: A moving bus, when suddenly turns left or right, the passengers lean right or left respectively. It's because of the inertia of direction, i.e. their resistance to change in direction.

1.1.3 Linear Momentum and Impulse:

Linear Momentum: *It describes the motion of a body and is quantified by the product of its mass and velocity. The formula for momentum is given by:*

Momentum = mass $\times$ velocity

Or, $p = mv$

Clearly, change in momentum of a body is due to the change in velocity. A change in velocity, in turn, implies acceleration. A body of mass m hence has some force F acted upon it, and hence the change in momentum is the force according to Newton.

Impulse:

Do the activities that follow.

- i) You slapped a baby's face.
- ii) You slipped on a road and
- iii) Two vehicles suddenly collided.

Consider the common factors among these three events. What result would follow after these occurrences?

Certainly, the result of these three events is what we know as impulse. In all of these events, two common factors are involved: a significant force and a short duration. None of these actions—slapping, slipping, or collision—lasts for an extended period. *The effect of a significant force applied for a short duration of time on a body is known as impulse. Mathematically, impulse is calculated as the product of force and time, expressed as $F \cdot \Delta t$*
i.e., impulse = $F \cdot \Delta t$

According to Newton's second law of motion, force is defined as the rate of change in the momentum.

$$\text{i.e., } F = \frac{mv - mu}{\Delta t} = \frac{m(v - u)}{\Delta t}$$

$$\text{Or, } F \cdot \Delta t = mv - mu$$

$\therefore$ Impulse = momentum change.

Hence, impulse results in a change in the momentum of a body.

-When a running man slips, he can't continue his motion, leading to a change in momentum. Similarly, when a ball is struck, it exhibits a change in motion, which corresponds to a change in its momentum.

-Likewise, when two cars collide, they alter their motion, resulting in a change in momentum.

Hence the effect of the impulse is to bring a change in the momentum. *The fact that impulse is equal to the change in momentum is called as **impulse- momentum theorem**.*

More Examples:

While catching a falling ball, one lowers his hand, which increases the time of impact, thereby reducing the force to minimize injury.

i) *If we fall on a rock or a cemented floor, we come to an immediate stop, thereby increasing the force of impact, which causes more harm.*

ii) *China wares and glass wares are wrapped in paper to increase the time of impact, thereby reducing the force and protecting the items.*

Sample Example:

1. A ball of mass 0.500 kg with a speed of 15.0 ms⁻¹ collides with a wall and bounces back with a speed of 10.5 ms⁻¹. If the motion is in a straight line, calculate the initial and final momenta and impulse. If the ball exerted an average force of 1000 N on the wall, how long did the collision last?

Solution: Here, mass, $m = 0.500$ kg,

Initial speed, $u = -15.0$ ms⁻¹

Final speed, $v = 15$ ms⁻¹ (opposite to u)

Force, $F = 1000$ N,

Time for collision, $\Delta t = ?$

We have, initial momentum, $mu = 0.500 \times 15 = -7.5$ ms⁻¹

Final momentum, $mv = 0.500 \times (10.5) = 5.25$ ms⁻¹

$\therefore$ Momentum change = $mv - mu$

$$= 5.25 - (-7.5)$$

$$= 12.75 \text{ kg ms}^{-1} = \text{Impulse (Impulse- Momentum theory)}$$

Again, from the impulse – momentum theorem,

$$\begin{aligned}
 F \cdot \Delta t = mv - mu &\Rightarrow \Delta t = \frac{mv - mu}{F} \\
 &= \frac{12.75}{1000} \\
 &= 0.013 \text{ s}
 \end{aligned}$$

1.2 Principle of Conservation of Linear Momentum

In the realm of dynamics, the principle of conservation of momentum asserts that within a system comprising two colliding bodies, the aggregate momentum remains constant provided no external forces exert their influence on the system. This fundamental concept states that the combined momentum of two bodies before a collision equals the **total momentum after the collision**. This conservation of momentum arises from the absence of external forces, ensuring that initial and final momentum values remain identical.

Statement: For a system of two colliding bodies, the total momentum before the collision is equal to the total momentum after the collision if no external force acts on the system.

Let us consider two bodies X and Y of masses m_1 and m_2 moving initially with velocities u_1 and u_2 respectively in a direction such that $u_1 > u_2$.

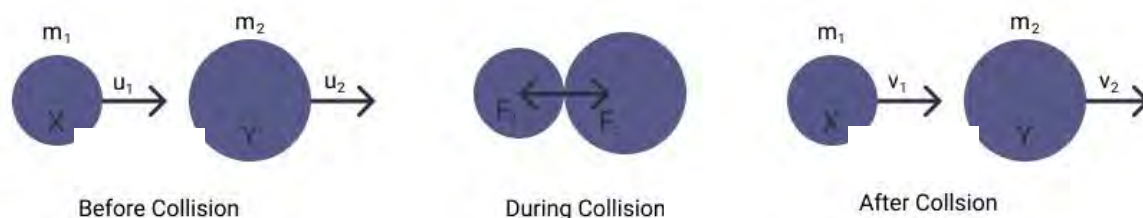


Fig 4.1: one dimensional collision between two bodies

As $u_1 > u_2$, they undergo a collision for a short time interval Δt . During the collision, let F_1 and F_2 be the forces exerted by the first and the second bodies respectively on each other. Finally, the bodies will move with velocities v_1 and v_2 respectively in the same direction such that $v_2 > v_1$.

Since force is the rate of change of momentum, $F = \frac{mv - mu}{\Delta t} = \frac{m(v - u)}{\Delta t}$

$$F_1 = \frac{m_2 v_2 - m_2 u_2}{\Delta t}$$

i.e, force by the first body is the rate of change of momentum of the second body.

Similarly, $F_2 = \frac{m_1 v_1 - m_1 u_1}{\Delta t}$ is the rate of change of momentum of the first body by the second body.

By the third law, $F_1 = -F_2$,

$$\text{Therefore, } \frac{m_2 v_2 - m_2 u_2}{\Delta t} = \frac{m_1 v_1 - m_1 u_1}{\Delta t}$$

$$m_2 v_2 - m_2 u_2 = -(m_1 v_1 - m_1 u_1)$$

$$\therefore m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

i.e, total momentum before collision = total momentum after collision

Hence the total momentum of an isolated system remains conserved and the principle on conservation of linear momentum

Worked Out Examples:

1. A ball A of mass 0.1 kg moving with a velocity of 6 ms^{-1} collides directly with a ball B of mass 0.2 kg at rest. Calculate their common velocity if both balls move off together. If A had rebounded with a velocity of 2 ms^{-1} in the opposite direction after collision, what would be the new velocity of B?

Solution: Here, mass of ball A, $m_A = 0.1 \text{ kg}$ mass of ball B, $m_B = 0.2 \text{ kg}$

Velocity, $u_A = 6 \text{ ms}^{-1}$ velocity, $u_B = 0$

After collision, $v_A = v_B = v$

We have, from the principle of conservation of linear momentum,

$$\begin{aligned} m_A u_A + m_B u_B &= (m_A + m_B)v \Rightarrow v = \frac{(m_A u_A + m_B u_B)}{(m_A + m_B)} \\ &= \frac{(0.1 \times 6 + 0.2 \times 0)}{(0.1 + 0.2)} = 2 \text{ ms}^{-1} \end{aligned}$$

In the second case,

$v_A = -2 \text{ ms}^{-1}$, $v_B = ?$

We write,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$= (0.1 \times 6 + 0.2 \times 0) = 0.1 \times (-2) + 0.2 v_B \Rightarrow v_B = 4 \text{ ms}^{-1}.$$

2. A ball of mass 0.05 kg strikes a smooth wall normally 4 times in 2 seconds with a velocity of 10 ms^{-1} . Calculate the average force on the wall.

Solution: Here, mass, $m = 0.05 \text{ kg}$

Velocity, $v = 10 \text{ ms}^{-1}$ = returning velocity, v'

Time, $t = 2 \text{ s}$

Force on the wall, $F = ?$

$$\begin{aligned} \text{We have, } F &= \text{rate of change of momentum in 4 times} = 4 \times m \frac{(v - v')}{t} \\ &= 4 \times 0.05 [10 - (-10)] / 2 \\ &= 2 \text{ N} \end{aligned}$$

3. The mass of a gas emitted from the rear of a toy rocket is initially 0.1 kg . If the speed of the gas relative to the rocket is 50 ms^{-1} and the mass of the rocket is 2 kg , what is the initial acceleration of the rocket?

Solution:

Here rate of mass = 0.1 kg/s

Mass of the rocket, $m = 2 \text{ kg}$

Relative velocity = $v - 0$ = velocity change, the rocket is supposed to be at rest

The force on the rocket, F = rate of change of momentum of the gas emitted

$$= 0.1(v - 0)/1$$

$$= 0.1 \times 50 = 5 \text{ N}$$

$$\text{Now, } a = F/m = 5/2 = 2.5 \text{ ms}^{-2}$$

1.3 Newton's Laws of Motion

In the exploration of particle motion, Sir Isaac Newton meticulously employed the scientific method to derive three foundational principles, famously recognized as Newton's Laws of Motion. These laws serve as the bedrock of classical mechanics, encapsulating the fundamental dynamics governing the behavior of objects in motion. The trio comprises the First Law of Motion, which delineates the inertia-based nature of objects; the Second Law of

Motion, explicating the relationship between force, mass, and acceleration; and the Third Law of Motion, illuminating the reciprocal action and reaction between interacting bodies. These laws collectively form an interconnected framework, providing a comprehensive understanding of motion and serving as a cornerstone in the study of classical mechanics.

4.3.1 Newton's First Law of Motion:

Force is involved during changes in the state of your body. Unless there is a change in your state, no force is necessary, and you remain intact. In other words, force is required to change direction, come to rest, start motion, accelerate, or decelerate. If no force is applied during such changes, you remain in your original state.

Sir Isaac Newton studied these phenomena in the motion of bodies and formulated a law known as Newton's first law of motion.

Statement: *Everybody remains in its own state unless an external force is acted upon.*

According to it, a body at rest will remain at rest, and a body in motion will continue moving with the same speed along a straight line unless acted upon by an external force. To change from rest to motion, or vice versa, to accelerate, decelerate, or change direction, an external agency, called force, is required.

If the force isn't acted, the body remains in its own state i.e, either at rest or in motion in a straight line.

Since the force appears as a role model to change the state of the body in the first law, the first law hence defines the force.

The law is also known as the law of inertia because a body remains inert unless acted upon by a force. This property exhibited by the body is called inertia, hence the term 'law of inertia'.

Examples:

- i) *When a bus at rest suddenly begins to move, the passengers lean backwards. According to the first law of motion, a force is exerted on the bus, causing it to move forward. However, the upper part of the passengers, not directly affected by the force, exhibit the property of inertia of rest. As a result, the passengers lean backwards.*
- ii) *When a moving bus suddenly stops, the passengers lean forward due to the inertia of motion, or as per the first law, a force acts on the bus causing it to come to a stop with the lower parts of the passengers attached to their seats. However, due to the absence of a force acting on the passengers directly, they tend to remain in motion and thus lean backward.*
- iii) *When a moving bus suddenly turns into a direction, the passengers lean in the opposite direction. This is due to the inertia of direction, or according to the first law, because no force is acting directly on the passengers. As the passengers continue their motion in the same straight line, they oppose the change in direction and consequently lean in the opposite direction.*
- iv) *Mangoes fall on shaking the mango tree.*
- v) *One takes a short run before jumping.*

- vi) *If in an emergency, we need to jump off a moving bus while jumping off, an external force is applied to our legs (or feet), causing them to reach the targeted point, while no force is exerted on the upper part of the body. Consequently, the upper part of the body, continuing with the motion of the bus, tends to fall in the direction of the bus. Therefore, for a safe jump, one should jump in the direction of the moving bus.*
- vii) *A moving fan goes on slowing down and finally stops. It doesn't violate the first law.*
- viii) *A rolling ball on the ground goes on slowing down and finally stops. It also doesn't violate the first law.*

4.3.2 Newton's second law of motion

Activity 4.2

Do and think about the following activities.

- i) Give a push or a pull to a man walking along the road and observe what happens to him. Determine the change in his momentum.
- ii) Apply more force to the same activity and observe how the motion changes. Determine the change in momentum here as well.
- iii) Consider how the momentum changes if you suddenly fall on the road while walking.
- iv) Consider or experience whether momentum changes when someone slaps you.
- v) Consider the similar change when two vehicles suddenly collide.

In all these activities, there is a change in momentum. The change is greater and faster if a larger force is applied, and the change occurs in the direction of the force applied. In other words, the greater the mass of a body, the larger the force required to produce a change in momentum. If a larger force is applied, the resulting acceleration is also greater. Through the study of such phenomena using scientific facts, Newton formulated a new law known as **Newton's second law of motion.**

Statement:

The rate of change of momentum is directly proportional to the applied force and the change takes the direction of the force.

Let a body of mass m moving with the velocity u attains a velocity v due to the force F applied in a time interval t .

$$\text{Hence rate of change of momentum} = \frac{mv - mu}{t}$$

$$\text{Now } \frac{mv - mu}{t} \propto F$$

$$\text{Or, } \frac{m\vec{v} - m\vec{u}}{t} \propto F$$

$$\text{OR, } F \propto \frac{m(v - u)}{t}$$

$$\text{OR, } F \propto ma$$

$$\therefore F = kma \dots \dots \dots (4)$$

To accelerate a body of mass 1 kg through 1 ms^{-2}

Hence the equation implies that, we need 1 N force by definition.

$$1 = k.1.1$$

$$\Rightarrow k = 1$$

On recalling the same equation above,

$$\boxed{F = ma}$$

It's the second law of motion.

If we know the mass of a body and its acceleration, we can evaluate the force. Hence the second law gives the **measurement of the force**.

Worked Out Examples:

1. A 6 kg object is under the action of the three forces $\vec{F}_1 = 20\hat{i} + 30\hat{j}$, $\vec{F}_2 = 8\hat{i} - 50\hat{j}$ and $\vec{F}_3 = 2\hat{i} + 2\hat{j}$. Find the acceleration of the object.

Solution: Here net force is the algebraic sum of all the three forces.

$$\text{Therefore, } \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\text{And from } \vec{a} = \vec{F}/m = 5\hat{i} - 3\hat{j}$$

2. A block of mass 3 kg is in contact with a second block of mass 2 kg, both resting on a frictionless floor. A horizontal force $F = 10\text{N}$ is applied to push the first block. Find the force with which the first block pushes the second block.

Solution: Since two bodies are in touch, 10N force applied goes to push both the blocks.

Therefore, $a = F/m \Rightarrow 10/(3+2) = 2 \text{ ms}^{-1}$. Now the pushing force to the second block
 $= m \times a = 2 \times 2 = 4\text{N}$

3. A car of mass 1000 kg is accelerating at 2 ms^{-1} . What resultant force acts on the car? If the resistance to the motion is 1000N, what is the force exerted by the engine?

Solution:

Here, mass, $m = 1000 \text{ kg}$

Acceleration, $a = 2 \text{ ms}^{-2}$

$\therefore$ Resultant force, $F = ma = 1000 \times 2 = 2000\text{N}$

Resistance, $F_R = 1000\text{N}$

Force due to engine, $F' = ?$

We have, $F' - F_R = ma \Rightarrow F' = 1000 \times 2 + 1000 = 3000\text{N}$

4. A lift moves (i) up and (ii) down with an acceleration of 2 ms^{-2} . In each case, calculate the reaction of the floor on a man of mass 50 kg standing on the lift.

Solution:

Here, mass, $m = 50 \text{ kg}$

Acceleration, $a = 2 \text{ ms}^{-2}$ (The man has the same acceleration as the lift because they move together.)

(i) Up $\Rightarrow T > mg \Rightarrow T - mg = \text{net force, } F$

[Note that, it is the reaction force of the floor that makes the man accelerate upwards, not the force applied on the lift.)

$$\therefore T = mg + ma = m(g+a) = 50 \times 12 = 600\text{N}$$

$$(ii) \text{Down} \Rightarrow T < mg \Rightarrow mg - T = ma$$

$$\therefore T = m(g-a) = 50 \times 8 = 400\text{N}$$

4.3.3 Newton's Third Law of Motion:

Do and feel the following activities and phenomena:

- i) Go for a swim and observe how you propel yourself forward by pushing water with your hands or legs in one direction while moving forward in the opposite direction.
- ii) Go for a jump and observe how you push the ground backward while propelling yourself upwards, resulting in two types of motion: the backward motion of the ground and your forward movement.
- iii) When firing a gun, you support it on your shoulder and lean slightly forward. As a result, you experience a backward force during the recoil of the bullet.
- iv) You feel more hurt to go for a day-long walk along a pitched road rather than along the grassy runway.
- v) You feel safer sitting on a sofa rather than on a wooden bench. You get severely hurt while beating the cemented wall rather than a soft wall.
- vi) In all these outlined tasks, two forces arise in opposite directions and always occur in pairs. The backward force initiated first in each task, such as throwing water, recoil force on the body during firing, pushing back the ground on jumping and walking, and pressing backward against benches, are the action forces. The forward-oriented forces in motion are the reaction forces. The pitched road and the cemented walls exert sufficient reaction forces that cause more severe injury.

From the study of such activities, Newton again formulated a new law which is known as the **Newton's third law of motion**.

Statement: *To every action, there is always an equal and opposite reaction.*

Newton's third law states that when two bodies interact, they apply forces to one another that are equal in magnitude and opposite in direction. The third law is also known as the law of action and reaction.

In the fig, mg is the downward weight of the load on the table and R is the reaction force.

By the third law, $R = -mg$

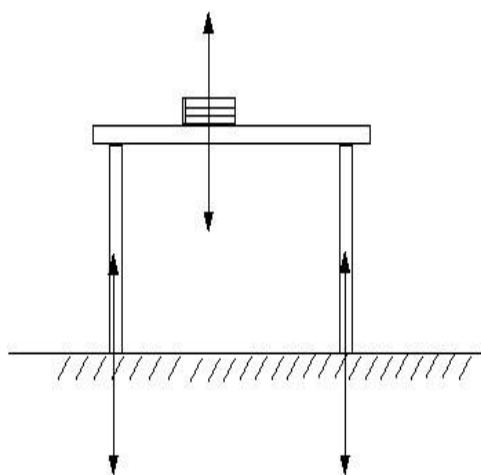


Fig4.2: Showing the Third Law

In the third law, action generally occurs in backwards motion while reaction proceeds onwards.

Action is given to a body and the body in turn provides the reaction to the action giving body.

Hence the action and reaction act on two different bodies and they never cancel.

Q: Action and reaction forces are equal and opposite but they never cancel, why?

Let's do the following tasks:

- i) Have you ever done boating? Go for boating and drive the boat yourself or watch carefully how it is being driven. You throw the water backwards and the boat is seen marching onwards. Think what's being over there.
- ii) Have you ever watched the flying rocket in YouTube? A heavy mass of the burnt gas is seen coming out and the rocket heading to the opposite direction.
- iii) Have you ever fired a gun, or seen someone firing one? Take a gun and fire it as a test. When you fire it, the bullet goes away, while simultaneously you experience a backward push (recoil) due to the reaction force.
- iv) Consider jumping. When you jump, you first push against the ground, and then you propel yourself forward.

The discovery of a rocket engine exemplifies Newton's third law of motion. The action occurs as the engine expels gaseous fuel backward, while the reaction is observed as the rocket moves forward.

The recoil of a gun represents the action, while the forward journey of the bullet serves as the reaction.

The action of pushing against the ground initiates the jumping motion, while the forward movement is a result of the reaction.

Things to Consider:

Newton's second law is the real law of motion. Why?

The second law is written as $F = ma = \frac{mv - mu}{t}$

If no force is applied, $F = 0$

$$\therefore \frac{m(v-u)}{t} = 0$$

Since $m \neq 0$, $v - u = 0 \Rightarrow v = u$

i.e the bodies remain in their own state unless the force is acted upon them.

If force is applied, $F \neq 0$,

$$\therefore \frac{m(v-u)}{t} \neq 0 \Rightarrow v \neq u.$$

i.e, the bodies change the state when force is applied.

Hence the first law is deduced from the second law, and hence the second law is the real law of motion.

4.4 Applications of Newton's Laws of Motion

Horse- Cart System:

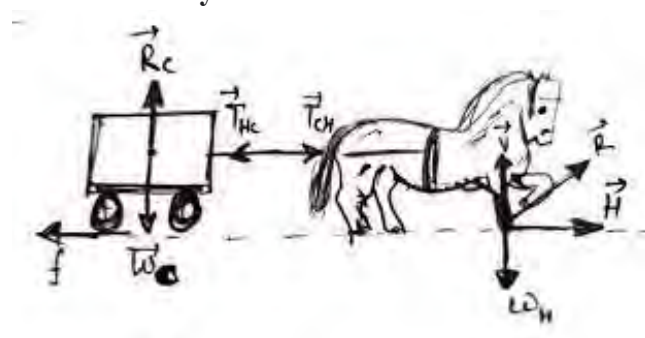


Fig4.3 An Illustration of the 2nd Law (Horse- Cart System)

When a horse pulls a cart, its legs press the ground in an inclined direction opposite to which there is a reaction R . The vertical component of R balances the weight of the horse while the forward horizontal component H acts against the tension on the rope pulling the cart.

Since the system moves forward, net force = $H - T_{HC}$

$$\Rightarrow H - T_{HC} = ma \dots \dots \dots (4.1)$$

For the cart,

Let T_{CH} be the tension on the rope that the horse pulls the cart and f be the frictional force on the wheel of the cart acting oppositely. Since the system is moving in the direction of T , net force = $T_{CH} - f$.

$$\text{Therefore, for the mass } m' \text{ of the cart, } T_{CH} - f = m'a \dots \dots \dots (4.2)$$

Here, the weight w of the cart is balanced by the normal reaction R_C on the cart.

For the combined system, T_{CH} is cancelled by T_{HC} .

Therefore, net horizontal force, $F = H - f$

$$\text{Or, } H - f = (m + m') a$$

$$\Rightarrow a = \frac{H - f}{m + m'}$$

Alternatively,

If H is the force exerted by the horse on the whole system, and f is the horizontal frictional force directed oppositely to H , the net force on the system is $H - f$ i.e, net force, $F = H - f$.

If m and m' be the masses of the horse and the cart, then $F = (m + m') a$.

Therefore, for the entire system, $H - f = (m + m') a$.

$$\Rightarrow a = \frac{H - f}{m + m'}$$

When a horse pulls a cart, the force helping the horse to move is provided by the ground on the horse as per the third law.

Pulley and Mass System

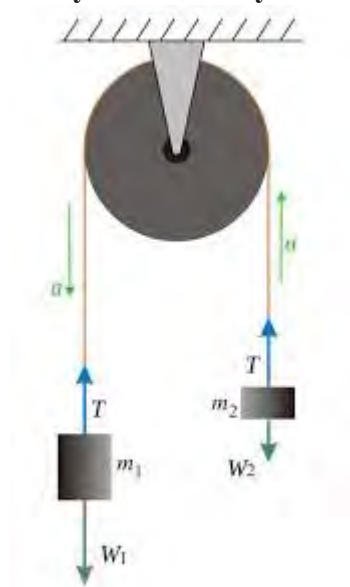


Fig 4.4: Pulley –Mass System (Application to the 2nd Law)

The pulley- mass system has been outlined in the fig.4.4.

Let $m_1 > m_2$ and for the mass m_1 , net force = $W_1 - T$

Therefore, $W_1 - T = \text{net force} = m_1 a$ (4.3)

And for the mass m_2 , net force = $T - W_2$

Therefore, $T - W_2 = m_2 a$ (4.4)

On adding the two equations, $W_1 - W_2 = (m_1 + m_2) a$

$$\Rightarrow a = \frac{W_1 - W_2}{m_1 + m_2}$$

Alternatively,

For $m_1 > m_2$, $W_1 > W_2$ and the net motion is towards the heavier weight, W_1 .

Therefore, $W_1 - W_2 = (m_1 + m_2) a$

$$\Rightarrow a = \frac{W_1 - W_2}{m_1 + m_2}, \text{ where the two } T\text{s cancel.}$$

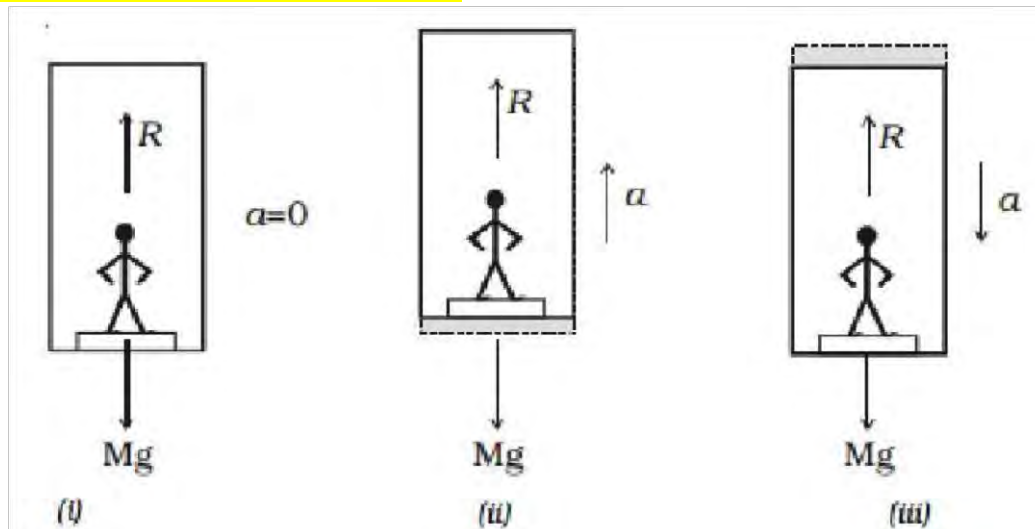


Fig 4.5: Motion in the Lift

For a man on a lift moving up and down, let 'Mg' be the downward weight of the man and R be the upward directed force.

Followings are the cases that arise:

i) **For the uniform speed of the lift, $a = 0$**

For the uniform speed, there is no acceleration and hence $R - mg = ma = 0$.

$$\Rightarrow R = mg$$

ii) **For the lift accelerating up:**

In this case $R > mg$.

Therefore, net force, $F = R - mg$

$$\text{OR, } R - mg = ma$$

$$\Rightarrow R = m(g + a)$$

iii) **For the lift accelerating down:**

In this case, $R < mg$

Therefore, net force, $F = mg - R$

$$\text{OR, } mg - R = ma$$

$$\therefore R = m(g - a)$$

iv) **Free fall of the lift**

For this $a = g$. For the falling motion, $mg - R = F$

$$\text{But } F = ma = mg$$

$$\text{Therefore, } mg - R = mg$$

$$\Rightarrow R = 0$$

v) **When the downward acceleration is greater than g i.e, $a > g$,**

Here, for the downward motion, $mg > R$.

Therefore, net force, $F = mg - R$

$$\text{Or, } ma = mg - R$$

$$\text{Or, } R = m(g - a).$$

Since $a > g$, R becomes negative.

In this case the man appears oneself moving in the upward direction sticking on the ceiling of the lift.

4.5 Moment, Torque and Equilibrium

Activity 4.3

Do the following tasks:

i) Close or open the door in your home and remember the point on the door your hand holds. You may hold the door at the middle section or at a point on the opposite side.

ii) Overturn a huge load or a massive stone either by your hand or by using a long bar.

Q i. Which option do you prefer for handling the door: at the middle section or a point on the side?

Q ii. Which method do you find easier for overturning the load: using your hand alone or using a bar?

We find it easier to push the door by holding it on the side rather than in the middle part. Similarly, we find it easier to overturn the load using a bar (rod).

In either case, the object rotates about a fixed axis, known as the axis of rotation.

The product of the force and the perpendicular distance of the line of action of the force from the axis of rotation is called the moment of force or torque. The effect of torque is to rotate a body around an axis.

$$\therefore \text{Torque } (\tau) = r F$$

In vector form, $\vec{\tau} = \vec{r} \times \vec{F}$. The higher the value of r , the less is the force required. Hence, it is easier to move the door holding it on a side.

Clockwise and Anticlockwise Moment

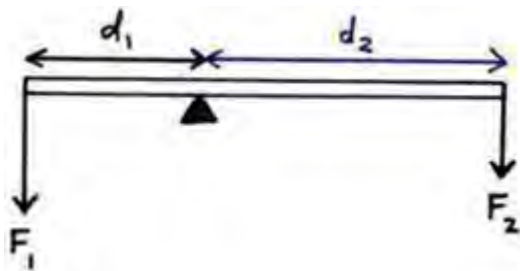


Fig 4.6: (a) Showing Principle of Moment

For a given rod pivoted at the middle section, force F_1 seeks to rotate it anticlockwise, while force F_2 seeks to rotate it clockwise.

Hence, the moment of the force F_1 is called the anticlockwise moment. Likewise, the moment of the force F_2 is called the clockwise moment.

$$\therefore F_1 \times r_1 \rightarrow \text{anticlockwise moment and } F_2 \times r_2 \rightarrow \text{clockwise moment}$$

Principle of Moment

Let a rod be in equilibrium about a point O under the action of a number of forces $F_1, F_2, F_3, \dots, F_5$.

Let F_1 & F_2 provide anticlockwise moments while F_3, F_4 and F_5 produce clockwise moments.

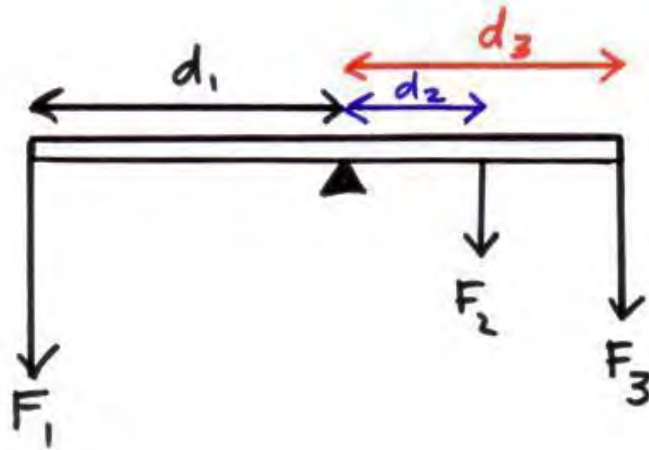


Fig 4.6: (b) Showing Principle of Moment

Therefore, sum of anticlockwise moment = $F_1 \times OA + F_2 \times OB$

And, the sum of the clockwise moment = $F_3 \times OC + F_4 \times OD + F_5 \times OE$

In the equilibrium, it is found that

$$F_1 \times OA + F_2 \times OB = F_3 \times OC + F_4 \times OD + F_5 \times OE$$

i.e., *sum of the anticlockwise moments = the sum of the clockwise moments.*

It is the principle of the moment.

Hence, in a body in *equilibrium* under the action of multiple forces, the sum of clockwise moments equals the sum of anticlockwise moments.

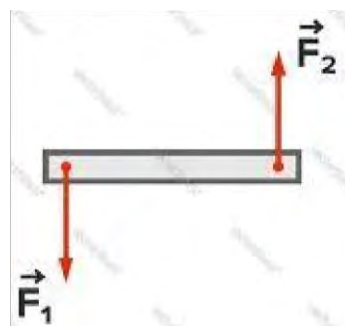
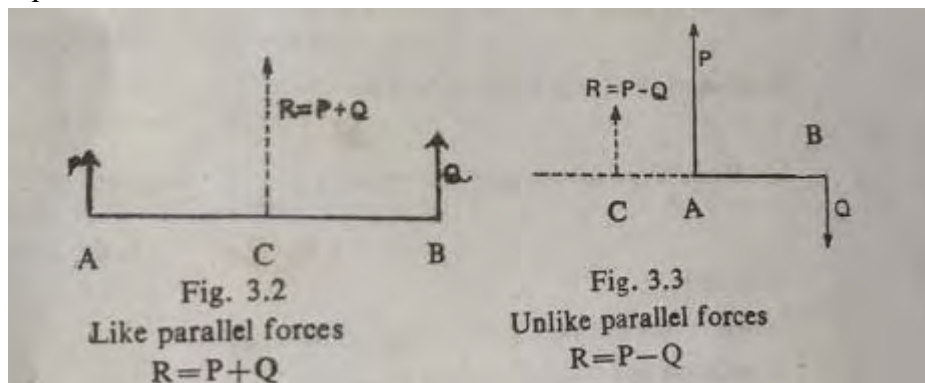


Fig 4.6: (c) Showing the principle of torque

Worked Out Examples:

1. A rectangular plate ABCD has two forces of 100N acting along BC and DA in opposite directions. If $AB = 5\text{ m}$, $BC = 3\text{ m}$, what is the moment of the couple or torque acting on the plate? What forces, acting along CD and AB respectively, are required to keep the plate in equilibrium?

Solution:

Here, two parallel and opposite forces each of 100N act at $BC = r = 5\text{ m}$ apart.

Therefore, moment of couple (or torque) = $F \times r = 100 \times 5 = 500\text{ Nm}$

Also, let F be the force for which, $F \times BC = 100 \times AB \Rightarrow F = 500/3 = 166.7\text{ N}$

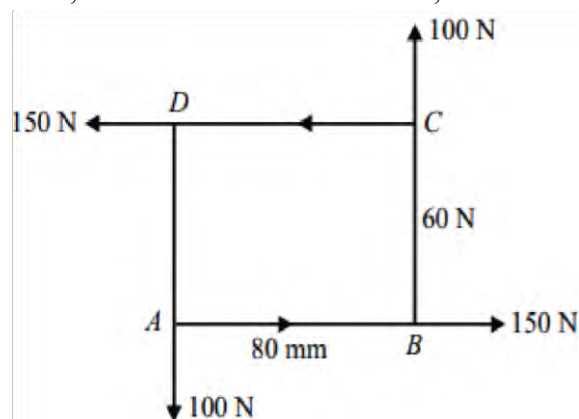


Fig 4.7(a)

4.5.1 Torque of a Couple

Two equal and opposite forces acting on a body with non-coincidence of their lines of action are called a couple. The effect of the couple is to generate rotational motion on the body. In the fig, F, F constitutes a couple with anticlockwise rotation on the rod AB.

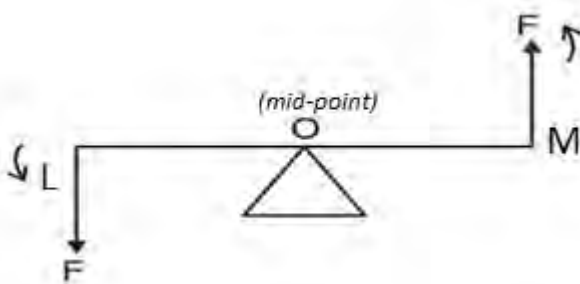


Fig 4.7: (b) Torque due to a couple

Here, moment of the couple = sum of the moment of the two forces.

$$= F \times OL + F \times OM$$

$$= F \times (OL + OM) = F \times LM$$

Hence the moment of a couple is the product of the either force times perpendicular distance between the lines of action of the forces.

Examples:

Turning a screw, turning a tap, turning a screw driver and compass needle in the Earth's magnetic field.

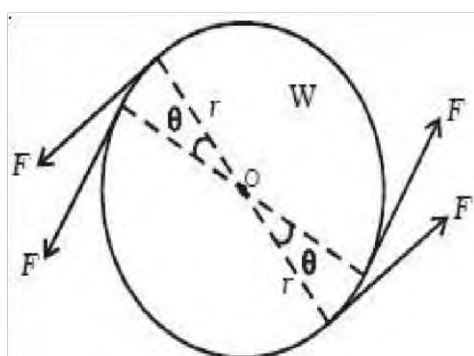
1.5.2 Work Done by a Couple

Let F, F be the two forces acting tangentially at two opposite ends of a diameter of a circle of radius r . The wheel now rotates through an angle θ .

Therefore, the work done by each force, $w = \text{Force} \times \text{distance}$
 $= F \times r\theta$

The total work done by both the forces $= 2Fr\theta$
 $= F \times 2r \times \theta = \text{Torque} \times \theta = \tau\theta$

Finally, the work done by the couple $= \text{torque} \times \text{angular displacement}$.



1.5.3 Equilibrium *Fig 4.8: Work done by a couple*

If a body under the action of forces is at rest, the body is said to be in equilibrium.

Based on the nature of the motion, the body can be in either translational equilibrium or rotational equilibrium.

For a body to be in translational equilibrium, the resultant forces acting on the body should be zero, i.e., $\sum F = 0$.

This term means that a body is not accelerating in any direction; it is either at rest or moving with a constant velocity. In other words, there is no net change in its state of motion.

The resultant force is the vector sum of all the individual forces acting on the body. If the resultant force is zero, it means all the forces balance each other out.

Similarly, for a body in rotational equilibrium, the sum of the torques acting on the body must be zero, i.e., $\sum \tau = 0$.

This means that the clockwise and counterclockwise torques balance each other out, resulting in no net rotational motion.

A group of forces that all act at the same point are called concurrent forces. If the resultant of these concurrent forces acting at a point on a body is zero, the body is said to be in equilibrium.

4.5.3 Equilibrium of the Rigid Bodies.

In a rigid body, the position of the two constituent points remains unaltered throughout the motion.

This intrinsic quality of rigidity implies that despite the body's dynamic movement or displacement, the spatial relationship between these two points remains constant and unaltered. This coherence in position preservation throughout the motion underscores the rigidity of the body, emphasizing its resistance to deformation or distortion. As the body navigates its trajectory, the unchanging relative positions of these points serve as a testament to the enduring stability inherent in rigid structures, contributing to the fundamental understanding of their mechanical behavior.

For the rigid body to be in equilibrium, the following are the conditions:

- iv) The sum of all the forces (vector sum) acting on a body must be zero i.e., $\sum F = 0$.

But $F = ma \Rightarrow ma = 0 \Rightarrow a = 0$ since $m \neq 0$.

$$\therefore a = 0 \Rightarrow \frac{dv}{dt} = 0 \Rightarrow v = \text{constant}$$

Hence, the constant value of v yields zero acceleration, which is the condition of equilibrium, and the body is said to be in translational equilibrium. To elaborate further, for a body to be in translational equilibrium, it must either be at rest or in motion with constant velocity.

- v) The resultant of the torques acting at a point on a body must be zero i.e., $\sum \tau = 0$.

Now $\tau = I\alpha$ and $\sum \tau = 0 \Rightarrow I\alpha = 0$

$$\text{Or, } \alpha = 0 \Rightarrow \frac{d\omega}{dt} = 0 \Rightarrow \omega = \text{constant}$$

Here, the constant value of ω implies for a body to be in rotational equilibrium, it must either be at rest or in motion with constant angular velocity.

1.6 Solid Friction

The concept of friction arises when two solids come into contact and relative motion ensues, resulting in the emergence of a counteracting force. This natural resistance to motion between solid surfaces is aptly termed friction. Within this phenomenon, the force responsible for opposing the motion is designated as the frictional force.

4.6.1 Laws of Solid Friction and their Verification:

Introduction:

Observe the following phenomena:

i) You can easily walk along the pitched road while you find it hard to walk along a muddy way.

ii) You easily write it down on paper but it feels difficult on a piece of plastic.

iii) You can easily hold a nut on a wall.

iv) A moving fan finally stops after the electric switch is off, disobeying Newton's first law.

v) A rolling ball on a ground finally stops disobeying the first law.

Considering the events occurring above, there is some phenomenon in each activity that opposes our motion called friction.

The phenomenon of arising an opposing force against the motion between two solids in touch is called friction. The force on the friction is called frictional force.

vi) It acts against the motion.

vii) It opposes the relative motion between the solids.

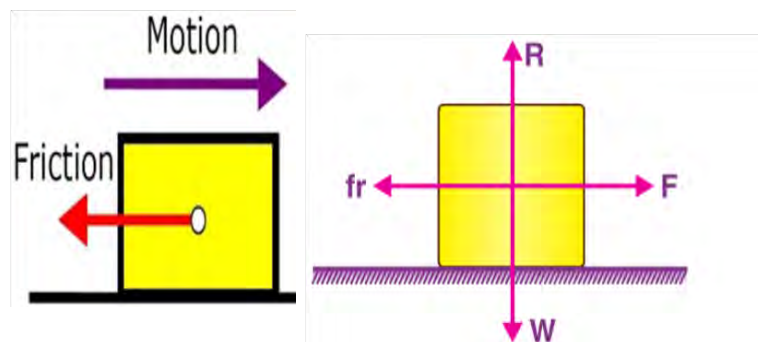


Fig 4.9 (a): Showing Frictional Phenomena

Cause of Friction

In the intricate interplay of interacting surfaces, frictional force materializes as a consequence of the microscopic interactions between atoms and molecules at the point of contact. This force acts tangentially to the direction of motion, creating a resistance that impedes the smooth glide of one surface over the other.

The cause of how friction arises is described in the following two ways:

- viii) **Modern Approach:** The phenomenon of adhesion between two solid surfaces due to the molecular attraction of the molecules present on the solid surface is the cause of arising friction.

4.7 Types of Friction:

Static and Kinetic Friction

Activity 4.4

Do the activities that follow:

Place a heavy mass on a horizontal surface.

ix) Try to slowly pull or push the object.

x) Seek assistance from your friend and attempt the same on the object; however, you still observe no movement in the object.

xi) Continue by adding more friends—one, two, three, and so on—and have them participate in the same activity. Eventually, the object reaches a state where it begins to move over the ground

xii) After moving, remove one, two, or more friends, but the body continues to move.

- Why were so many people involved in moving the body?

- Why does the motion continue even after removing one or two people once the body is in motion?

Before initiating motion, there exists the frictional force between the body and the ground surface. This type of friction, occurring between the bodies before they come into motion, is referred to as static friction

More clearly, to say, static friction is the friction between the solids in contact but at rest. As we continue to apply additional force to pull the body, the cumulative force endeavors to overcome static friction, allowing the body to initiate movement over the surface. The maximum value of the frictional force a body just begins to move over the other body is called limiting friction. This force reaches its maximum value.

After the body is in motion, there is no need of such a great force as the body has been set into motion from the rest, and the body continues its motion even with a lesser force. Hence the friction between two solids during the motion is called kinetic (or dynamic) friction.

And, Static friction > Kinetic friction.

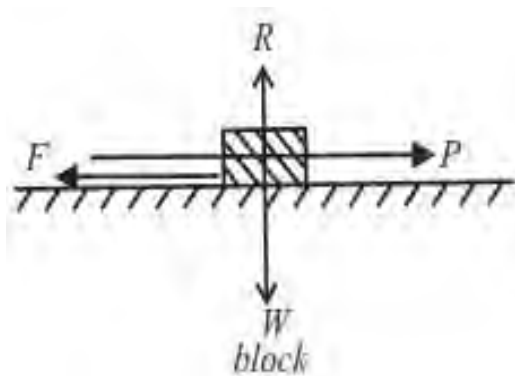


Fig 4.9 (b): Showing Frictional Phenomena

Sliding and Rolling Friction

The phenomenon of arising a counteracting force when a body is sliding over another solid surface is called sliding friction.

In the same way, the phenomenon of arising a resisting force between two solids when one body is rolling over the surface is called rolling friction.

These distinct forms of friction elucidate the dynamic interactions between surfaces in different modes of motion, offering insights into the diverse forces at play in the realm of mechanical interactions.

4.8 Law of Friction

Followings are the laws:

- i) The phenomenon of friction between two surfaces opposes the relative motion between the surfaces.
- ii) The friction depends up on the roughness of the surface.
- iii) The frictional force acts parallel to the surface in touch
- iv) The friction is independent on the area of the surface in touch.
- v) The kinetic friction is independent of the relative velocities of the surfaces.
- vi) The magnitude of the frictional force is directly proportional to the normal reaction i.e. $F \propto R$.

$$\therefore F = \mu R$$

Here, $\mu = F/R$ is called the coefficient of friction.

For static friction, $\mu_s = F/R$ and for kinetic (dynamic) friction, $\mu_k = F/R$

And $\mu_s > \mu_k$

Friction is a necessary evil. Justify your answer.

Friction is necessary to us for walking, putting a nail on a wall, writing on a piece of paper, driving and stopping a vehicle, and in so many phenomena. In every observed phenomenon, the presence of friction leads to energy loss, as it staunchly opposes the motion of objects and contributes to wear and tear. Consequently, the efficiency of the system diminishes, never reaching the ideal 100%. This consistent interplay between friction, energy dissipation, and reduced efficiency underscores the real-world challenges in achieving perfect mechanical performance across various processes and systems.

Hence, friction is necessary in so many events, but due to energy loss, it appears as an evil.

Some Facts:

- a. **To show limiting friction is directly proportional to the normal reaction ($F \propto R$):**

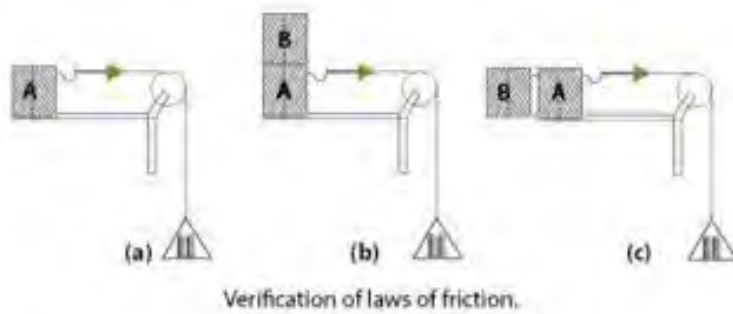


Fig 4.10: Showing Sliding Friction

Let a block be kept on a horizontal smooth surface. A long string is taken whose one end is attached to the block while the other end passes over a pulley system, and attaches to a pan. Keep adding weights to the pan until the block begins to move. Take another similar block and place it over the first block. Go on adding the weights on the pan till it moves. It is noted that the weights on the pan double those of the first block, and hence, one claims that the limiting friction is directly proportional to the normal reaction.

i.e. $F \propto R$.

Similar results will be obtained if we keep more blocks.

b. To show that limiting friction is independent of the area of contact:

Now, continue stacking the blocks one on top of the other. Estimate the load on the pan required to just make the blocks move, as shown in Figure 4.10 (b).

Next, arrange the two blocks one after another, as depicted in Figure 4.10 (c), and observe that the load required to just make the blocks move remains the same as in Figure 4.10 (b)

Hence the same weight on the pan for the blocks in the two cases indicates that the limiting friction is independent of the surface area.

4.9 Angle of Friction:

It is the angle made by the resultant of the normal reaction and the frictional force with the normal reaction.

In the fig, θ is the angle of friction.

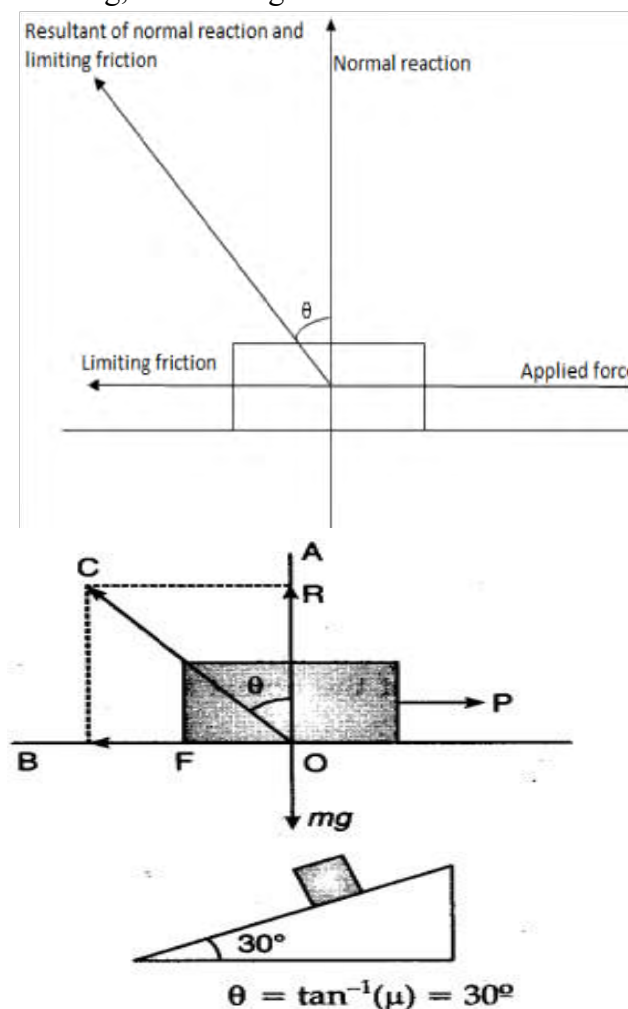


Fig 4.11: Showing angle of friction and angle of repose

Let $R = mg$ represent the normal reaction to a block of mass m , and let F be the frictional force opposing the motion. Therefore, $OC = R \sin \theta$ represents the resultant of R and F , making an angle θ with the vertical. By definition, θ is the angle of friction.

Now, from the fig 4.12;

$$\tan \theta = \frac{RC}{OA} = \frac{F}{R} = \mu$$

i.e., the tangent of the angle of friction is equal to the *coefficient of the friction*.

4.10 Angle of Repose

The angle of inclination that a solid kept on the inclined plane just begins to slide down is called the angle of repose.

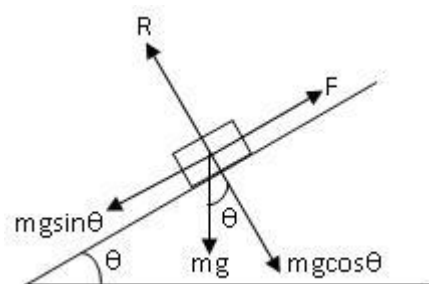


Fig 4.12: Showing angle of repose

In the fig, mg is the weight of the block of mass m placed on an inclined plane to which R is the normal reaction.

The normal reaction R is balanced by the component $mg \cos \theta$, while the component $mg \sin \theta$ down the inclined plane contributes to the block sliding down, which is balanced by the upward-directed frictional force F .

Therefore, we write-

$$F = mg \sin \theta$$

$$R = mg \cos \theta$$

$$\Rightarrow \frac{F}{R} = \tan \theta$$

$$\therefore \tan \theta = \mu$$

i.e. the tangent of the angle of repose is equal to the coefficient of friction.

Moreover, from the study of the angle of friction and angle of repose, it is found that;

Angle of friction = angle of repose i.e., $\alpha = \theta$.

Project Work:

Project 4. Determine the Surface Roughness Using Coefficient of Limiting Friction on an Inclined Plane

Materials: inclined plane (adjustable angle), various surfaces (e.g., wood, metal, plastic), a block (same material for consistency), protractor or inclinometer, weights (optional, for consistent normal force), and a measuring tape or a ruler.

Theory: Coeff. of friction, $\mu = \frac{F}{R} = \tan \theta$.

Procedure:

- i) Place the inclined plane on a stable surface.
- ii) Attach one of the surfaces to the inclined plane.
- iii) Place the block on the inclined plane

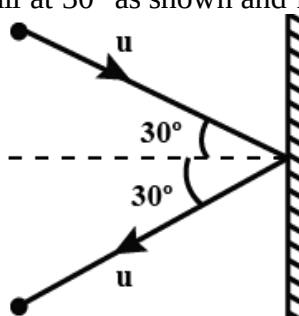
Gradually increase the angle of the inclined plane until the block just starts to slide. Find the angle θ at which the block begins to slide using the protractor or inclinometer and take the records for the angle in a *tabular form*. Repeat the above steps for each different surface. Finally, compare the values for different surfaces and draw your conclusion.

Exercise

1. What causes the mangoes fall on shaking the branch of a mango tree?
 - a. inertia of rest
 - b. inertia of motion
 - c. inertia of direction
 - d. effect of gravity
2. A body is moving in a constant speed along a straight path. In which case is no force required?
 - a. increase its speed
 - b. change its direction
 - c. keep it moving with uniform velocity
 - d. decrease its momentum
3. When two surfaces are coated with lubricant oil, which of the following cases will happen?
 - a. slide up on each other
 - b. stick to each other
 - c. roll up on each other
 - d. repel to each other.
4. What is the mass for a force $\vec{F} = 6\hat{i} + 8\hat{j}$ acting on a body that produces an acceleration of 1 ms^{-2} ?
 - a. 8 kg
 - b. 9 kg
 - c. 10 kg
 - d. 12 kg
5. Which is the direction of the fall for a person jumping from a running bus?
 - a. in the direction of the bus?
 - b. opposite of the direction of the bus
 - c. onwards at the right angle to the direction of the bus
 - d. at the point of touch by the leg
6. What is the acceleration for a 6 kg object under the influence of the three forces $\vec{F}_1 = 20\hat{i} + 30\hat{j}$, $\vec{F}_2 = 8\hat{i} - 50\hat{j}$ and $\vec{F}_3 = 2\hat{i} + 2\hat{j}$?
 - a. $3\hat{i} - 5\hat{j}$
 - b. $5\hat{i} - 3\hat{j}$
 - c. $5\hat{i} + 3\hat{j}$
 - d. $3\hat{i} + 5\hat{j}$

7. What force does a 40 kg man exert on a 50 kg man while the 50 kg man exerts a 40 N force on the 40 kg man?
- 40N
 - 60N
 - 50N
 - 100N
8. A force of 6 N acts on a body at rest with a mass of 1 kg. The body attains a velocity of 30 ms^{-1} . How much time did the force act on the body?
- 10 s
 - 5 s
 - 8 s
 - 6 s
9. Which one of the following is not a force?
- tension
 - impulse
 - thrust
 - weight
10. Which of the following is the correct option for the frictional force?
- conservative force
 - non-conservative force
 - path independent force
 - unnecessary force
11. A lift is moving up with an acceleration equal to $\frac{1}{5}$ of gravity. What is the apparent weight of a 60 kg man standing on the lift that is moving up with an acceleration equal to $\frac{1}{5}$ of gravity?
- 720N
 - 480N
 - 600N
 - 120N

12. What is the average force on the wall for the overall time t when a ball of mass m strikes a wall at 30° as shown and reflects with the same speed following the law



of reflection?

- $m \cos 30^\circ / t$
 - $m \sin 30^\circ / t$
 - $2m \cos 30^\circ / t$
 - $2m \sin 30^\circ / t$
13. What is the angle between an action and the reaction when a book is on a table?
- 180°
 - 90°
 - 0°
 - 45°

14. Two blocks of mass 8 kg and 12 kg are connected at the two ends of a light inextensible string. The string passes over a frictionless pulley. What is the acceleration of the system?

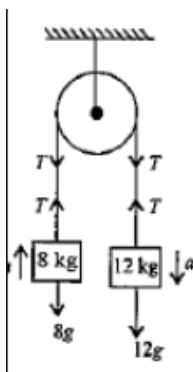


Fig 4.14

- a. 2 ms^{-2}
 - b. 2.5 ms^{-2}
 - c. 1.5 ms^{-2}
 - d. 1.75 ms^{-2}
15. The coefficient of the friction of an inclined plane is μ , and the angle of inclination is θ . Which case below represents the angle of repose for angle θ ?
- a. $\theta = \tan^{-1}\mu$
 - b. $\theta = \cot^{-1}\mu$
 - c. $\theta = \sin^{-1}\mu$
 - d. $\theta = \sec^{-1}\mu$
16. What is the acceleration produced when a horizontal force of 100N is applied on a block of mass of 10 kg placed on a rough horizontal surface with a coefficient of friction of 0.5?
- a. 10 ms^{-2}
 - b. 5 ms^{-2}
 - c. 15 ms^{-2}
 - d. 0.5 ms^{-2}
18. What will be the value of the limiting friction on a block of mass 2 kg resting on a plane inclined at an angle of 30° with the horizontal, where the coefficient of friction between the block and the surface is 0.7?
- a. 10.2N
 - a. 11.2N
 - b. 11.9N
 - c. 9.8N
19. What is the acceleration down an inclined plane with a 30° inclination and $g = 10 \text{ ms}^{-2}$?
- a. 2.5 ms^{-2}

- b. 10 ms^{-2}
 - c. 5 ms^{-2}
 - d. 9.8 ms^{-2}
20. Which factor best supports the instability of an empty bucket on a stormy wind?
- a. low value of inertia
 - b. high value of inertia
 - c. low value of friction
 - d. high value of air resistance
21. (a) State Newton's first law of motion.
- (b) How does it define the force?
 - (c) It is called the law of inertia. How?
22. (a) State impulse-momentum theorem.
- (b) How is an impulse equal to the momentum change of a body?
 - (c) Why are china wares wrapped in straw before packing?
23. (a) The second law is the real law of motion, how?
- (b) We use $k=1$ in proving $f = ma$, why?
 - (c) A body of mass of 2 kg moving at 8 ms^{-1} finally stops in 2 s due to the force applied on the body. Find the value of the force applied. (Ans: 8N)
24. (a) Define the term couple of force.
- (b) If a wheel of radius 0.5 m is turned through an angle 30° by applying two forces 100N each, find the value of the work done. (Ans: 52.33 J)
 - (c) In evaluating the work above, why is distance double that of the radius?
25. (a) Friction is a necessary evil. Justify it.
- (b) Show that angle of friction is equal to the angle of repose.
 - (c) A ball rolling on ice with a velocity of 14 ms^{-1} finally comes to rest after travelling the distance of 40 m. Find the coefficient of friction. Take $g = 9.8 \text{ ms}^{-2}$ (Ans: 0.25)
26. (a) State how you verify the principle of moment in the laboratory.
- (b) A uniform meter rod weighing 100 g is loaded with 10 g, 30 g, 50 g, 70 g, 90 g at 10cm, 30 cm, 50 cm, 70 cm, 90 cm marks respectively. Find the position of the center of gravity. (Ans: 0.63 m).
27. (a) State and explain the principle of conservation of linear momentum on the basis of Newton's third law of motion.
- (b) Superman throws a 2400N boulder at an adversary. What horizontal force must Superman apply to the boulder to give it a horizontal acceleration of 12.0 ms^{-2} (Ans: 2940 N)

(c) If there is 60N frictional force against the motion, what horizontal force is necessary to provide the same acceleration? (Ans: 3000N)

28. (a) Define impulse.

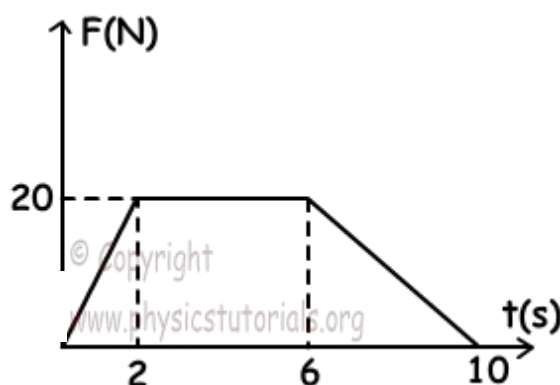


Fig 4.15

(b) Applied force vs. time graph of an object is given in fig. 4.15. Find the impulse of the object between 0-10s. (Ans: 140 kg ms⁻¹)

(c) If the mass of the object is 2kg, find the acceleration it gained in i. 2s and ii. 6s. (Ans: 1. 10 ms⁻², ii. 10 ms⁻²)

29. (a) Define the angle of friction and the angle of repose, and demonstrate that these two angles are equal.

(b) A block of mass of 1 kg resting on an inclined plane of angle 30° just starts moving down. If the coefficient of friction is 0.2, find its velocity after 5 s. (16.35 ms⁻¹)

(c) What will be the final velocity after the next s in the above problem? (Ans: 19.62 ms⁻¹)

30. A little wagon of mass of 7 kg moves in a straight line on a frictionless horizontal surface. It has an initial speed of 4 ms⁻¹ and then is pushed 3m in the direction of the initial velocity by a force of 10N. Calculate the wagon's final speed and the acceleration produced by the force. (Ans: 4.96 ms⁻¹)

31. What would be the acceleration of a block sliding down an inclined plane that makes an angle of 45° with the horizontal if the coefficient of sliding friction between two surfaces is 0.3? (Ans: 4.93 ms⁻²)

32. A block of wooden mass of 150g rests on an inclined plane. If the coefficient of static friction between the surfaces in contact is 0.30, find (a) the greatest angle to which the plane may be tilted without the block slipping, (b) the force parallel to the plane necessary to prevent slipping when the angle of the plane with the horizontal is 30°. (Ans: a. 16. 7°, 0.36N)

33. What is the average force exerted by a shot-putter on a 7 kg shot if the shot is moved through a distance of 2.8 m, and is released with a speed of 13 ms⁻¹. (Ans: 2.1 × 10²N)

34. A block of mass 5 kg resting on a horizontal surface is connected by a cord, passing over a light frictionless pulley to a hanging block of mass of 5 kg. The coefficient of kinetic friction between the block and the surface is 0.5, calculate the acceleration and the tension in the cord. (Ans: $2.5 \text{ ms}^{-1} \text{ }^2$, 37.50 N)

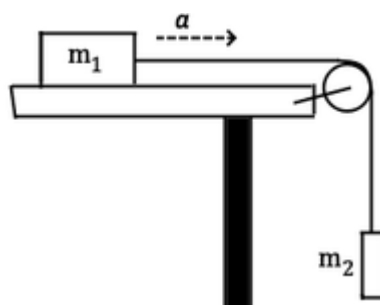


Fig 4.15

35. In a certain men's track and field event, the shot put has a mass of 7.30 kg and is released with a speed of 15.0 ms^{-1} at 40° above the horizontal over a man's straight left leg. What are the initial horizontal and vertical components of the momentum of this shotput? (Ans: 83.9 kg ms^{-1} , 70.4 kg ms^{-1})
36. Two skaters collide and grab onto each other on frictionless ice. One of them, of mass 70.0 kg, is moving to the right at 2.0 ms^{-1} , while the other, of mass 65.0 kg, is moving to the left at 2.50 ms^{-1} . What are the magnitude and direction of the velocity of these skaters just after they collide? (Hint; apply the law of conservation of linear momentum.) (Ans: -0.167 ms^{-1} , the negative sign indicates that the skater moves to the left)
37. A 22-rifle bullet, travelling at 350 ms^{-1} , strikes a large tree, which it penetrates to a depth of 0.130 m. The mass of the bullet is 1.80 g. Assume a constant retarding force, (a) how much time is required for the bullet to stop? And (b) what force, in newton does the tree exert on the bullet? (Ans: $7.4 \times 10^{-4} \text{ s}$, 848 N)
38. Two dogs pull horizontally on ropes attached to a post, the angle between the ropes is 60.0° . If dog A exerts a force of 270N and dog B exerts a force of 300N, find the magnitude of the resultant force and the angle it makes with dog A's rope (Ans: 494 N , 31.8°)
39. A hockey puck with a mass of 0.160 kg is at rest at the origin ($x=0$) on the horizontal and frictionless surface of the rink. At time $t=0$, a player applies a force of 0.250N to the puck, parallel to the x-axis, he continues to apply this force until $t= 2.00 \text{ s}$, (a) what are the position and speed of the puck at $t = 2.00 \text{ s}$? (b) If the same force is again applied at $t= 5.00 \text{ s}$, what are the position and speed of the puck at $t= 7.00 \text{ s}$? (Ans: a. 3.12 m and 3.12 ms^{-1} , b. 21.875 m and 6.25 ms^{-1}).
40. A chair of mass 12.0 kg is sitting on the horizontal non frictionless floor. You push the chair with a force $F = 40.0 \text{ N}$ that is directed at the angle of 37.0° below the horizontal and the chair slides along the floor. (a) Draw clearly a labelled free-body diagram for the chair. (b) Use your diagram and Newton's laws to calculate the normal force that the floor exerts on the chair. (Ans: 142 N , draw yourself the diagram showing the pushing force at 37° with horizontal)
41. A baseball with a mass of 0.145 kg is moving in the + Y direction with a speed of 0.130 ms^{-1} , and a tennis ball with a mass of 0.0570 kg is moving in the – Y

direction with a speed of 7.80 ms^{-1} . What are the magnitude and direction of the total momentum of the system consisting of the two balls? (Ans: $-0.256 \text{ kg ms}^{-1}$ in the $-Y$ direction.).

42. A wireless aerial attached to the top of a mast 20 m high exerts a horizontal force of 600 N upon it of. The mast is supported by a stay wire running to the ground from a point of 6 m below the top of the mast and inclined at 60° to the horizontal. Assuming that the action of the ground on the mast can be regarded as a single force, determine the force in the stay wire. (Ans: 1714.30 N).
43. A typical car weighs about 1200N. If the coefficient of rolling friction is 0.015, what horizontal force is needed to make the car move with a constant speed of 72 km/hr on a level road? Also, calculate the power developed by the engine to maintain this speed. (Ans: 18 N , 360 W)
44. A roller whose diameter is 1.0 m weighs 360 N. What horizontal force is necessary to pull the roller over a brick 0.1 m high when the force is applied at the center? (Ans: 270 N)
45. Two masses m_1 and m_2 are connected on the inclined plane over a pulley as shown in the fig;

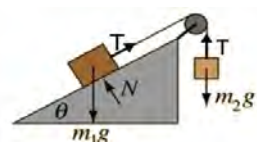


Fig 4.16

- (a) Find the acceleration of the system in terms of m_1 , m_2 , θ , and g .
 (b) Find the tension in the string
 (c) If $m_1 = m_2$, then determine the acceleration of m_1 and m_2 at angle 25° . [Ans: (a) $a = \frac{(m_2 - m_1 \sin \theta)g}{m_1 + m_2}$, (b) $T = \frac{m_1 m_2 g}{m_1 + m_2} (1 + \sin \theta)$, (c) $a = 0.28 \text{ ms}^{-2}$]

Answer to the Multiple Choice Questions:

Q.N	1.	2.	3.	4.	5.	6.	7.	8.	9.
Ans	a	c	a	c	a	b	a	b	b
Q.N	10.	11.	12.	13.	14.	15.	16.	17.	18.
Ans	b	a	c	a	a	b	c	c	a

Chapter 5

Work, Energy & Power

The French mathematician Gaspard-Gustave Coriolis introduced the term work in 1826 saying "weight lifted through a height", similar to the case of early steam engines to lift buckets of water out of flooded ore mines. The modern statement: "work is force times path" is due to him and gave the correct formula for change in kinetic energy associated with work although he spoke of force vive (including the modern factor $\frac{1}{2}$), instead of kinetic energy. The word energy is derived from the [Greek word](#) ἐνέργεια (*energeia*) which appears for the first time in the 4th century BCE works of Aristotle.

[Leibniz](#) defined the product of the mass of an object and its velocity squared from the idea of [Vis viva](#) (living force), as a modern concept of the energy.

In 1802 lectures to the Royal Society, [Thomas Young](#) was the first to use the term *energy* in its modern sense, instead of [Vis viva](#). The term finally was first invented by him in 1807 after the Greek word *Energeia* that roughly translates to "activity." Energy is the ability for an object to perform work on another object".

Moreover, [Gustave-Gaspard Coriolis](#) described the term [kinetic energy](#) in 1829 in its modern sense, and Thomas Rankine introduced the term [potential energy](#) in 1853.



Gaspard-Gustave Coriolis(1792-1843)



[*Thomas Young\(1773-1829\)*](#)

Learning Outcomes

5.1 Introduction

Do or observe the following tasks:

- i) Visit a bank or some company, or some industrial area, and notice a guard standing at the gate.
- ii) Visit the fields and watch a farmer digging or ploughing the field.
- iii) Note some people carrying loads on their backs.

- iv) Watch a man lifting a bucket of water and walking along the road.
- v) Observe a man with loads on his head and walking along.
- vi) Remember, the people involved in all these tasks receive payment accordingly.

Q: Do all these phenomena refer to a specific type of work? If they all imply the act of doing something, what is the definition of this work? In case the nature of the work varies, how would you define work to encompass all these tasks?

In a common sense, all of these tasks exemplify work. However, in physics, the concept of work carries a distinct meaning. *Work is defined as the product of a force acting upon a system and the displacement of that system occurring in the direction of the force.*

Mathematically, work (w) = force (f).displacement (d)

i.e, $w = f \cdot d$

If the force acts at an angle θ , then $w = fdcos\theta$

For $d = 0$, $w = 0$

Hence for work to occur, there must be displacement in the direction of the force. If the displacement is zero, no work is done. Therefore,

- i) A gateman or guard standing all day is often considered as doing no work, as there is no net displacement.
- ii) A man carrying a load while standing also does not perform any work.
- iii) Additionally, the work done by static friction exerted by a body on a surface is zero because there is no displacement.

If $\theta = 0^\circ$, $w = fd$ i.e, they act in the same direction.

If $\theta = 90^\circ$, $w = 0$

- iv) A man walking while carrying a water bucket does zero work because the load acts vertically downwards while he moves forward. In this case, the load and the displacement are perpendicular, forming a 90-degree angle. According to the formula $w = fdcos\theta$, where θ represents the angle between force and displacement, the cosine of 90 degrees is 0, resulting in zero work being done.
- v) The work done by the centripetal force around a circle is zero because the force and the displacement act at 90 degrees.
- vi) The work done by the tension in a string when a body undergoes a circular path is also zero.
- vii) A coolie carrying a load on his head while traveling on a platform is said to have done zero work.
- viii) The work done due to the movement of a body on a frictionless surface, when the force (F) is equal to zero, is also zero.

But, a porter moving vertically up the stairs with a suitcase on his head does work.

Why?

→ The porter applies the force upwards against the load that is acting vertically downwards. Since the displacement here is in the direction of the force applied by the porter, there is work done.

Units and Dimension of Work:

1 joule=?

It is the work done by a force of 1N to produce 1m displacement.

i.e, 1 joule = 1N×1m. The joule refers to the name of James, an English physicist.

SI Units: kilogram weight meter (or kilogram force meter⇒ kg wt m (or kgf m)

And 1 kgf m = 9.8 joule meter

For the dimensional formula for work, $W = Fd\cos\theta = [M LT^{-2}] \times [L] = [ML^2 T^{-2}] \times [L] = [M L^2 T^{-2}]$

a. Work Done by a Constant Force:

If a force is applied on a system or on a particle, and displacement is caused in the direction of the force, then work done, $W = Fd$

If the force makes an angle θ with the displacement, we consider the effective part of the force in the direction of the displacement, known as the component of the force $F\cos\theta$. Therefore, the work done, W , is calculated as $W = F\cos\theta d = Fd\cos\theta$. Hence $W = Fd\cos\theta$ is the general formula.

If $\theta = 0$, then $W = Fd$ i.e F and d act in the same direction.

If $\theta = 90^\circ$, then $W = 0$ as $\cos 90^\circ = 0$

Up to this point, the examples we have studied are all the instances of work done by a constant force.

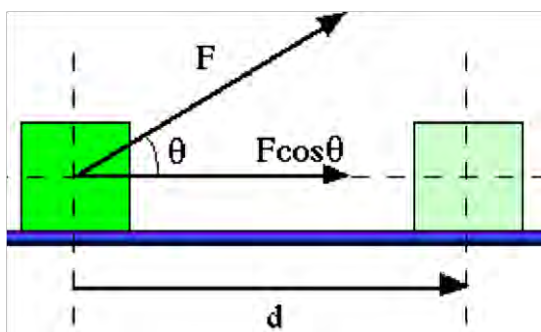
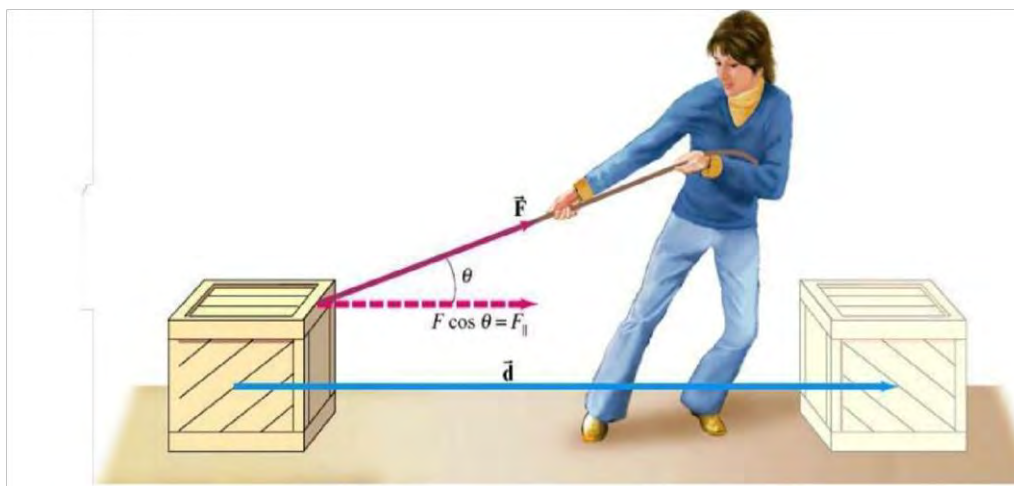


Fig 5.1: A concept of work

And for such a constant force, $w = F.d$

If the force acts at an angle θ , then $w = Fd\cos\theta$.

- i) We push a vehicle when its engine doesn't start and write $W = Fd$ as the force acts in the direction of the displacement.
- ii) We pull or push the desks in the classroom to arrange them properly. The force acts at some angle θ with the displacement, and therefore, we write $F \cos\theta$ instead of F to calculate the work done, expressed as $W = F \cos\theta d = F d \cos\theta$

b. Work Done by a Variable Force

In this case, the value of the force applied to do work isn't constant. Either the magnitude, the direction of the force, or both change. The force acting on a spring is an example of a variable force, where the force changes with the extension produced in the wire. Other examples include magnetic force, electrostatic force, and gravitational force. The work done by a variable force can be analyzed both analytically and graphically.

Analytical Method

Let $\vec{F}$ be a varying force that makes an infinitesimal distance $\vec{dS}$.

Hence the small amount of work done, dW is written as $\vec{dW} = \vec{F} \cdot \vec{dS}$

Therefore, total work done, $W = \int_{x_1}^{x_2} \vec{F} \cdot \vec{dS} = \int_{x_1}^{x_2} (F \cos\theta) ds$

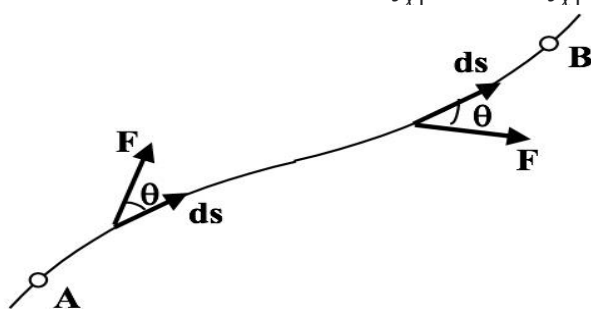


Fig 5.2a Analytical representation of work done by a variable force

- If you use a wheelbarrow along a curved path, the force you apply varies. The work done is evaluated by integrating the product of the force (F) over a small increment (ds) and the cosine of the angle between them.

Graphical Method

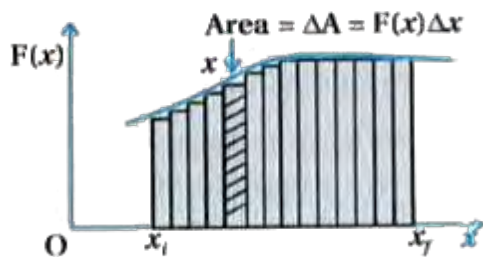
Let F be a variable force that varies along the x -axis.

Here, we have to calculate the work done due to this variable force along the x -axis from the initial position, x_i to the final position, x_f .

For the purpose, a plot of force against displacement, $F(x)$ vs x has been plotted.

Let the displacement be divided into small increments Δx_i such that the value of $F(x)$ for each elementary area is constant.

Therefore, work done for each interval, $\Delta W = F \Delta x$



The total work done from x_i to x_f is written as

Fig 5.2b Graphical representation of work done by a variable force

$$W = \sum_{x_i}^{x_f} F \Delta x$$

For infinitesimal increment, $\Delta x \rightarrow 0$, $W = \lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F \Delta x = \int_{x_i}^{x_f} F dx$

Therefore, work done due to the variable force, $W = \int_{x_i}^{x_f} F dx$

Worked Out Examples:

1. A block of mass 3kg is attached to a spring with a spring constant of 5N/m. If it is stretched to a distance of 2m from equilibrium, what is the work done to return to the equilibrium position for the first time?

Solution:

Here, the initial position of the block, $x_i = 2\text{m}$

The final position of the block, $x_f = 0\text{m}$

Spring constant, $k = 5\text{N/m}$

Force on the block at a distance x from the equilibrium position, $F(x) = -kx$

Work done to bring the block from 2m to the equilibrium position,

$$W = \int_{x_i}^{x_f} F dx = \int_2^0 -kx dx = \int_2^0 -5x dx = -5 \left[\frac{x^2}{2} \right]_2^0 = -5/2 \left[0 - 2^2 \right] = 10\text{J}$$

Sample Question:

1. A force $F = (10 + 0.50x)$ acts on a particle in X – direction, where F is in newton and x is in metre. Find the work done by this force during the displacement from $x = 0$ to $x = 2\text{m}$ (Ans: 21 joule)

Work done by a body: This refers to the work done by the body on its surroundings, often resulting in a decrease in its energy (e.g., kinetic energy).

Work done on a body: This refers to the work done on the body by an external force, often resulting in an increase in its energy (e.g., potential energy).

Work done by a body is the energy lost by the body and hence the work done is negative while work done on a body is the energy gained by the body and hence the work done is positive.

5.4 Power

Power is defined as the rate at which work is done or energy is transferred over time. It quantifies how quickly energy is used or converted from one form to another.

Mathematically, Power = work done / time taken.

$$\text{Or, } P = \frac{W}{T}$$

Where;

- i) P is the power,
 - ii) W is the work done or the energy transferred,
 - iii) t is the time over which the work is done or the energy is transferred.
- $t \rightarrow 0 \Rightarrow P \rightarrow \infty$ and $t \rightarrow \infty \Rightarrow P \rightarrow 0$.

$$\text{Also, } P = \frac{W}{T} = \frac{F \cdot d}{T} = F \cdot v = Fv \cos \theta$$

$$\therefore P = \frac{W}{T} = F \cdot v = Fv \cos \theta$$

The standard unit of power is watt (W), where 1 watt is 1 joule of work done in 1 s. It is after the Scottish physicist James Watt, an inventor of the steam engine, the unit is used.

Power can also be expressed in different contexts, such as electrical power, where it is defined as the product of voltage V and current I :

Horse power is another unit for power used earlier specially in power of vehicle.

And 1 horse Power (H.P) = 746 watt.

Worked Out Examples:

1. An elevator weighing 500 kg is to be lifted up at a constant velocity of 0.4 ms^{-1} .
What should be the minimum horse power of the motor to be used?

Solution:

Here, mass, $m = 500 \text{ kg}$,

Velocity, $v = 4 \text{ ms}^{-1}$

We have, $P = F \cdot v = mgh = 500 \times 10 \times 0.4 = 2000 \text{ watt} = 2000/746 = 2.68 \text{ H.P}$

Sample Exercise

1. A gun fires 8 bullets per second into a target X. If the mass of each bullet is 3 g and its speed 600 s^{-1} , calculate the power delivered by the bullets.
(Hint: $P = \text{work done per unit time} = \text{total K.E of 8 bullets} = 8 \times \text{K.E of each bullet}$ (Ans 4.320 Kw)

5.5 Energy

Energy is the capacity to do work. If work is performed, energy is expended, and if energy is gained, an equivalent amount of work has been done.

There are many kinds of energy, with the main category being the division of energy into mechanical energy and non-mechanical energy. Non-mechanical energies include heat energy, sound energy, light energy, nuclear energy, etc. Here, we will discuss mechanical energy, which is divided into two categories: kinetic energy and potential energy. This distinction between mechanical and non-mechanical energy, as well as the categorization of mechanical energy into kinetic and potential energy, aligns with fundamental concepts in physics. Let's explore this breakdown further.

5.5.1 Mechanical Energy:

Kinetic energy:

Imagine the following phenomena:

- i. You are sitting on the ground at some distance from your child (your nephew), who is playing nearby. You are suddenly informed that the child is playing with an explosive ball that is going to explode within a few minutes. What do you do then?

You run as fast as you can to save the child from the possible danger. All the energy within you is used up in running fast. This total energy within you is what we call *Kinetic Energy*.

- ii. You took part in a district-level race competition organized by your college. How did you run during your turn in the competition? Can you describe it? You definitely answer that you ran as fast as you could. Indeed, all the energy you possessed was converted into kinetic energy due to such a terrific race.
- iii. You are walking and talking with your friends. Some of the energy you possess is in the form of kinetic energy.
- iv. You are plowing the field. Here, you aren't utilizing all the energy. Part of the energy you are using is in the form of kinetic energy.

Kinetic Energy:

Kinetic energy is the energy possessed by a body due to its motion. According to the classical mechanics, a body of mass m moving with a velocity v has kinetic energy given by the formula $K.E = \frac{1}{2}mv^2$. This means that the kinetic energy is directly proportional to the mass of the object $K.E = \frac{1}{2}mv^2$ the square of its velocity.

Proof: Consider an object of mass m at rest that begins to move due to the application of a force F . The body attains a velocity v on covering a displacement s .

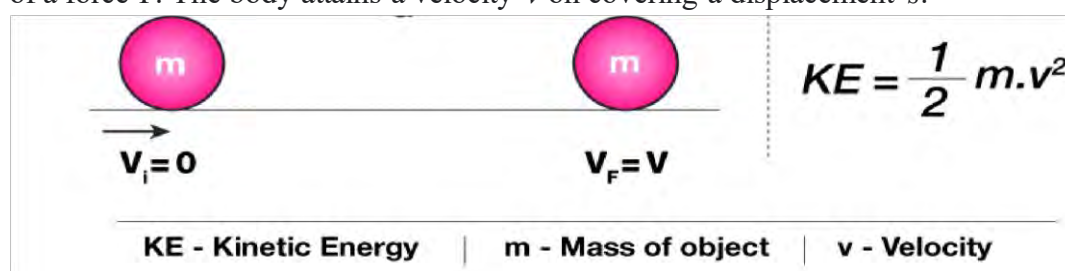


Fig 5.3 A pictorial diagram showing how a body gains KE

Hence, one writes, $v^2 = u^2 + 2as = 2as$

$$\text{Or, } \frac{v^2}{2} = as \quad \dots \quad (i)$$

The work done by the object, $w = f.s = mas = \frac{mv^2}{2} = \text{Energy gained} = K.E$

Therefore, $K.E = \frac{1}{2}mv^2$ proved.

Since mass and v^2 are always positive, K.E is always positive and can't be negative as suggested by classical approach.

5.3.3 Work- Energy Theorem

Consider the phenomena:

Have your meal as usual, then go to the field to work.

After a daylong work, you feel tired and hungry. This is due to the loss of energy you experienced throughout the day. Clearly, the energy loss is attributed to the work you performed. Conversely, when you engage in work, the amount of work done on an object is equal to the change in its kinetic energy. These activities and feelings illustrate a well-known theorem called the *Work-Energy theorem*. As already mentioned, an energy gain is from the work done and the loss of energy goes to make a work. *Hence the energy and work must be related and the solution for this relation is the **Work –Energy theorem**.*

Let's consider a body moving on a frictionless surface with an initial velocity u , which attains a final velocity v after traversing a distance s due to the force F acting on it.

The work done, $w = F.s$

$$= ma.s$$

$$\text{But, } v^2 = u^2 + 2as$$

$$\text{Or, } \frac{v^2 - u^2}{2} = as$$

$$\begin{aligned} \therefore \text{Work done, } w &= ma.s = m \left(\frac{v^2 - u^2}{2} \right) \\ &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= K.E_f - K.E_i \end{aligned}$$

$$\begin{aligned} \text{Now from above, } w &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \Delta E_k \\ \therefore \frac{1}{2}mv^2 &= w + \frac{1}{2}mu^2 \end{aligned}$$

It implies that final K.E is work done added to the initial K.E. Hence the K.E increased is equal to the work done, known as **work- energy theorem**.

Worked Out Examples:

1. A bullet has a mass of 2.60 g and a muzzle velocity of 335 ms^{-1} . It can penetrate eight pine boards, each with thickness 0.019 m. What is the average stopping force exerted by the wood.

Solution:

Here, mass of the bullet, $m = 2.60 \text{ g} = 2.60/1000 \text{ kg}$

Velocity, $v = 335 \text{ ms}^{-1}$

Total thickness, $s = 8 \times 1.90 = 15.2 \text{ cm} = 15.2/100 \text{ m}$

According to work- energy theorem,

$$W = -E = -F_{av} \cdot \Delta s_{\text{stop}}$$

$$\Rightarrow F_{av} = \frac{\frac{1}{2}mv^2}{\Delta s_{\text{stop}}} = \frac{1}{2} \times 2.60 \times 10^{-3} \times (335)^2 / 15.2 \times 10^{-2}$$

$$= 959.8 \text{ N}$$

2. An object A moving horizontally with KE of 800J experiences a constant horizontal opposing force of 100N while moving from a place X to another place Y, where XY is 2 m. What is the energy of A at Y? In what further distance will A come to rest if this opposing force continues to act on it?

Solution:

Here, KE, $E = 800 \text{ J}$

Opposing force, $F = 100\text{N}$

Distance, $s = 2\text{m}$

Now, work done from X to Y, $W = F.s = 100 \times 2 = 200\text{J}$

This much amount of energy is supplied from the KE of the body.

Therefore, remaining energy saved $= 800 - 200 = 600\text{J} \Rightarrow$ Energy at Y

Further distance is travelled in the expense of this KE, 600J

Therefore, KE consumed = work done

Or, $600 = F.s$

$$\Rightarrow s = \frac{600}{100} = 6\text{ m}$$

3. The position x of a particle of mass 1 kg moving along X –axis at time t is $(x = \frac{1}{2}t^2)\text{ m}$. Find the work done by the force acting on it in the time interval from $t = 0$ to $t = 3\text{ s}$.

Solution:

Here,

Mass, $m = 1\text{ kg}$

Work done, $W = ?$

$$\text{We have, } x = \frac{1}{2}t^2 \Rightarrow \frac{dx}{dt} = t$$

$$\therefore t = 0 \Rightarrow v_1 = 0 \text{ and } t = 3\text{ s} \Rightarrow v_2 = 3\text{ ms}^{-1}$$

$$\begin{aligned}\text{Now work done, } W &= \text{change in KE} = \frac{1}{2}m(v_2^2 - v_1^2) \\ &= \frac{1}{2} \times 1 \times 3^2 \\ &= 4.5\text{J}\end{aligned}$$

Sample Question:

1. How much work must be done to stop a 1200 kg car moving at 99 km/h in a straight path?
(Ans: 453750 joule)

5.3.4 Relation between K.E and Momentum:

Let a stationary mass, M explodes into two parts of masses m_1 and m_2 and move with the respective velocities v_1 and v_2 respectively.

Therefore, we write here $0 = m_1 v_1 + m_2 v_2$

$$\text{Or, } m_1 v_1 = -m_2 v_2 \Rightarrow p_1 = -p_2 \dots\dots\dots (5.)$$

$$\text{The kinetic energy, } E = \frac{1}{2} m v^2 = \frac{1}{2} \frac{m v^2}{m} = \frac{p^2}{2m}$$

$$\text{Therefore, } \frac{E_1}{E_2} = \frac{\frac{p_1^2}{2m_1}}{\frac{p_2^2}{2m_2}} \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1} \Rightarrow E \propto 1/m$$

Hence a body of smaller mass has greater K.E.

Further,

$$\frac{E_1}{E_2} = \frac{m_2}{m_1} + 1 \Rightarrow \frac{E_1 + E_2}{E_2} = \frac{m_1 + m_2}{m_1}$$

$$\text{Therefore, } \frac{E_1}{E_2} = \frac{M}{m_1} \Rightarrow E_2 = \frac{m_1 E_1}{M}$$

The relation helps to find the K.E of either the smaller or of the larger body.

Q: A light body and a heavy body have the same kinetic energy. Which one will have the greater momentum?

Ans: Here we have,

$$\text{K.E, } E = \frac{p^2}{2m} \Rightarrow p^2 = 2mE$$

$$\therefore p = \sqrt{(2mE)} \Rightarrow p \propto \sqrt{m}$$

Hence the heavier body has greater momentum.

Worked Out Examples:

1. A 20 units mass explodes into two parts, 4 units and 16 units. If the K.E of the larger mass is 100J, find it for the smaller mass.

$$\text{Solution: We have K.E, } E = \frac{p^2}{2m} \Rightarrow E_1 = \frac{p_1^2}{2m_1} \text{ and } E_2 = \frac{p_2^2}{2m_2}$$

$$\text{Since the mass explodes, } p_1 = -p_2 \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$$

$$\therefore E_1/E_2 = \frac{m_2}{m_1} \Rightarrow E_1 = \frac{m_2}{m_1} \times E_2 = \frac{16}{4} \times 100 = 400\text{J}$$

Sample Question:

1. If the linear momentum of a body increases by 20%, what will be the % changed in the K.E?

(Ans: 44%)

5.5.2 Potential Energy:

Potential energy (P.E) is defined as the energy stored in an object due to its position relative to other objects or its configuration. It is a form of mechanical energy and can be converted into kinetic energy (the energy of motion) or other forms of energy.

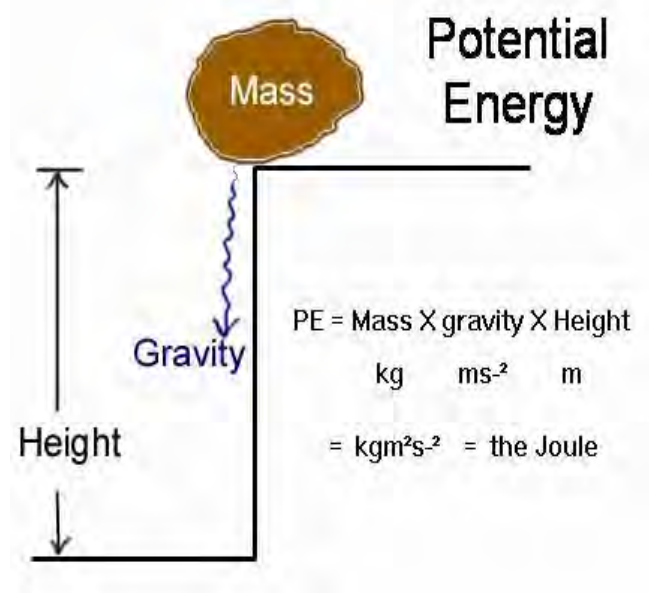


Fig 5.4: A concept of P.E

The compressed string, a catapult about to set, a body at certain height above the ground are said to have P.E.

If a body is raised to a height h above the ground surface against gravitational pull, then work done, $w = f.h = mgh = \text{gain in P.E.}$

$\therefore \text{P.E.} = mgh$, is the gravitational P.E

Examples:

- i) A stretched rubber or a compressed spring possesses P.E.
- ii) A bent bow possesses P.E.
- iii) A compressed gas possesses P.E.
- iv) Water in a dam and a ball raised to height have gravitational P.E. of primary focus.

Facts on K.E. &P. E.

Finally, we write, $\text{K.E.} + \text{P.E.} = \text{Total mechanical energy}$ If K.E is maximum, P.E is zero and vice versa.

- i) A man resting on his bed has potential energy (P.E.), while a man running at his top speed possesses only kinetic energy (K.E.). A man, walking normally, possesses a combination of kinetic energy and potential energy, with a portion being kinetic and the remainder being potential.
- ii) The water in a pond does possess potential energy (P.E.), and the distribution of this pond water to an urban area involves the transformation of potential energy into kinetic energy (K.E.).

- iii) The energy stored in a stretched string is potential energy (P.E.). When a stone is released from such a string, as in a catapult, there is a conversion of this potential energy into kinetic energy (K.E.).
- iv) The release of the arrow from a stretched bow exemplifies the transformation of potential energy (P.E.) into kinetic energy (K.E.).
- v) When the compressed gas in a bicycle tire bursts, the potential energy stored in the compressed gas is transformed into kinetic energy and other forms of energy such as sound and heat. A body can possess potential energy (PE) due to its positional state but may not have momentum. Furthermore, a body with no energy also has no momentum.

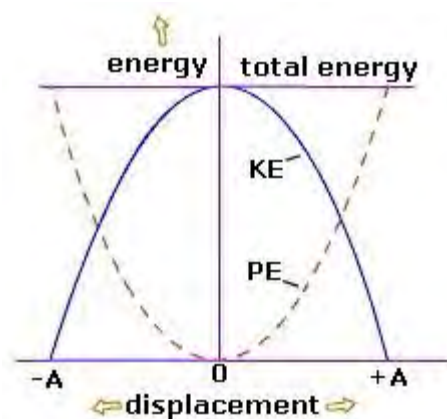


Fig 5.5 Variation of KE and PE

What about the concept of change in PE = work done by/against a force?

Worked Out Examples:

Q 5.5 A mass of 10Kg is taken from the ground for 10m uphill on the wedge. The wedge makes an angle of 30° with the ground. Find the potential energy of the block.

Solution:

Here, mass, $m = 10 \text{ kg}$

Length of the wedge, $L = 10\text{m}$

Angle of inclination of the wedge with the ground, $\theta = 30^\circ$

We have, $P.E = mgh = 10 \times 10 \times h = 100 \times L \sin 30^\circ = 100 \times 10 \times \frac{1}{2} = 500 \text{ joule.}$

Sample Question:

1. Water is pumped vertically upwards through a distance of 100m and the work done is 294 kilo joule. Calculate the volume of the water pumped. (Given density of water = 10^3 kg/m^3)
(Ans: $2.94 \times 10^{-5} \text{ m}^3$.)

5.6 Principle of Conservation of Energy

We have studied different forms of energy in our previous classes. These include mechanical energy, heat energy, sound energy, chemical energy, light energy, electrical energy, and so on.

When one form of energy disappears, the next form of energy is evolved, and vice versa. There is no destruction or creation of energy; it only changes its form.

- i) When we turn on a radio, the chemical energy in the battery or the electrical energy supplied is converted into sound energy. Here, the loss of the former type of energy is balanced by the production of the second kind of energy.
- ii) When a fan is turned on, the electrical energy supplied is converted into mechanical energy. The amount of electrical energy expended is balanced by the generation of mechanical energy.

In summary, the principle of conservation of energy can be stated as:

“Energy can neither be created nor be destroyed, but it can be changed from one form to another form”

- iii) The principle of conservation of energy is an empirical principle and cannot be proved purely through mathematics.
- iv) The law is also believed to be universally valid, as it has not been contradicted in any experiment or observation of nature.
- v) The law connects mechanics to all other branches of physics, making the concept of energy one of the unifying ideas of physics.
- vi) The mechanical energy is conserved only when the forces are conservative in nature.
- vii) The energy spent against the friction dissipates in the form of heat energy which isn't available for useful work.

The conservation of energy is best understood by taking the example of a pendulum motion.

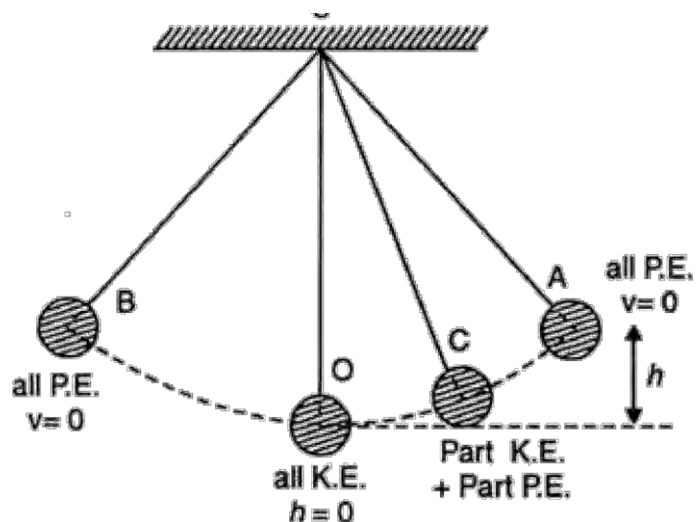


Fig 5.6a: Conservation of Energy

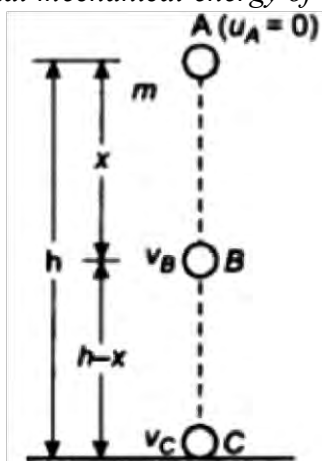
5.6.1 Principle of Conservation of Mechanical Energy (falling body case)

In the case of a falling body, assuming air resistance is negligible (so that only gravitational force is acting), mechanical energy is conserved.

Statement: *Energy can neither be created nor be destroyed, but it can be changed from one form to another form.*

OR

The total mechanical energy of an isolated system remains conserved.



Proof:

Fig 5.6b: Showing conservation of mechanical energy

Let us consider a body of mass m allowed to fall freely under gravity from a height h above the ground level at point A.

The body begins to fall from its zero value ($u_A = 0$) at point A, reaches first to point B ($AB = x$), and finally on the ground at point C ($BC = h - x$). v_B and v_C are the velocities at the respective points B & C.

Now at point A,

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}mu_A^2 = \frac{1}{2}m \times 0 = 0$$

$$P.E = mgh = mg \times h = mgh$$

$$\text{Therefore, total energy} = K.E + P.E = 0 + mgh = mgh$$

At point B,

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}mv_B^2 = \frac{1}{2}m(u_A^2 + 2gx) = \frac{1}{2}m \cdot 2gx = mgx$$

$$P.E = mgh = mg(h - x) = mgh - mgx$$

$$\text{Therefore, total energy} = K.E + P.E = mgx + mgh - mgx = mgh$$

At point C,

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}mv_C^2 = \frac{1}{2}m(u_A^2 + 2gh) = \frac{1}{2}m \cdot 2gh = mgh$$

$$P.E = mgh = mg \times 0 = 0$$

$$\text{Therefore, total energy} = K.E + P.E = mgh + 0 = mgh$$

Hence, in the above three points, the total mechanical energy remains conserved. Only it is changing from P.E. to K.E.

Hence the total mechanical energy remains conserved. Only it is changing from one form to another form and the principle of conservation of mechanical energy.

Do you Know?

- i) *A body falling towards the Earth in air has frictional force due to the air resistance in addition to the conservative force such as gravitational force and hence the total mechanical energy isn't conserved.*
- ii) *A driver accelerates his vehicle while moving up a hill. As the vehicle speeds up, the height from the ground level increases, thereby increasing the potential energy. Hence, kinetic energy decreases. To maintain kinetic energy and to speed up the vehicle, the driver accelerates his vehicle.*

Worked out Examples:

Q 5.6: A 0.125 kg steel ball with a K.E of 0.25 J rolls along a horizontal track. How high up an inclined track will the ball roll if friction is neglected?

Solution:

Here mass of the ball, $m = 0.125 \text{ kg}$

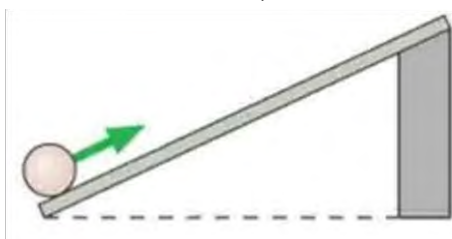


Fig 5.7

We have, total energy, $E = \text{P.E.} + \text{K.E.}$

Or, $0.25 = 0 + 0.25$, all the energy is K.E at the bottom.

At the top, all K.E is converted into P.E. and $v = 0$ there.

And total energy, $E = \text{P.E.} + \text{K.E.}$

Or, $0.25 = 0.25 + 0 \Rightarrow \text{P.E.} = mgh$

$= 0.25$

Or, $0.125 \times 9.8 \times h = 0.25 \Rightarrow h = 0.20 \text{ m}$

Sample Questions:

A roller coaster car with a mass of 200 kg starts at A and rolls down a track as shown in the fig. It reaches B with a speed of 12 ms^{-1} . Calculate the speed with which the car started at A, assuming no friction between A and B. (Ans: 10.2 ms^{-1})

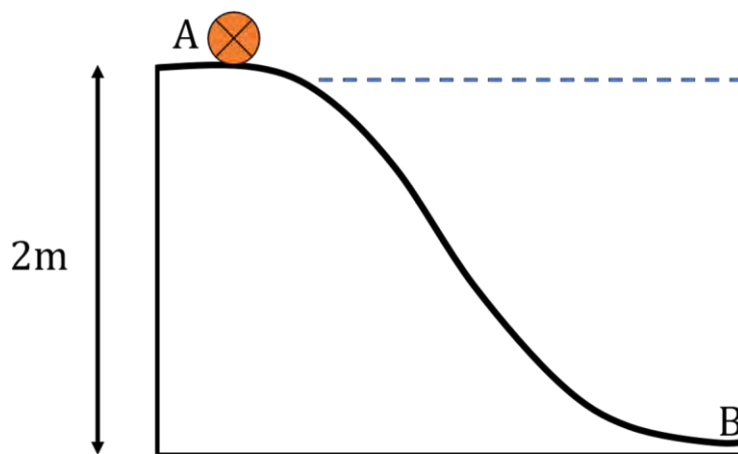


Fig 5.8

5.7 Conservative and Non- conservative Forces

5.7.1 Conservative force:

It is the force under the action of which total mechanical energy remains conserved.

Gravitational force, elastic force, magnetic force, electrostatic force, and all the field forces are examples of the conservative force. A conservative force has the following properties:

- i) It is independent of the path, but depends on the initial and the final position of the body.

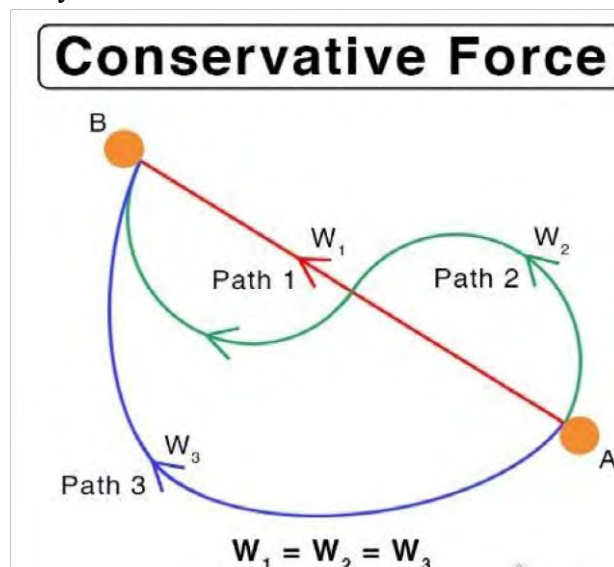


Fig 5.9: Showing the nature of conservative force

- ii) Work done by a conservative force around a closed path is zero.
- iii) Work done by it is completely recoverable and is central in nature.
- iv) Conservative forces permit one to write elegant conservation laws, such as the conservation of mechanical energy.

$$E_f = E_i$$

5.7.2 Non- Conservative Force:

It is the force under the action of which total mechanical energy isn't conserved. This is because a part of the energy is dissipated in the form of sound, heat, etc. Frictional forces, tension forces, push and pull actions, applied forces, and viscous forces are examples of non-conservative forces. A non-conservative force has the following properties:

- i) It is path-dependent force. More work will be done on a longer path.
- ii) The work done by a non-conservative force around a closed path is not zero. For example, consider going from home to college and back. The work done in both the paths is against friction and differs, resulting in a non-zero value of the work done. The longer the path, the more work friction does.
- iii) Work done by it isn't completely recoverable and is retarding in nature.
- iv) The non-conservative forces disrupt the fundamental conservation laws, such as the conservation of mechanical energy

$$E_f < E_i$$

Unless one adds the explicit factor for work, they do:

$$E_f = E_i + W$$

One can improve the law of conservation of mechanical energy to work in real-world situations by accounting for friction:

$$KE_f + GPE_f = KE_i + GPE_i + W_f, W_f \rightarrow \text{work done by friction.}$$

Worked out Examples:

1. A roller coaster with a mass of 2500 kg has a speed of 10 ms^{-1} as it goes over a 13 m hill. When it reaches the crest of an 8 m hill, it is moving at 13 ms^{-1} . How much work did friction do on the coaster?

Solution:

Here,

Mass, $m = 2500 \text{ g}$

Speed, $v_i = 10 \text{ ms}^{-1}$

Speed, $v_f = 13 \text{ ms}^{-1}$

Height, $h_i = 13 \text{ m}$

Height, $h_f = 10 \text{ m}$

$$KE_f + GPE_f = KE_i + GPE_i + W_f$$

$$\text{OR, } mv_f^2 + mgh_f = \frac{1}{2}mv_i^2 + mgh_i + W_f$$

$$\text{OR, } \frac{1}{2} \times 2500 \times 169 + 2500 \times 9.8 \times 8 = \frac{1}{2} \times 2500 \times 100 + 2500 \times 9.8 \times 13 + W_f$$

$$\Rightarrow W_f = -36250 \text{ joule}$$

Friction usually does perform negative work and it slows down them.

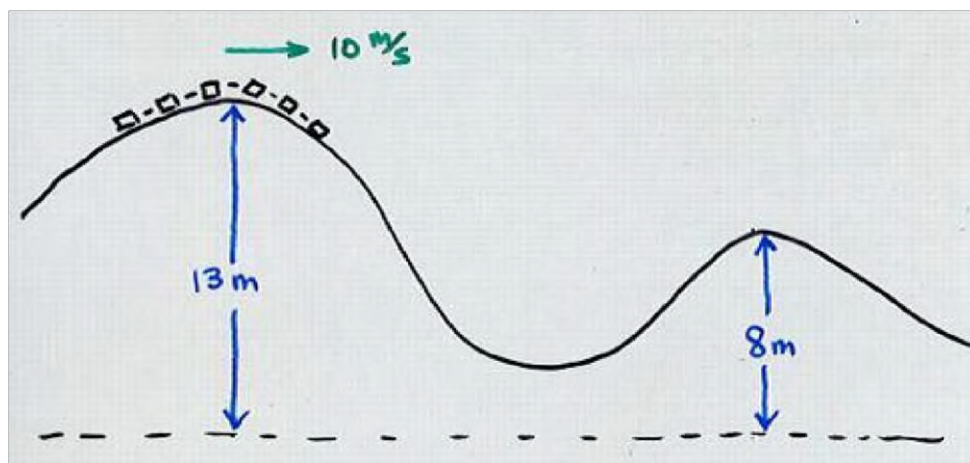


Fig 5.10

3. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with the coefficient of kinetic friction = 0.1. Compute the;
- Work done by the applied force in 10 s.
 - Work done by friction in 10 s.
 - Work done by the net force on the body in 10 s.
 - Change in kinetic energy of the body in 10 s.

Solution:

The mass of the body = 2 kg

Horizontal force applied = 7 N

Coefficient of kinetic friction = 0.1

Acceleration produced by the applied force, $a_1 = F/m = 7/2 = 3.5 \text{ ms}^{-2}$

Force of friction, $f = \mu R = \mu mg = 0.1 \times 2 \times 9.8 = 1.96 \text{ N}$

Retardation produced by friction, $a_2 = -f/m = -1.96/2 = -0.98 \text{ ms}^{-2}$

Net acceleration with which the body moves,

$$a = a_1 + a_2 = 3.5 - 0.98 = 2.52 \text{ ms}^{-2}$$

Distance moved by the body in 10 seconds,

$$s = ut + \left(\frac{1}{2}\right) at^2 = 0 + \left(\frac{1}{2}\right) \times 2.52 \times (10)^2 = 126 \text{ m}$$

(a) The time at which work has to be determined is $t = 10 \text{ s}$

Work = Force $\times$ displacement

$$= 7 \times 126 = 882 \text{ J}$$

(b) Work done by the friction in 10 s

$$W = -f \times s = -1.96 \times 126$$

(c) Work done by the net force in 10 s

$$W = (F - f) s = (7 - 1.96) \times 126 = 635 \text{ J}$$

(d) From $v = u + at$

$$v = 0 + 2.52 \times 10 = 25.2 \text{ ms}^{-1}$$

$$\text{Final Kinetic Energy} = \frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times (25.2)^2 = 635 \text{ J}$$

$$\text{Initial Kinetic Energy} = \frac{1}{2} mu^2 = 0$$

$$\text{Change in Kinetic energy} = 635 - 0 = 635 \text{ J}$$

The work done by the net force is equal to the final kinetic energy.

Sample Question:

5.7 A conservative force acts on a 4 kg particle in the X – direction. The potential energy $V(x)$ is given by $V(x) = 40 + (x-6)^2$, where x is in metre. If it is given that at $x = 8 \text{ m}$, KE is 30J, then find the maximum KE of the particle.

(Ans: 34J, Hint: Find $V(x)$ at $x = 8 \text{ m} \Rightarrow 44\text{J}$, then find $T.E = KE + V(x) = 74\text{J}$. Find minimum $V(x)$ by $dV/dx = 0 \Rightarrow x = 6 \text{ m} \Rightarrow V_{\min} = 40\text{J} \Rightarrow T.E = V_{\min} + KE_{\max} \Rightarrow 34\text{J}$)

5.8 Collision; Elastic and Inelastic Collision

In physics, the term "collision" typically implies an interaction between two or more objects where they exert forces on each other, resulting in a change in their motion. While physical contact is the most common scenario in collisions, it is not strictly necessary for a collision to occur. For example, the collision of α -particles with the nucleus involves no physical contact, whereas the collision between two billiard balls or a cricket ball and a bat does.

Though most common examples of collisions involve physical contact, it is true that, in a broader sense, a collision can occur without direct physical contact if there is an interaction through forces such as gravitational or electromagnetic forces, leading to a change in motion. However, in everyday usage and most introductory physics contexts, collisions are typically associated with physical contact.

During a collision, the total energy and momentum of the system of colliding objects remain conserved.

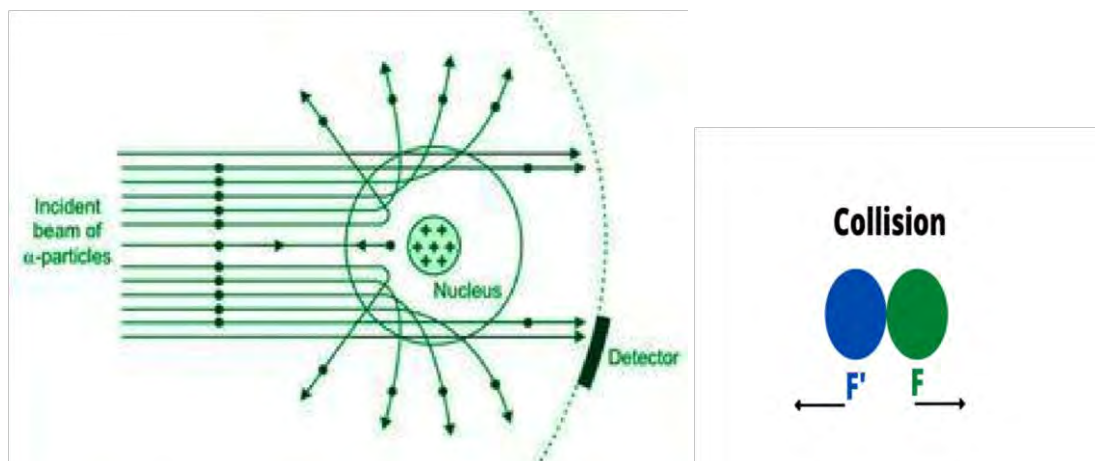


Fig 5.14: Showing how α -particles collide with nucleus.

Coefficient of Restitution

For any two colliding bodies, the ratio of the relative velocity after the collision to the relative velocity before the collision is called the coefficient of restitution.

i.e., Coefficient of Restitution, $e = \text{Relative velocity after collision} / \text{Relative velocity before collision}$.

The value of e lies as $0 < e < 1$, $e = 1$

for perfectly elastic collision. $e = 0$,

for perfectly inelastic collision.

5.8.1 Types of Collision

Elastic Collision

In an elastic collision, both the momentum and KE are conserved

i.e., $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$$\text{And } \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Such collisions occur among atomic or subatomic particles, nuclear particles, and gaseous molecules, where there is no conversion of mechanical energy into sound, heat, or other forms of energy during the collision. Consequently, the forces involved are called conservative forces.

When a ball on a billiard table hits another ball, it is often approximated as an elastic collision, although in reality, it is nearly but not perfectly elastic due to minor energy losses.

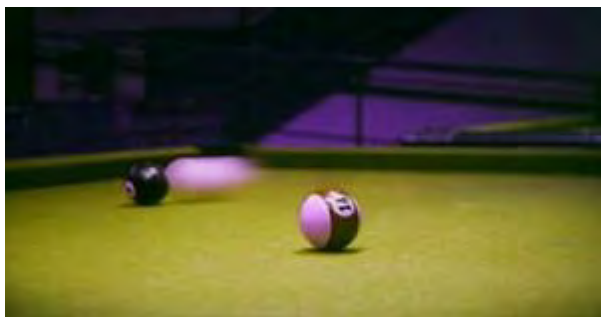


Fig5.15: Elastic collision when a ball at a billiard table hits another ball.

Hence in elastic collision,

- i) Momentum is conserved
- ii) KE is conserved
- iii) Total energy is conserved
- iv) Mechanical energy isn't converted into another form of energy. And, hence forces are the conservative forces.

Inelastic Collision

In this collision, only the momentum is conserved but not the KE.

i.e, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$\text{And } \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

There is a loss of KE in the form of sound, heat, or other forms of energy.

$$\text{The loss of energy, } \Delta E = \left\{ \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \right\} - \left\{ \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \right\}$$

Since the total mechanical energy in an inelastic collision is converted into other forms of energy such as heat, sound light energy, etc., the forces involved in it are called the nonconservative forces.



Fig 5.16: Inelastic collision, car crash

Hence, in an inelastic collision,

- i) Momentum is conserved
- ii) KE isn't conserved
- iii) Total energy is conserved
- iv) Mechanical energy is converted into other form of energies.

Hence forces involved are non- conservative in nature.

For example, when two cars crash and crumple together, a ball of clay is dropped and sticks to the floor, or a tackle in football where players move together after impact. These illustrate how kinetic energy is transformed into other forms of energy during inelastic collisions.

One Dimensional Elastic Collision:

Let us consider two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 respectively in a direction such that $u_1 > u_2$. As $u_1 > u_2$, they suffer a collision for a small time

interval and finally move with the velocities v_1 and v_2 respectively in the same direction such that $v_2 > v_1$.

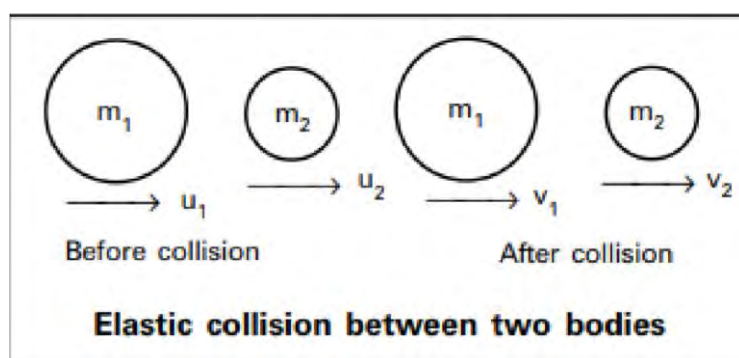


Fig 5.17: one dimensional elastic collision between two bodies

For an elastic collision,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \quad \dots\dots (5.21)$$

And $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$\Rightarrow m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2) \quad \dots\dots(5.22)$$

Dividing eqⁿ (5.) by eqⁿ (5.),

$$u_1 + v_1 = v_2 + u_2$$

Or, $u_1 - u_2 = v_2 - v_1 \quad \dots\dots\dots (5.23)$

Hence in one-dimensional elastic collision, the relative velocity of the approach before the collision is equal to the relative velocity of recession after the collision.

Multiplying Eq. (5.23) by m_2 and then subtracting from (5.21), we have

$$(m_1 - m_2) u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$$

Or, $v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \quad \dots\dots\dots (5.24)$

In the same way multiplying eqn (5.23) by m_1 and then adding to eqn (5.21), one gets

$$2m_1 u_1 + (m_2 - m_1) u_2 = (m_1 + m_2) v_2.$$

$$\Rightarrow v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 \quad \dots\dots\dots (5.25)$$

Eqns. (5.24) and (5.25) give the velocities of the two bodies after collision in terms of the initial velocities.

Special Cases:

Case (i):

If the colliding bodies are of equal masses i.e, $m_1 = m_2 = m$, then $v_1 = u_2$ and $v_2 = u_1$

Hence if two bodies of equal masses collide, then, their velocities simply interchange.

When $u_2 = 0$, then $v_1 = 0$ and $v_2 = u_1$

i.e. when a body collides with another body of the same mass at rest, then, the first body stops and the second body continues with the velocity of the first body. The ejection of the proton with the collision with the neutron is an example of it.

Case (ii):

If a body in motion is of negligible mass as compared to the body at rest ($m_1 \ll m_2$), then $m_1 = 0$ and $u_2 = 0$.

The Eqns. (5.24) and (5.25) $\Rightarrow v_1 = -u_1, v_2 = 0$

i.e. *when a light body suffers a head-on collision with a massive body at rest, the light body recoils with the same speed while the massive body remains at rest.*

Case (iii):

When the body at rest has a negligible mass as compared to the body in motion ($m_1 \gg m_2$), then $u_2 = 0$ and $m_2 = 0$.

Eqs (5.24) and (5.25) $\Rightarrow v_1 = u_1, v_2 = 2u_1$

When a heavy body undergoes a head-on elastic collision with a light body at rest, the heavy body moves at the same speed, while the light body moves at a speed double the velocity of the heavy body.

The motion of α - particles in an α - particles scattering experiment remains unaltered when there is a collide with the electrons of the targeted atoms.

Project Work

Project 5.1 Study the nature of collision between a ball and a floor with a coefficient of restitution.

Hint: The coefficient of restitution (COR) is defined as the ratio of the relative velocity of separation to the relative velocity of approach between two colliding bodies.

i.e, $e = \frac{v_2 - v_1}{u_1 - u_2}$, v_2, v_1 are the velocities of the two bodies after the collision.

u_1, u_2 are the velocities of the two bodies before the collision.

For a ball colliding with a stationary floor, $u_2 = 0 \Rightarrow e = \frac{v}{u} = \sqrt{\frac{h_2}{h_1}}$ as $v^2 = u^2 + 2gh$, for a falling body.

- i) Take a ball (e.g., a rubber ball or a tennis ball) and a measuring tape, choose a smooth or a hard floor, and use a measuring device (e.g., a high-speed camera or a motion sensor) as *materials required*.
- ii) Measure the initial height from which the ball is dropped.
- iii) Drop the ball from the measured height and record the bounce.
- iv) Measure the height to which the ball rises after the first bounce.
- v) Calculate the initial and final velocities using the kinematic equations.
- vi) Determine the coefficient of restitution.
- vii) Repeat the observations on varying heights, collect the data in a tabular form, and draw your conclusion on the nature of collision.

Here :

$e = 1 \Rightarrow$ perfectly elastic collision, $e = 0 \Rightarrow$ perfectly inelastic collision and $0 < e < 1 \Rightarrow$ inelastic collision.

Remember that if a body goes up through the same height as that during the fall, it is an elastic collision.

Project 5.2: Study the validity of the work energy theorem for a heavy solid under free fall motion.

Materials you require: A heavy solid ball, a ruler, a measuring tape, a stopwatch or a high-speed camera, and a mass measuring scale.

Theory: Use $W = \Delta K = K_f - K_i$ (work-energy theorem), $W = mgh$, $K_f = \frac{1}{2}mv^2$ and $K_i = 0$ for $u = 0$ initially before fall. Therefore, $mgh = \frac{1}{2}mv^2$

Procedure you may follow: Measure the mass of the ball and the height through which the ball is allowed to fall. Find the time of fall using the stopwatch or the high-speed camera. Record the data in a tabular form and calculate v using, $v = u + gt$ or $v^2 = u^2 + 2gh$.

And, show $W = K_f$

Or, $mgh = \frac{1}{2}mv^2$

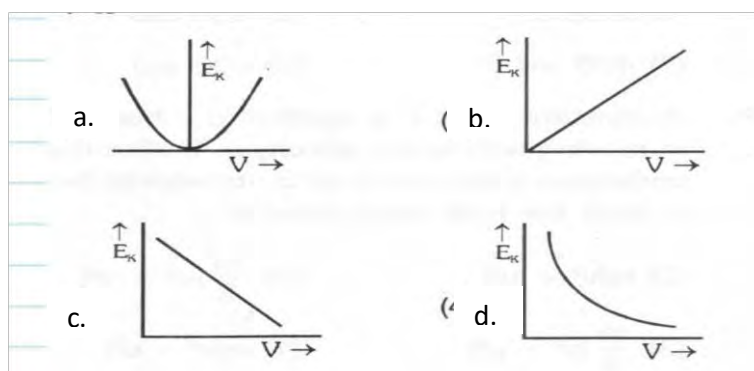
Finally, draw your conclusion about the validity of the theorem

Exercise

1. A ball of mass m collides normally with a wall at speed u and rebounds with the same speed. What is the change in the momentum of the ball?
 - a. $2mu$
 - b. ero
 - c. mv^2
 - d. $\frac{1}{2}mv^2$
2. A man hanging a water full bucket of 5 kg mass on his hand follows a 50 m level road. What is the work done by him?
 - a. 250 joule
 - b. 10 joule
 - c. 0 joule
 - d. 5 joule.
3. The power of a water pump is 2 kW. How much amount of water can it raise to a height of 10 m in 1 minute ($g = 10 \text{ ms}^{-2}$)?
 - a. 1000 liters
 - b. 100 liters
 - c. 1200 liters
 - d. 2000 liters
4. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. Which is the best alternative so that the power delivered to the body at t is proportional to it?
 - a. t

- b. t^2
 - c. $\sqrt{t}$
 - d. $\sqrt[3]{t}$
5. A body is acted upon by a force which is proportional to the distance covered, s . Which is the best alternative so that the work done by the force is proportional to it?
- a. s
 - b. s^2
 - c. $\sqrt{s}$
 - d. s^3
6. The water in a dam at an altitude is distributed to an urban area. Which of the following best represents the transformation of energy?
- a. PE to KE
 - b. KE to PE
 - c. chemical to light
 - d. none of the above
7. A meter of mass 800g pivoted at one end and is displaced through an angle of 60° . What is the increase in potential energy?
- a. 2J
 - b. 4J
 - c. 20J
 - d. 0.20J
8. If two balls, each with a mass of 0.06 kg, are moving in opposite directions at a speed of 4 ms⁻¹, collide, and rebound with the same speed of 4 ms⁻¹, what is the impulse imparted to each ball due to the collision?
- a. 0.48 ms⁻¹
 - b. 0.53 ms⁻¹
 - c. 0.81 kg ms⁻¹
 - d. 0.92 kg ms⁻¹
9. Which is the best alternative so that the work done by the external forces on a system changes?
- a. total energy
 - b. kinetic energy
 - c. potential energy
 - d. chemical energy

10. Which of the following represents the best suited relation between K.E and velocity?



11. A body of weight W_1 is suspended from the ceiling of a room through a chain of weight W_2 . What is the force exerted by the ceiling on the chain?
- W_1
 - W_2
 - $W_1 + W_2$
 - $W_1 - W_2$
12. Two bodies of masses 3 kg and 4 kg are acted upon by the same forces. If the acceleration of the lighter body is 2 ms^{-2} , what will be the acceleration of the heavier body?
- 1 ms^{-2}
 - 1.5 ms^{-2}
 - 2 ms^{-2}
 - 2.5 ms^{-2}
13. (a) Explain what is meant by work.
 (b) Obtain an expression for the work done by a constant force.
 (c) A man carrying a load of 200N on his head climbs up a hill through a height of 500 m. Do you agree he is doing a work? Justify it.
14. (a) Explain what you mean by the term work- energy theorem.
 (b) Explain how the principle of conservation of energy applies to a man sliding from rest down a vertical pole if there is a constant force of friction acting on him.
15. (a) How would you employ the law of conservation of mechanical energy to work in real-world situations by accounting for friction?
 (b) An 850 kg roller-coaster is released from rest at Point A of the track shown in the figure. Assume there is no friction or air resistance between Points A and C. How fast is the roller-coaster moving at Point B? (Ans: 30 ms^{-1})

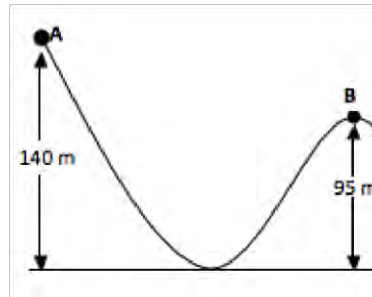


Fig 5.19

16. (a) Can the KE of a body ever be negative?
- (b) The momentum of a body is increased by 100%. What is the percentage increase in KE?
17. (a) Explain how the mechanical energy of a freely falling body remains conserved before reaching the ground.
- (b) When a constant force is applied to a body moving with constant acceleration, is the power of the force constant? If not, how would the force have to vary with speed for power to be constant?
18. (a) Define the coefficient of friction.
- (b) A 68.5-kg skater moving initially at 2.40 ms^{-1} on rough horizontal ice comes to rest uniformly in 3.52 s due to friction from the ice. What force does friction exert on the skater? (Ans: -46.7 N)
19. (a) Show that mechanical energy is conserved in a given system.
- (b) Show the principle of conservation of mechanical energy.
- (c) A block of weight 150N is pulled 20 m along a horizontal surface at constant velocity. Calculate the work done by the pulling force if the coefficient of the kinetic friction is 0.20 and the pulling force makes an angle of 60° with the vertical. (Ans: 537.89J, Hint: **The vertical component of the pulling force reduces the normal reaction.**) (3)
20. (a) Differentiate between elastic and inelastic collision.
- (b) Explain the elastic collision in one dimension.
- (c) A 650 Kw power engine of a vehicle of mass $1.5 \times 10^5 \text{ kg}$ is rising on an inclined plane of inclination 1 in 100 with a constant speed of 60 km/hr. Find the frictional force between the wheels of the vehicle and the plane. (Ans: $2.4 \times 10^4 \text{ N}$)
21. (a) State work – energy theorem.
- (b) A stationary mass explodes into two parts of mass 4 units and 40 units respectively. If the larger mass has an initial KE of 100J, find the initial KE of the smaller mass. (Ans: 1000J)
- (c) Comment on the increase in energy for the smaller mass in the above problem.

22. A ball with a mass of 0.1 kg is thrown vertically upwards with a velocity of 20 ms^{-1} . What is the potential energy at the maximum height? What is the potential energy of the ball when it reaches three-quarters of the maximum height while moving upwards? (Ans: 20J, 15J)
23. A block with a mass of 20 kg slides down the surface of a bowl with a diameter of 0.5 meters, moving from the top to the bottom. Calculate the speed of the ball as it reaches the bottom. (Ans: 2.24 ms^{-1})
24. An object moves along a certain path from point A with position vector $\vec{r}_1 = 3\hat{i} + 2\hat{j}$ to point B with position vector $\vec{r}_2 = 2\hat{i} + 3\hat{j}$ under the force $\vec{F} = \hat{i} + \hat{j}$. Find out the work done by the force. (Ans: 0 J)
25. A particle moves along the x-axis from $x=0$ to $x=5$ m under the influence of a force given by $F = 3x^2 - 2x + 7$. Find the work done in this force. (Ans: 135J)
26. A car of mass 1000 kg moves at a constant speed of 20 ms^{-1} along a horizontal road where the frictional force is 200N. Calculate the power developed by the engine. If the car, now, moves up an incline at the same constant speed, calculate the new power developed by the engine. Assume that the frictional force is still 200N and that $\sin \theta = \frac{1}{20}$, where θ is the angle of the inclination to the horizontal. (Ans: 4 kW, 14Kw)
27. While playing a football match, player A collided and got entangled with player B who was playing for the opposite team and running from the opposite side. The mass of player A was 40 kg and the mass of player B was 60 kg. If player B was running with a velocity of 3 ms^{-1} and player A was running with a velocity of 4 ms^{-1} , find the velocity and direction of both players after collision assuming other forces were negligible. (Ans: -0.2 ms^{-1})
28. A train of mass $2.0 \times 10^5 \text{ kg}$ moves at a constant speed of 72 km h^{-1} up a straight incline against a frictional force of $1.28 \times 10^4 \text{ N}$. The incline is such that the train rises vertically 1.0 m for every 100 m traveled along the incline. Calculate (i) the rate of increase per second of the train and (ii) the necessary power developed by the train. (Ans: (i) 400kW (ii) 656kW)
29. A stationary radioactive nucleus of mass 210 units disintegrates into an alpha particle of mass 4 units and a residual nucleus of mass 206 units. If the kinetic energy of the alpha particle is E, calculate the kinetic energy of the residual nucleus. (Ans: $\frac{2}{103}E$)
30. A railway truck of mass $4 \times 10^4 \text{ kg}$ moving a velocity of 3 ms^{-1} collides with another truck of mass $2 \times 10^4 \text{ kg}$ which is at rest. The coupling joins and the trucks move off together. What fraction of the first truck's initial kinetic energy remains as kinetic energy of two trucks after the collision? (Ans: $\frac{2}{3}$)
31. A bullet of mass 0.010 kg is fired vertically with a velocity of 100 ms^{-1} into a wooden block of mass 0.190 kg suspended by a long string above the gun. If the bullet is embedded in the block, through what height does the block move? (Ans: 1.25 m)

32. A vehicle of mass 2000 kg travelling at 10ms^{-1} on a horizontal surface is brought to rest at a distance of 12.5 m by the action of its brakes. Calculate the average retarding force. What power must the engine develop in order to take the vehicle up an incline of 1 in 10 at a constant speed of 10ms^{-1} , if the frictional resistance is equal to 200N? (Ans: 8000N, 22kW)
33. You push your physics book 1.50 m along a horizontal tabletop with a horizontal force of 2.40 N. The opposing force of friction is 0.600 N. (a) How much work does your 2.40 N force do on the book? (b) What is the work done on the book by the friction force? (c) What is the total work done on the book? (Ans: (a) 3.6J, (b) 0.9J, (c) 2.7J)
34. How many joules of energy does a 100-watt light bulb use per hour? How fast would a 70 kg person have to run to have that amount of energy? (Ans: $3.6 \times 10^5\text{J}$, 100ms^{-1})
35. A ball with a mass 0.420 kg is initially moving with a speed of 2.00ms^{-1} . A player kicks the ball, exerting a constant force of magnitude 40.0N in the same direction as the ball's motion. Over what distance must her foot be in contact with the ball to increase the ball's speed to 6.00ms^{-1} ? (Ans: 0.168 m)
36. A worker applies a constant horizontal force of 80N to a block of ice on a smooth horizontal floor. The frictional force is negligible. The block starts from rest and moves 11 m in 5 s. (i) What is the mass of the block of ice? (ii) If the worker stops pushing at the end of 5 s, how far does the block move in the next 5 s? (Ans: 90.9 kg, 22m)
37. A 6.0 kg box moving at 3.0ms^{-1} on a horizontal, frictionless surface runs into a light spring of force constant 75 N/cm. Use the work-energy theorem to find the maximum compression of the spring. (Ans: 0.085m)
38. A tandem (two-person) bicycle team must overcome a force of 165N to maintain a speed of 9.00ms^{-1} . Find the power required per rider, assuming that each contributes equally. Express your answer in watts and in horsepower. (Ans: 742.5 watt) Arrange questions as per the units unless they are integrated.

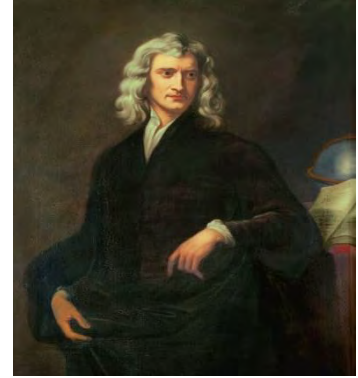
Answer to the Multiple Choice Questions:

Q.N	1.	2.	3.	4.	5.	6.
Ans	a	c	c	a	b	a
Q.N	7.	8.	9.	10.	11.	12.
Ans	a	a	b	a	c	b

Chapter 6

Circular Motion

Think of circular motion as the result of a balance between two forces—one centrifugal, the other centripetal (toward the center)—rather than as the result of one force, a centripetal force, which constantly deflects the body away from its inertial path in a straight line. Newton extended the dynamics of intermittent colliding bodies to his analysis of bodies subject to a continuous force. He determined the nature of the force directed toward the center of a circle that will move the body uniformly around the circle (i.e., the solution to the direct problem of uniform circular motion). In this analysis, the circular path is approximated by a polygonal path of n sides, where the number of sides of the polygon will ultimately increase without limit. As the number of sides of the polygon was increased, the polygonal path approached a circular path.



Sir Isaac Newton (1642-1726)

6.1 Introduction to Circular Motion

Let's engage in the activity to gain an understanding of circular motion:

Activity 6.1

- Take a ball or a stone weighing approximately 100 grams and tie it with an inextensible string.
- Hold one end of the rope and allow it to move in a circular path.
- Try to identify the force on the stone and the rope separately with the help of your teacher.
- Guess the velocity if the rope breaks at A, B, C and D as indicated in the figure.
- Find the change in velocity of the ball between the points A and B, A and C and C and D if $|V_A| = |V_B| = |V_C| = |V_D| = 5\text{ms}^{-1}$.

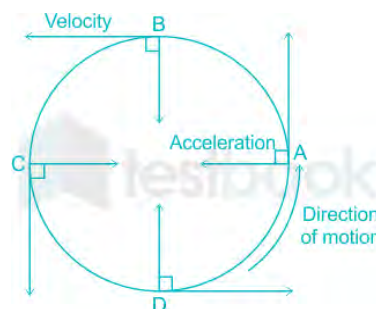


Fig 6.1: Circular motion

A body is said to be in a circular motion if it remains at an equidistance from a fixed point. This fixed point is called the center of the circular path, and the distance is known as the radius vector.

In circular motion, there is a change in velocity at every point along the path. The velocity at each point is given by the tangent drawn at that point.

One writes,

$$\begin{aligned}\text{Velocity} &= \text{magnitude} + \text{direction} \\ &= \text{speed} + \text{direction}.\end{aligned}$$

At each point, there is a change in the direction of the motion. Therefore, the change in velocity is due to the change in direction.

Since there is a change in velocity, acceleration is present, and thus any object undergoing circular motion experiences a force. The force directed towards the center is called the centripetal force, while the force directed away from the center is sometimes referred to as the centrifugal force. However, it's important to note that centrifugal force is more accurately described as a pseudo-force resulting from the inertia of the object. Both forces have equal magnitudes but are directed oppositely.

A body moving at a constant speed in circular motion has no net work done on it.

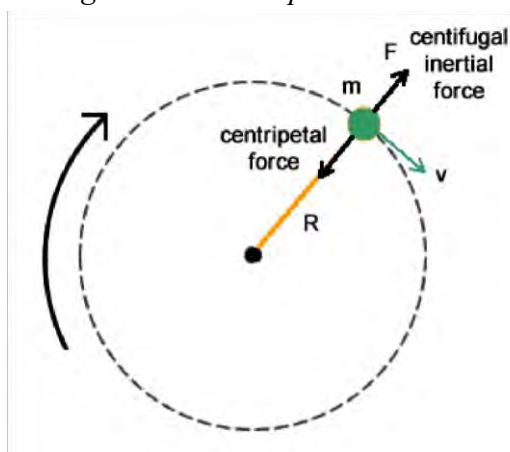


Fig 6.2: Concept on centripetal and centrifugal force.

Examples:

- i) If you whirl a bucket with some water in it in a vertical circle, the water doesn't fall down due to the centrifugal force that pushes it away from the centre, which is balanced by the inwards centripetal force.
- ii) While travelling in a bus we lean outwards when the bus undergoes a circular path due to the centrifugal force that finally is balanced by the inwards centripetal force.
- iii) When a small piece of stone tied to a string is whirled in a circular path, the stone tries to go away by detaching the string due to the action of the centrifugal force and the string is pulling the stone towards the centre due to the centripetal force.
- iv) Mud flying off a tire is an example of centrifugal force, while a satellite orbiting the Earth is an example of centripetal force.

- v) Due to the Earth's orbiting motion, it experiences centrifugal force, resulting in flattening at the poles and bulging at the equator.

6.1.2 Angular Displacement and Velocity

Activity 6.2

- i) Create the largest possible circular path on your school ground using a chalk mark.
- ii) Stand at a point marked as A on the path with a stopwatch, and begin moving around the circle.
- iii) Stop at a point on the circle, marking it as B, before completing the circle, and note the time it took to reach there as 't'.
- iv) Measure the distance between points A and B using a string, naming it as 's'. Then, connect points A and B with the center O of the constructed circle. Measure the angle subtended by AB at the center and name it as θ .
- v) Begin your new journey from the initial point A once more, completing a cycle. Note the time taken and designate it as T. Repeat this process for two complete cycles, noting the total time as 2T. Then, continue by completing three full cycles, recording the total time taken as 3T, and so forth.
- vi) The time T has a special feature and try to recognize it.

Imagine the swift motion involving multiple complete cycles within a second and attempt to comprehend it. Here, the distance 's' between two points represents the linear distance, while the angle ' θ ' denotes the angular displacement. The ratio of angular displacement to time (θ/t) yields the angular velocity ' ω ', and the time 'T' required for a complete oscillation defines the period. Furthermore, the frequency 'f' corresponds to the number of cycles completed in one second.

Angular Displacement:

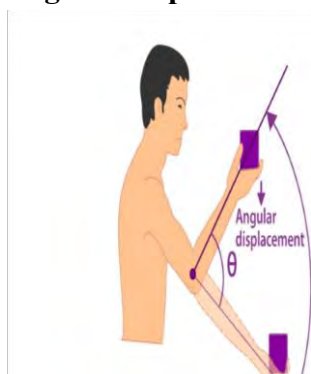


Fig 6.3: Showing how angular displacement arises.

It is the total angle traced out by the radius vector of a circle at its center. It is denoted by θ .

Let a particle initially at point A reaches to point B subtending an angle $\text{BOA} = \theta$ at its centre.
Therefore, θ is the angular displacement.

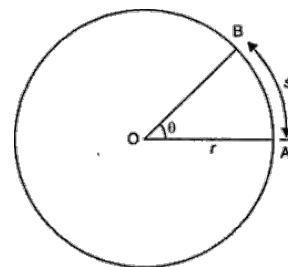


Fig 6.4: Showing how angular displacement relates with linear displacement.

Angular Velocity:

It is the rate of change of angular displacement and is denoted by ω , read as omega.

Therefore,

Angular Velocity = angular displacement / time

$$\therefore \omega = \theta/t$$

For the instantaneous value of ω ,

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Unit of ω

$$\omega = \theta/t = \text{rad/second} = \text{rads}^{-1}$$

Angular Acceleration:

The rate of change of angular velocity is the angular acceleration.

i.e Angular Acceleration,

$$\alpha = \frac{\text{Angular Velocity Change}}{\text{time}} \\ = \frac{\omega - \omega_0}{t}$$

$$\gg \omega = \omega_0 + \alpha t.$$

Time Period:

The time taken by a particle to make a complete oscillation is the time period, T.

From $\omega = \frac{\theta}{t}$, take $\theta = 2\pi$ for a complete oscillation and $t = T$ for that.

$$\therefore \omega = \frac{2\pi}{T}.$$

Frequency:

Frequency refers to the number of cycles made in 1 second, denoted by f.

Sample Question:

1. From the definition of time period and frequency just above, try to deduce that frequency is the reciprocal of the time period i.e. $f = \frac{1}{T}$.

Worked out Example:

Q: What is the angular displacement of each of the following hands of a clock in 1.00 h?

Solution: Here, clockwise direction is taken as the negative angle.

For a second hand,

24. h corresponds to 60 minutes that requires a 60 complete rotations by the second hand.

Therefore, required angle, $\theta = -60 \times 2\pi = -377 \text{ rad}$.

For the minute hand, 1 h corresponds to 1 complete rotation $\equiv -2\pi \text{ rad} = -6.28 \text{ rad}$

And for the hour hand, 12 h $\approx$ 1 complete rotation

Hence $1 \text{ h} \approx \frac{1}{12} \text{ rotation} \Rightarrow -\frac{1}{12} \times 2\pi = -0.52 \text{ rad}$

Sample Question:

1. Find the angular displacement that a 0.52 m wide door rotates to make a linear distance of 46.7 cm. (Ans: 0.90 rad)

6.2 Relation between Linear Velocity and Angular Velocity

vi) Linear velocity is the rate of change of linear distance i.e, $v = \frac{s}{t} \approx \frac{ds}{dt}$

vii) Angular velocity is the rate of change of angular displacement i.e $\omega = \frac{\theta}{t} \approx \frac{d\theta}{dt}$

The relationship between linear velocity (v) and angular velocity (ω) is defined by the radius of rotation (r). It can be expressed by the equation, $v = r \times \omega$

Let a particle of mass m is undergoing a circular motion in a circle of radius r and centre at O . The particle reaches to a point B during the time t from a point A initially.

As referred to the outlined figure,

$$\theta = \frac{s}{r}$$

$$\Rightarrow s = r\theta$$

Differentiating above equation with respect to time.

$$\frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\therefore v = r\omega$$

It's the relation between the linear velocity and the angular velocity

Again, differentiating $v = r\omega$,

$$\frac{dv}{dt} = r \frac{d\omega}{dt} = r\alpha$$

$$\therefore a = r\alpha, \alpha \text{ being the angular acceleration.}$$

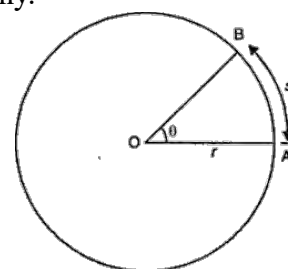


Fig 6.5

Worked Out Example:

1. The angular displacement of a particle is given as $\theta(t) = 2e^{0.75t} + 3t^2 - t$. Find the angular velocity in 2 s elapse.

Solution:

$$\text{Here, } \theta(t) = 2e^{0.75t} + 3t^2 - t.$$

$$\text{Therefore, } \omega = \frac{d\theta}{dt} = 2 \times 0.75 e^{0.75t} + 6t$$

$$\begin{aligned} \Rightarrow \text{At } t = 2 \text{ s, } \omega &= 2 \times 0.75 \times e^{0.75 \times 2} + 6 \times 2 = \\ &= 1.50 \times e^{1.50} + 12 \\ &= 18.72 \text{ rad/s} \end{aligned}$$

Uniform Circular Motion:

It is the circular motion in which a particle undergoing such a motion has equal angular displacement in equal time interval.

Note: The work done in a uniform circular motion is zero because the angle between force and displacement is zero.

6.3 Centripetal Acceleration

Since there is a change in velocity in a circular motion, there is some acceleration present that is known by the name centripetal acceleration. The centripetal acceleration is the acceleration experienced by an object moving in a circular path. It is always directed towards the center of the circle along the radius and is responsible for keeping the object moving in its circular track. The magnitude of centripetal acceleration can be calculated using the formula: $a = v^2/r$ where v is the linear velocity of the object and r is the radius of the circular path.

Let us consider a body of mass m undergoing a uniform circular motion in a circle of radius r and center at O .

Initially, the body is at point A ($t=0$) that reaches to a point B in a small time Δt such that V_A and V_B are the velocities at the respective points and $|V_A| = |V_B| = v$. The angular displacement subtended at the center is θ .

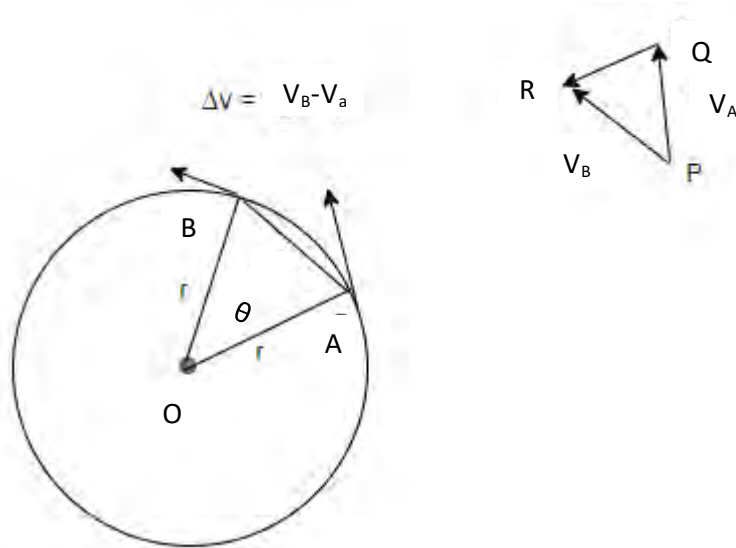


Fig 6.7: Showing how centripetal acceleration arises.

Let $\overrightarrow{PQ} = V_A$ and $\overrightarrow{PR} = V_B$ as indicated in the figure acting at an angle θ .

Hence one writes:

$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$

$$V_A + \overrightarrow{QR} = V_B$$

$$\therefore \overrightarrow{QR} = V_B - V_A$$

$= \Delta V$ is the change in velocity.

In the limiting case of $\Delta t \rightarrow 0$, OAB is a triangle.

Therefore, in the similar triangles

PQR and OAB ,

$$\frac{QR}{PQ} \text{ and } \frac{AB}{OA}$$

$$\text{Or, } \frac{\Delta V}{\Delta t} = \frac{v \Delta t}{r} \quad \because PQ = V_A$$

$$\Rightarrow PQ = |V_A| = V$$

And distance $AB = \text{velocity} \times \text{time}$

$$\text{Or, } \frac{\Delta V}{\Delta t} = v^2/r$$

$$\text{But } \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = a = v^2/r$$

$$\Delta t \rightarrow 0$$

Hence, $a = v^2/r = r\omega^2$ is the expression for the centripetal acceleration.

$$\text{Since } \omega = \frac{2\pi}{T} = 2\pi f,$$

$$\begin{aligned} a = v^2/r = r\omega^2 &= r (2\pi/T)^2 \\ &= r (2\pi f)^2 \\ &= 4\pi^2 r f^2. \end{aligned}$$

In the limiting case, $\Delta t \rightarrow 0$

$$\Rightarrow B \rightarrow A$$

$\Rightarrow R \rightarrow Q$ and $\rightarrow QR = \Delta V$ becomes normal to PQ.

Hence, centripetal acceleration acts normally to the velocity and has a changing direction.

The radial acceleration ($a = \frac{v^2}{r} = r\omega^2$) can be defined as the acceleration of the object along the radius of the curved path. It is a vector quantity. The direction of the radial acceleration is always towards the center.

Worked Out Examples:

25. In a test of a “g-suit”, a volunteer is rotated in a horizontal circle of radius 7m. What is the period of rotation at which the centripetal acceleration has a magnitude of i) 3g? ii) 10g?

Solution: Here,

Radius, $r = 7\text{m}$,

- i. Radial acceleration, $a = 3g$,
- ii. Radial acceleration, $a = 10g$

We have, i. $a = r\omega^2 = r4\pi^2/T^2$

$$T^2 = r4\pi^2/a = 7 \times 4\pi^2/3g$$

$$\text{Or, } T = 3.07\text{s}$$

$$\text{For ii. } T^2 = 7 \times 4\pi^2/10g$$

$$\therefore T = 1.68\text{s}$$

26. A model of a chopper rotor has four blades, each 3.40m in length from the central shaft to the blade tip. The model is rotated in a wind tunnel at 550rev/min. i) what is the linear speed of the blade tip, in ms^{-1} ? ii) What is the radial acceleration of the blade tip expressed as a multiple of the acceleration of gravity, g ?

Solution: Here,

Radius, $r = 3.40\text{m}$

Frequency, $f = 550\text{rev/min}$

$$= 550/60 \text{ rev/s} = 9.17\text{rev/s}$$

Linear velocity, $v = ?$

Radial acceleration, $a = ?$

We have, $v = r\omega = r2\pi f$

$$= 3.40 \times 2\pi \times 9.17$$

$$= 196\text{ms}^{-1}$$

Radial acceleration, $a = r\omega^2$

$$= 3.4 \times 4\pi^2 (9.17)^2$$

$$= 1.13 \times 10^{-4} \text{ ms}^{-2}.$$

$$= 1.17 \times 10^7 \text{ kg is the radial acceleration.}$$

Worked Out Example

1. A child of 15 kg is rotating a merry-go-round at an angular velocity of 1.97 rad/s. If the distance from the child to the center of the merry-go-round is 1.75 m, what is the centripetal acceleration of the child and the centripetal force it experienced?

Solution:

Here,

Mass of the child, $m = 15 \text{ kg}$

Angular velocity, $\omega = 1.97 \text{ rad/s}$

Radius, $r = 1.75 \text{ m}$

Centripetal acceleration, $a = ?$

Centripetal force, $F = ?$

$$\begin{aligned}\text{We have, } a &= r\omega^2 = 1.75 \times 1.97^2 \\ &= 6.79 \text{ ms}^{-2}\end{aligned}$$

$$\begin{aligned}\text{And centripetal force, } F &= m r \omega^2 = 15 \times 1.75 \times 1.97^2 \\ &= 101.87 \text{ N}\end{aligned}$$

6.4 Centripetal Force

Centripetal force is the force directed towards the center of a body undergoing uniform circular motion. As we have derived an expression for centripetal acceleration, any object in circular motion experiences a force directed towards the center to maintain uniform circular motion.

The centripetal acceleration,

$$\therefore a = \frac{v^2}{r} = r\omega^2$$

$$\text{Therefore, } F = ma = mv^2/r = mr\omega^2 \quad \because \omega = 2\pi/T = 2\pi f$$

When a body is whirled in a circular path, the tension produced in the string acts as the centripetal force. Upon detaching the string immediately, the body moves away due to the centrifugal force.

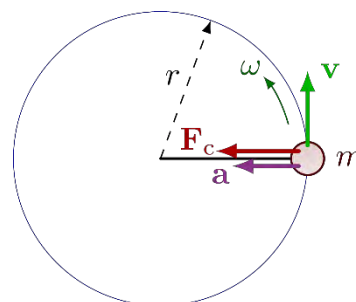


Fig 6.8: Showing centripetal force

When we travel in a vehicle along a highway road with innumerable bends, we lean outward at every moment along the bends. On a circular track near the bends, the vehicle is subjected to a centripetal force, creating a similar sensation on the lower part of our body in contact with the vehicle. The upper part, therefore, leans outwards in opposition to this force, which is compensated for by the centrifugal force.

While undergoing a circular path, a body experiences centrifugal force, given by

$$F = mr\omega^2$$

For a longer string, r increases, thereby requiring more force to maintain the circular motion.

There is a change in velocity due to the change in direction. In this case, the force acts perpendicular to the displacement or velocity, as given by $\mathbf{F} \cdot d\mathbf{s} = 0$, resulting in no change in speed.

6.5 Motion in a Horizontal Circle: Conical Pendulum

A conical pendulum is a simple pendulum that whirls in a circle on a horizontal plane tracing a right circular cone at a constant angle with the vertical. The spherical pendulum and the vertical pendulum are its examples.

A swing ride at the carnival, where people sit in seats attached to the



Fig 6.9 A swing ride, horizontal circle.

long chains, and they are swung around in circles is the real life example of a conical pendulum. The main application of the conical pendulum was used as a fly ball governor (centrifugal governor) in a steam engine during the Steam Age in the 1800s.

Let us consider a body of mass 'm' attached to one end of a string of length l and whirled in a horizontal circle with a uniform velocity v. The string makes an angle θ with the vertical.

The tension T produced on the string is resolved as $T\cos\theta$ directed vertically upward which balances the weight of the body and $T\sin\theta$ as an inward-directed component that provides the necessary centripetal force.

I.e. $T\cos\theta = mg$ ---- (6.1)

And $T\sin\theta = \frac{mv^2}{r}$ ---- (6.2)

(6.1) ÷ (6.2) $\Rightarrow$

Fig 6.10

$$\tan\theta = \frac{v^2}{rg}$$

But $\tan\theta = \frac{r}{h}$ in the figure.

$$\therefore \frac{r}{h} = \frac{v^2}{rg}$$

$$\text{Or, } \frac{g}{h} = \frac{v^2}{r^2} = r\omega^2 = \left(\frac{2\pi}{T}\right)^2$$

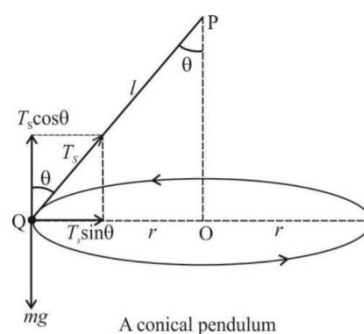
$$\begin{aligned} \therefore T &= 2\pi \sqrt{\frac{h}{g}} \\ &= 2\pi \sqrt{\frac{l\cos\theta}{g}} \quad \because h = l\cos\theta \end{aligned}$$

It is the expression for the time period of the conical pendulum.

Squaring (6.1) and (6.2) and on adding.

$$T^2 (\cos^2\theta + \sin^2\theta) = (mg)^2 + \left(\frac{mv^2}{r}\right)^2$$

$$\therefore T = mg \sqrt{1 + \left(\frac{v^2}{rg}\right)^2}, \text{ the equations hence give the time period.}$$



- viii) The constant physical quantities in a circular motion on a horizontal plane are speed, kinetic energy, and angular momentum

Worked Out Examples:

27. An object of mass 0.5kg is rotated in a horizontal circle by a 1m long string. The maximum tension in the string before it breaks is 50N. What is the greatest number of revolutions per s of the object?

Solution: Here,

Mass, $m = 0.5\text{kg}$,

Length, $l = 1\text{m} = \text{radius, } r$

Maximum tension, $t = 50\text{N} = F$

Frequency, $f = ?$

We have, $F = m r \omega^2 = m r 4\pi^2 f^2$

$$\text{Or, } f = \frac{1}{2\pi} \sqrt{\left(\frac{F}{mr}\right)}$$

$$= \frac{1}{2\pi} \left(\frac{50}{0.5 \times 1}\right)$$

$$\text{Or, } 50 = 0.5 \times 1 \times f^2$$

$$= 1.6 \text{ rev/s}$$

28. A certain string breaks when a weight of 25N acts on it. A mass of 500g is attached to one end of the string of 1m long and is rotated in a horizontal circle. Find the greatest number of revolutions per minute which can be made without breaking the string?

Solution: Here,

Maximum tension in the string, $T_{\max} = 25\text{N}$

Mass, $m = 500\text{g} = 0.5 \text{ kg}$

Radius, $r = 1\text{m}$

Greatest number of revolution per minute =?

We have,

$$T_{\max} = m r \omega^2$$

$$= 0.5 \times 1 \times 4\pi^2 f^2$$

$$\text{Or, } f = \frac{1}{2\pi} \sqrt{(T_{\max}/0.5)} = 1.125 \text{ rev/s}$$

$$\therefore f = 67.5 \text{ rev/min is the greatest number of revolution.}$$

6.6 Motion in a Vertical Circle

Activity 6.2

Take a moderately heavy mass and whirl it in a uniform vertical circular motion using a weak string. (The mass so chosen that it is heavy enough to provide noticeable tension but not so heavy that it immediately breaks the weak string.)

Determine the position in the circular path where the tension in the string is minimized to understand the safest conditions for whirling the object. Remember, the string is most likely to break at the point of maximum tension, which occurs at the bottom of the path.

In vertical circular motion, the tension varies significantly with the position of the object in its path due to the addition of gravitational forces. The tension is typically minimized when the object is at the top of the circular path.

The motion of a particle in a vertical circle has been outlined in fig 6.12. The particle sometimes lies at the lowest point C and sometimes at the highest point A. Roller Coaster rides and Ferris wheel are two examples of motion in a vertical circle.

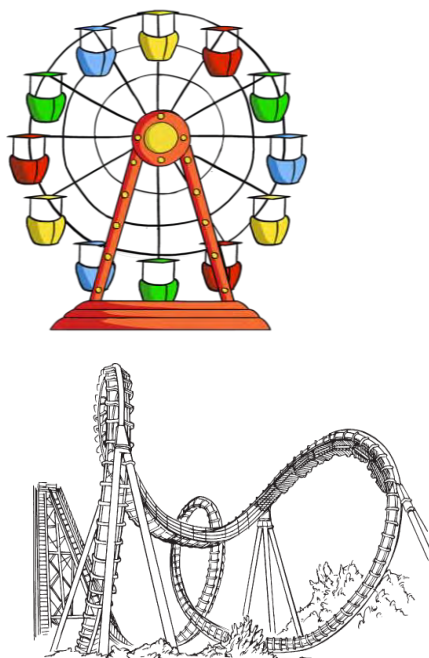


Fig 6.11: A Roller coaster

Let's consider that a body of mass m is attached to one end of a string and whirled in a vertical circle. During circulating, the path is 'p' where ' mg ' is the weight of the body directing vertically downwards and T is the tension on the string. On resolving ' mg ', $mg\cos\theta$ acts opposite to T and $mg\sin\theta$ normal to the length as shown and $|V_A| = |V_B| = |V_C| = |V_D| = v$

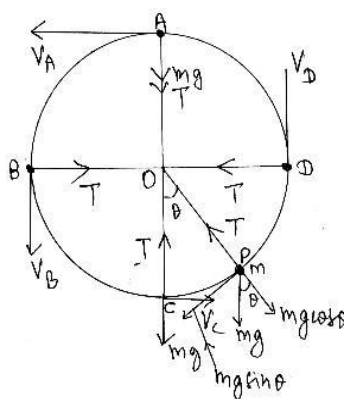


Fig 6.12: Motion in a vertical circle

Here we see the net force at p,

$$T - mg\cos\theta = F$$

$$T - mg\cos\theta = \frac{mv^2}{r}$$

$$\therefore T = \frac{mv^2}{r} + mg\cos\theta$$

Cases:

At the lowest point A, $\theta = 0^\circ$

$$\therefore T_A = \frac{mv^2}{r} + mg \cos 0^\circ$$

$$\text{Or, } T_A = \frac{mv^2}{r} + mg = T_{\max}$$

At the highest point C, $\theta = 180^\circ$

$$\therefore T_C = \frac{mv^2}{r} + mg \cos 180^\circ$$

$$\text{Or, } T_C = \frac{mv^2}{r} - mg = T_{\min}$$

Hence there is a maximum tension at the lowest point and $T_A = T_{\max} = \frac{mv^2}{r} + mg$.

And, the string has a high chance of breaking here.

And, a minimum tension is forwarded at the highest point, $T_C = T_{\min} = \frac{mv^2}{r} - mg$.

At point B, $\theta = 90^\circ$

$$\therefore T_B = \frac{mv^2}{r} + mg \cos 90^\circ = \frac{mv^2}{r}$$

$$\text{I.e. } T = \frac{mv^2}{r}.$$

Hence at the horizontal points, the tension equals with the centripetal (or centrifugal force).

To evaluate the minimum tension in the vertical circle, let $T_c = 0$ at the highest point.

$$\text{Now, } T_C = \frac{mv^2}{r} - mg$$

$$\text{Or, } 0 = \frac{mv^2}{r} - mg$$

$$\gg v = \sqrt{(gr)}$$

It is the value of the minimum velocity at the highest point for the body to keep on a vertical circle and is called the critical velocity.

Below the critical velocity, the body can't go around the circle as the string loses and gets slacked.

In a similar sense, one can evaluate the value of the minimum velocity at the highest point.

For this,

Total M. Energy at A = Total M. Energy at C

$$\text{Or, } (K.E + P.E)_A = (K.E + P.E)_C$$

$$\text{Or, } \frac{1}{2}m(v_A)^2 = \frac{1}{2}m(v_C)^2 + (P.E_C - P.E_A)$$

$$\text{Or, } \frac{1}{2}m(v_A)^2 = \frac{1}{2}m \cdot gr + mg \cdot (2r)$$

$$\text{Or, } (v_A)^2 = gr + 4gr = 5gr$$

$$\therefore v_A = \sqrt{(5gr)}$$

It gives the value of the minimum velocity at the lowest point for the body to go around the circle safely.

Worked Out Examples:

Q: An object of mass 8kg is whirled round in a vertical circle of radius 2m with a constant speed of 6ms^{-1} . Calculate the maximum and minimum tension in the string.

Solution:

Here,

Mass, $m=8\text{kg}$

Speed, $v=6\text{ms}^{-1}$

Radius, $r=2\text{m}$

Maximum tension, $T_{\text{max}}=?$

Minimum tension, $T_{\text{min}}=?$

We have,

Tension is maximum at the lowest point i.e.,

$$\begin{aligned} T_{\text{max}} &= \frac{mv^2}{r} + mg \\ &= 8 \times 6^2 / 2 + 8 \times 10 \\ &= 224\text{N} \end{aligned}$$

And, tension is minimum at the lowest point i.e.,

$$\begin{aligned} T_{\text{min}} &= \frac{mv^2}{r} - mg \\ &= 8 \times 6^2 / 2 + 8 \times 10 \\ &= 64\text{N}. \end{aligned}$$

Q: A stone of mass 500g is attached to a string of length 50cm which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a height of 100cm above the ground. The angular speed is increased slowly until the string breaks. In what position is this break most likely to occur and at what angular speed? Where will the stone hit the ground?

Solution: Here,

Mass = 500g = 0.5 kg

Maximum tension, $T_{\text{max}} = 20\text{ N}$

Angular speed, $\omega = ?$

Radius, $r = 0.5\text{m}$

Range, $R = ?$

The maximum tension in the string is at the lowest point of the circle where it breaks.

When it breaks at the lowest point at a height of 50 cm above the ground, it acts like a projectile fired horizontally.

$$\text{Now, } T_{\text{max}} = \frac{mv^2}{r} + mg$$

$$\text{Or, } v = \sqrt{[(T_{\text{max}} - mg) \times \frac{r}{m}]}$$

$$\therefore v = 3.875\text{ms}^{-1}.$$

It is the initial velocity for the projectile.

$$\begin{aligned} \text{Now from } v = r\omega, \omega &= \frac{v}{r} = 3.875/0.5 \\ &= 7.75\text{ rad/s} \end{aligned}$$

Now for the projectile motion, $u = v = 3.875\text{ms}^{-1}$

One writes,

$$h = ut + \frac{1}{2}gt^2$$

$$\text{Or, } 0.5 = 0 + \frac{1}{2} \times 10 \times T^2$$

It implies to give $T = 0.32\text{ s}$

$$\begin{aligned}
 \text{To find the range, } R &= v \cdot T \\
 &= 3.875 \times 0.32 \\
 &= 1.24\text{m}
 \end{aligned}$$

6.7 Application of Centripetal Force

The application of centripetal force mainly focuses on the automobiles while making a turn, the turn is an arc of a circular path and the friction on the wheels provides the necessary centripetal force to balance the motion.

6.8.1 Motion of a car on a level curved path

While taking the round, the tires of the car tend to leave road and move away from the centre of the curve.

Let a vehicle be undergoing a horizontal curved track of radius r . In the motion of the car, frictional force on the tires provides the necessary centripetal force, and lack of it will cause it to be thrown outside.

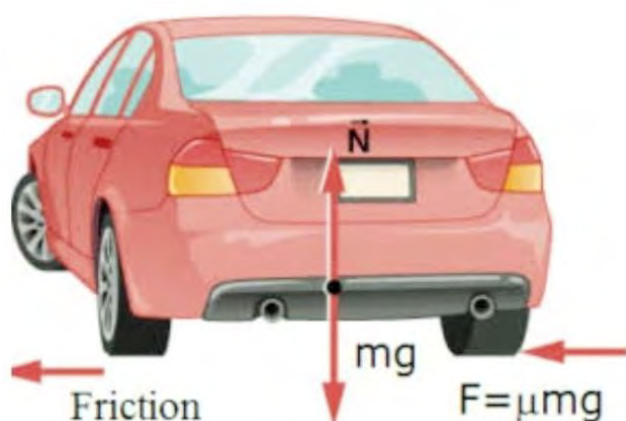


Fig 6.13 a: Rise of frictional force on a level road

Let F_1 and F_2 be the frictional forces on the two tires. Then the the road

From $f = \mu R$,

$$F_1 + F_2 = \mu (R_1 + R_2)$$

But $R_1 + R_2$ is balanced by mg .

$$\therefore F = F_1 + F_2 = \mu mg$$

$$\text{But } F = \frac{mv^2}{r} = \mu mg$$

$$\therefore v = \sqrt{(\mu gr)}$$

It is the maximum velocity for the vehicle to take a turn. If the velocity exceeds the above value, the vehicle may skid off and move out of the road.

6.8.2 Banking of Road

Generally, in the highway roads on the turnings, the outer edge of the road is raised through an angle with respect to the inner edge.

Such a raising is known as the banking angle. The purpose of such banking is to tilt the normal towards the center of the circular track in order to provide necessary

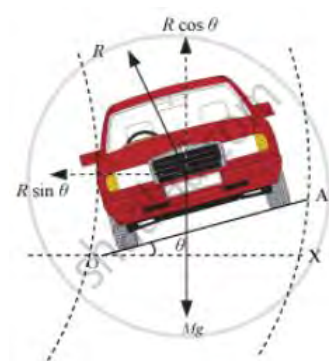


Fig 6.14 b: Banking angle

centripetal force by the inwards horizontal components and to save the tires.

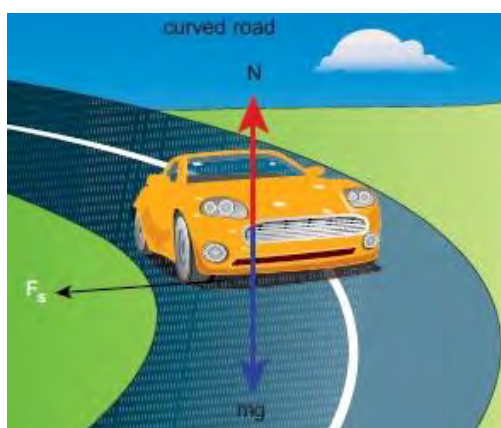


Fig 6.14 Illustrating banking angle

Sometimes the road may be icy or wet and may not have sufficient friction that saves the tires from weary, banking of the road is necessary in the case to take around the vehicle safely.

As referred to the figure, $R \cos \theta = (R_1 + R_2) \cos \theta$ balances the weight.

While $R \sin \theta = (R_1 + R_2) \sin \theta$ provides the necessary centripetal force.

i.e. $(R_1 + R_2) \cos \theta = mg$ ---6.3

And $(R_1 + R_2) \sin \theta = \frac{mv^2}{r}$ ---6.4

Now, $(6.4) \div (6.3) \Rightarrow \tan \theta = \frac{v^2}{rg}$

$\therefore \theta = \tan^{-1} \frac{v^2}{rg}$

Hence the value of banking is evaluated.

Worked Out Examples:

29. A racing car of 1000kg moves round a banked track at a constant speed of 108km/hr. Assuming the total reaction at the wheels is normal to the track and the horizontal

radius of the track is 100m, calculate the angle of inclination of the track to the horizontal and reaction at the wheels.

Solution:

Here,

Mass of the car, $m=1000\text{kg}$

Speed, $v=108\text{km/h}$

$$= \frac{108 \times 1000}{60 \times 60}$$

$$= 30\text{ms}^{-1}$$

Radius, $r=100\text{m}$

Banking angle, $\theta=?$

Reaction, $R=?$

We have,

$$\theta = \tan^{-1} \frac{v^2}{rg}$$

$$= \tan^{-1} \frac{30^2}{100 \times 10}$$

$$= \tan^{-1} \left(\frac{900}{1000} \right)$$

$$= \tan^{-1} 0.9$$

$$= 42^\circ$$

And $R \cos \theta = mg$

$$\therefore R = \frac{mg}{\cos \theta}$$

$$= (1000 \times 10) / \cos 42^\circ$$

$$= 13457\text{N}.$$

6.8.3 Bending of a Cyclist

When a cyclist goes round the circular track in a turn, it leans towards the center of the track by an angle θ as indicated in the figure.

Hence the normal reaction R from the ground to the cyclist along the line is at an angle θ at the center.

The R now can be resolved into two components.

The vertical component $R \cos \theta$ balances the weight while $R \sin \theta$ goes to balance the centripetal force.

Hence, $R \cos \theta = mg$

$$R \sin \theta = \frac{mv^2}{r}$$

$$\therefore \tan \theta = \frac{v^2}{rg}$$

$$\Rightarrow \theta = \tan^{-1} \frac{v^2}{rg}$$

Some Facts on Circular Motion

So far we have studied circular motion, there are three mathematical quantities i.e. Speed, Acceleration and Force as a primary matter of interest to analyze a circular motion.

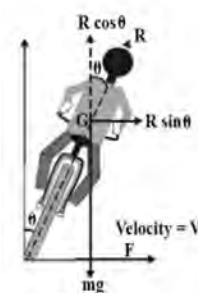


Fig 6.15: Motion of a cyclist in a banked

A repetitive closed motion is generally not allowed as a perfect circle. Hence the motion in a circle is very important in our daily life such as moon's revolution around the Earth, Earth's revolution around the sun, electron orbital motion round the nucleus, satellites going round the Earth and the mechanism of centrifuge used to segregate the unwanted impurities.

The angular displacement in a circular motion denoted by θ whose unit is radian is a vector quantity because $\theta = \frac{l}{r}$.

The direction of the angular displacement is given by the right-hand rule, according to which if the curled fingers represent the direction of motion, the thumb held perpendicular to the curvature of the fingers represents the direction of the angular displacement. Hence, by analyzing the same sense of the right-hand rule, the angular velocity has the direction of the thumb. For counterclockwise rotation, ω directs upwards, while it directs downwards for clockwise rotation.

According to Newton's first law of motion, a body can't change its direction itself without force. However, in circular motion, the change is due to the centripetal force. The body tries to gain its natural linear path, giving rise to what is called the centrifugal force. Hence, this is the reason why centripetal and centrifugal forces are equal but opposite in nature, respectively representing the forces of action and reaction. In cases of non-uniform circular motion, accelerations are defined in both the radial and tangential directions.

A body undergoing circular motion with increasing angular velocity experiences both centripetal and tangential accelerations resulting in a net acceleration.

$$a = \sqrt{a_c^2 + a_t^2}$$

Project Work:

Project 5.1 Find the angle of banking for a circular track of a highway for a safe turning of a vehicle.

Hint:

- ☐ Introduction and objective, for optimal banking angle.
- ix) Theoretical background, understanding of physics to enhance behind the banking angle and the forces involved such as centripetal force, gravitational force and the normal reaction force.
- ☐ Data collection methodology:
 1. Mark the edge of the track using a chalk or some color.
 2. Take two points on the arc marked, draw tangents and normal on the tangents at the points so that the normal meets at a point, giving rise to the radius, called the centre of the circle. (It can also be done by visual inspection or using a map/blueprint if available or by GPS device, smartphones or, using a mapping software.)
 3. Measure the radius of the possible circle.
- x) Take a specified value of velocity
 - ☐ Calculation steps and results: Use $\theta = \tan^{-1} \frac{v^2}{rg}$
 - ☐ Validation and discussion.
 - ☐ Conclusion and recommendations.
 - ☐ References and appendices.

Exercise

1. What type of motion does the moon execute around the sun?
 - a. uniform circular motion
 - b. linear motion
 - c. streamline motion
 - d. rotational motion
2. Which is the best alternative for a body completing $\frac{3}{4}$ cycle on a circular track of radius r with constant velocity v ?
 - a. Change in K.E. is $\frac{1}{2} mvr$.
 - b. Change in momentum is mvr .
 - c. Change in K.E. is zero.
 - d. Change in K.E. is mv^2 .
3. A fighter plane is pulling out for a dive at 900 km/h in a vertical circle of radius of 2 km. Its mass is 5000 kg. What is the force exerted by the air on it at the lowest point?
 - a. 2.0625×10^4 N upward
 - b. 2.0625×10^5 N downward
 - c. 2.0625×10^4 N downward
 - d. 2.0625×10^5 N upward
30. A mass of 2kg is whirled in a horizontal circle by means of a string at an initial speed of 5 rpm. Keeping the radius constant, the tension in the string is doubled. What would be the new speed?
 - a. 7 rpm
 - b. 10 rpm
 - c. 14 rpm
 - d. 12 rpm
31. A stone tied to the end of 80 cm long string is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the acceleration?
 - a. 9.9 ms^{-2}
 - b. 10.6 ms^{-2}
 - c. 11.2 ms^{-2}
 - d. 12.04 ms^{-2}
32. What is the acceleration produced on a motor car travelling at 30 ms^{-1} on a circular road with radius 500m?
 - a. 1.8 ms^{-2}
 - b. 2.2 ms^{-2}
 - c. 3.2 ms^{-2}
 - d. 4.5 ms^{-2}
33. A stone tied to the end of a 20cm long string is whirled in a horizontal circle. If the centripetal acceleration is 9.8 ms^{-2} , what would be its angular speed in radian/s?
 - a. 7
 - b. 14
 - c. 23
 - d. 15

34. The diameter of Earth is 12800 km and it takes 24 hours for the Earth to complete one revolution about its axis. What is the acceleration of a particle placed on the surface of the Earth at the equator due to Earth's rotation?
- 0.34 ms^{-2}
 - 0.28 ms^{-2}
 - 1.4 ms^{-2}
 - 3.2 ms^{-2}
35. A cyclist goes round a circle at 15 km/h. If the speed doubles, what is the force on it?
- doubled
 - halved
 - quadrupled
 - unchanged
36. An ant crawls around a 22cm long arc of a circular track.. What would be the radius of the circular track at an instant it makes an angular displacement of 45° ?
- 0.23 m
 - 0.28 m
 - 0.35 m
 - 0.44 m
37. In a uniform circular motion, which one will be the best alternative?
- Acceleration and speed change.
 - Acceleration and speed are constant.
 - Acceleration and velocity are constant.
 - Acceleration and velocity change.
12. What effect would the velocity vector of a particle moving in a circle describing equal angles in equal times produce?
- Changes direction.
 - Remains constant.
 - Changes magnitude.
 - Changes both direction and magnitude.
13. A body of mass 2kg is moving in a circle of radius 1m with angular velocity of 2 rad/s. What is the value for the centripetal force?
- 9N
 - 8N
 - 10 N
 - 7 N
38. A circular object of radius 25cm is revolving with a speed of 20rev/s. What is the acceleration produced?
- $400\pi^2 \text{ ms}^{-2}$
 - $200 \pi^2 \text{ ms}^{-2}$
 - $250 \pi^2 \text{ ms}^{-2}$
 - $450 \pi^2 \text{ ms}^{-2}$
39. What type of force is the centrifugal force in nature?
- gravitational force
 - centripetal force,
 - fictitious force

- d. viscous force
40. A car is racing on a circular track of radius 747m with a speed of 65ms^{-1} . What is the value of banking angle required?
- 25°
 - 30°
 - 35°
 - 32°
17. A body is describing circular motion with frequency f along circular path of radius r . What is the tangential acceleration?
- $4\pi^2 rf^2$
 - $4\pi rf$
 - $4\pi r^2 f$
 - $4\pi rf^2$
18. A uniform annular ring of inner and outer radii R_1 and R_2 is rolling down an inclined plane without slipping. What is the ratio of the centripetal force on two bodies of same mass lying at inner and outer surface?
- $\frac{R_2}{R_1}$
 - $1 + \frac{R_2}{R_1}$
 - $1 - \frac{R_2}{R_1}$
 - $\frac{R_1}{R_2}$
19. (a) Differentiate between linear and circular motion.
 (b) Show $f = \frac{1}{T}$, f is frequency and T is the time period.
 (c) There is a net force acting on a particle in a uniform circular motion though speed doesn't change, why?
20. (a) Define banking angle. Why is it necessary?
 (b) At what angle should a circular road be banked so that a car running at 50 km/h be safe to go round the circular turn of 200m radius? (Ans: 5.61°)
21. (a) What is the source for centripetal force in a conical pendulum?
 (b) Differentiate between simple pendulum and conical pendulum.
 (c) Find the angular velocity of a particle in a circular motion revolving 100 times in a minute. (Ans: 10.47 rad/s)
22. (a) Why is a body undergoing a circular motion accelerating, even though it is moving at a constant speed?
 (b) The positively charged nucleus of an atom attracts the electron in the orbits but electrons do not collapse into the nucleus. Why?
 (c) Calculate the force necessary to keep a mass of 0.2 kg moving in a horizontal circle of radius 0.5m with a period of 0.5 s. (Ans: 15.8 N)
23. (a) For a particle undergoing a circular motion with constant speed, is its acceleration also constant?
 (b) What is the source of centripetal force on a car undergoing a circular arc?
 (c) The radii of the two circular paths are in the ratio of 2:3. If they have same acceleration, find the ratio of their square of the speed. (Ans: 2:3)
41. (a) An airplane tilts while making curved flight. Why?

- (b) A racer moving with a constant tangential speed of 50 ms^{-1} , takes one lap around a circular track in 40 seconds. Calculate the magnitude of the acceleration of the car.
(Ans: 7.86 ms^{-2})
25. (a) Define angular displacement.
- (b) Derive an expression for the force directed towards the center of a circle of radius r and acting on a body of mass m undergoing circular motion with a constant speed v in the circle.
- (c) The extremity of the hour hand of a clock moves $1/20$ th as fast as the minute hand. What is the length of the hour hand if the minute hand is 10 cm long? (6 cm)
26. (a) What is a conical pendulum?
- (b) Deduce for the time period $T = 2\pi \sqrt{\frac{l \cos \theta}{g}}$ for such a pendulum of length l that inclines at θ with the vertical. .
- (c) An object of mass 10 kg is whirled round a horizontal circle of radius 4m by a revolving string inclined to the vertical. If the uniform speed of the object is 5 ms^{-1} , calculate i) The tension in the string ii) The angle of inclination of the string to the vertical. (Ans: 118 N , 32°)
27. (a) What is meant banking of track?
- (b) Find the angle through which the outer edge of a road is to be raised in a bend of a road with a circular arc of radius r .
- (c) A train of mass 10^5 kg rounds a curve of radius 150 m at a speed of 20 ms^{-1} . Find the horizontal thrust on the outer rail if the track is not banked. At what angle must the track be banked so that there is no thrust on the rail? $g = 9.8 \text{ ms}^{-2}$. (Ans: $2.67 \times 10^5 \text{ N}$, 15.22°)
28. (a) Define the motion in a vertical circle.
- (b) Find the expression for the critical velocity for the body to go safely to the circle.
- (c) A construction team is concerned that cars should be able to go through the turn without skidding off the road even if conditions are bad. Determine the minimum radius that can be used for the curve of the turn if a car traveling 110 km/h might be traveling in conditions that cause a coefficient of static friction of only 0.60 between the tires and the road. (Ans: 158.67 m)
29. A mass 100gm is suspended by a string 40cm long. Keeping the string always taut, the ball describes a horizontal circle of radius 10 cm. Find the angular speed of the ball.
(Ans: 5.03 rad/s)
30. What is the angular velocity of the minute hand of a clock? What is the angular displacement of the minute hand in 20 minutes? If the minute hand is 5 cm long, what is the linear velocity of its tip? (Ans: $1.746 \times 10^{-3} \text{ rad/s}$, 2.095 rad , $8.73 \times 10^{-5} \text{ ms}^{-1}$.)
31. A stone of mass 0.8kg is attached to one end of a string 0.9m long. The string will break if its tension exceeds 600N. The stone is whirled in a horizontal circle, the other end of the string remains fixed. Find the maximum speed the stone can attain without breaking the string.
(Ans: 25.98 ms^{-1})

32. A bob of mass 200gm is whirled in a horizontal circle of radius 50 cm by a string inclined at 30° . Calculate tension on the string and the speed of the bob. (Ans: $2.3N$, $1.7ms^{-1}$)
33. The radius of curvature of a metre gauge railway line at a place where the train is moving at 36 km/h is 50m. If there is no side thrust on the rails find the elevation of the outer rail above the inner rail. (Hint: a metre gauge railway implies 1 m distance between the rails).
(Ans: 0.2 m)
34. A stone tied to the end of a 80 cm long string is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone? (Ans: $9.9ms^{-2}$, the direction is along the radius vector)
35. A car is moving in a circular horizontal track of radius 10 m with a constant speed of $10 ms^{-1}$. A plumb bob is suspended from the roof of the car by a light rigid rod. Find the angle made by the rod with the vertical. (Ans : 45°)
36. A stone of mass 500g is attached to a string of length 50cm which will break if the tension in it exceeds to 20N. The stone is whirled in a vertical circle, the axis of rotation being at a height of 100cm above the ground. The angular speed is very slowly increased until the string breaks. In what position is this break most likely to occur and at what angular speed? Where will the stone hit the ground? (Ans: $7.75 rad/s$, $1.24 s$)
37. A spaceman in training is rotated in a seat at 5 m from the axis of rotation. If he can withstand acceleration up to 9g, what is the maximum number of revolutions per second permissible? (Ans: 0.675 rev/s)
38. A mass of 0.4kg is rotated by a string at a constant speed v in a vertical circle of radius 1m. If the minimum tension of the string is 3N, calculate i) v, ii) the maximum tension iii) the tension when the string is just horizontal. (Ans: $4.2 ms^{-1}$, $11.00N$, $7.00N$)

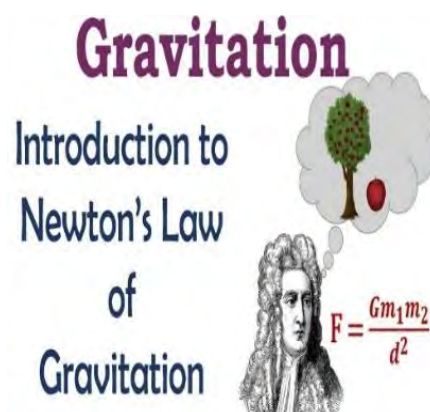
Answer to the multiple choice questions:

Q.N	1.	2.	3.	4	5.	6.	7.	8.	9.
Ans	a	c	d	a	a	a	a	a	c
Q.N	10.	11.	12.	13.	14.	15.	16.	17.	18.
Ans	b	d	a	b	a	c	b	a	d

Chapter 7

Gravitation

Numerous theories of gravitation emerged in ancient Greek philosophy and were further elaborated upon through the Middle Ages by scientists from various parts of the world. These ideas gained significant momentum during the Renaissance and Scientific Revolution, ultimately culminating in the formulation of Newton's law of gravity. Sir Isaac Newton discovered the existence of gravity. It is said that he realised about gravity when he saw an apple fall down from a tree. Newton came up with the idea that some unseen force must attract the apple towards the Earth. He named this force “gravity” from the Latin word “gravitas”, meaning “weight”. Finally in series of his work, Newton came to establish a famous law of gravitation, now called as Newton’s universal law of gravitation and it is his most renowned work.



Newton’s law of gravitation

7.1 Introduction:

Before understanding the insight of the chapter, let’s observe an activity.

Observe the following phenomena.

- i) The Earth is holding itself and rotating while the atmosphere surrounds it.
- ii) Binding the Earth and the other planets, along with the perennial source of energy, the sun forms a solar system.
- iii) Binding the stars together giving rise to galaxies.
- iv) The constant motion of the heavenly bodies throughout the time.

What will be the ultimate source to all these phenomena?

The study of all observed phenomena implies that there is an interaction between any two heavenly bodies, characterized by the weakest force known as the gravitational force. *If such a force acts on a body due to the Earth, it is called gravity; if it acts between any two bodies, it is referred to as gravitation. Gravity always refers to gravitation, but gravitation may or may not refer to gravity.*

7.2 Newton's Law of Gravitation

So far, we are observing the phenomenon described in activity 7.1, where there is a gravitational interaction between any two bodies, whether terrestrial or celestial. From the study of the apple's fall, Newton established the famous law known as Newton's Law of Gravitation. This law was announced by Newton in 1687 in his monumental work, 'Principia.'

According to this law, any two particles in the universe attract each other with a force.

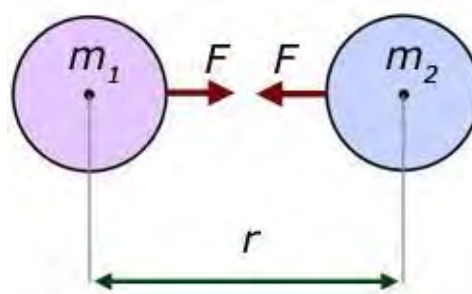


Fig 7.1: Gravitational force between two bodies

The force is directly proportional to the product of the masses of the bodies, and inversely proportional to the square of the distance apart them.

$$\text{i.e, } F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore F = G \frac{m_1 m_2}{r^2}$$

Here, G is the universal gravitational constant.

If $m_1 = m_2 = 1 \text{ kg}$ and $d = 1 \text{ m}$

Then, $F = G$

Hence, G represents the force of attraction between two bodies of unit masses when they are kept at a unit distance apart.

7.3 Acceleration due to Gravity

The acceleration produced on a body due to Earth's gravity is called acceleration due to gravity. It is symbolized by 'g,' and its value is 9.8 ms^{-2} . It is independent of the size and shape of the body.

A body thrown up is found to have its velocity decreased by 9.8 ms^{-1} each second, while for the falling body, it is found to be increased by the same value each second.

For a body of mass 'm' lying on the surface of the Earth, one writes-

$$F = \frac{GMm}{R^2}$$

$M \rightarrow$ mass of the Earth

$R \rightarrow$ radius of the Earth

From Newton's Second Law of motion,

$$F = mg.$$

$$\therefore mg = \frac{GMm}{R^2}$$

$$\Rightarrow g = \frac{GM}{R^2}$$

Since G and M are constants, $g \propto 1/R^2$

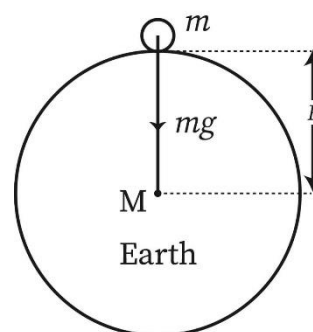


Fig 7.2: Force due to gravity

From the relation, the acceleration due to gravity is inversely proportional to the square of the radius. Since the Earth is not perfectly spherical in nature (bulged near the equator), the value of g changes according to the places on the Earth.

At pole, $g_{\text{pole}} = 9.83 \text{ ms}^{-2}$

At equator, $g_{\text{eq}} = 9.78 \text{ ms}^{-2}$.

xi) Water flows faster along the downhill.

xii) We find it difficult to control the motion when running downhill and may experience knee pain.

These all phenomena are in accordance with the relation $g \propto 1/R^2$.

And from the expression, $g = GM/R^2$, we can estimate the mass and density of the Earth.

Worked Out Examples:

Q: If the acceleration of the free fall of the Earth's surface is 9.8 ms^{-2} and radius of the Earth is 6400 km , calculate the value for the mass of the Earth.

$$(G = 6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}).$$

Solution:

$$\text{Here, } g = 9.8 \text{ ms}^{-2}$$

$$\text{Earth's radius, } R = 6400 \text{ km} = 6400 \times 10^3 \text{ m} = 6.4 \times 10^6 \text{ m}$$

$$\text{Universal Gravitational constant, } G = 6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$$

Mass of the Earth, $M = ?$

We have,

$$\begin{aligned} g &= \frac{GM}{R^2} \Rightarrow M = g R^2 / G \\ &= 9.8 \times (6.4 \times 10^6)^2 / 6.67 \times 10^{-11} \\ &= 6.02 \times 10^{24} \text{ Kg.} \end{aligned}$$

Q: The gravitational force on a body of mass of 1 kg on the Earth's surface is 10 N . Assuming the Earth is a sphere of radius R , calculate the gravitational force on a satellite of mass 100 kg in a circular orbit of radius $2R$ from the center of the Earth.

Solution:

Here mass of the body, $m = 1 \text{ kg}$.

Gravitational force, $F = 10 \text{ N}$

Radius of the Earth = R

We have,

$$F = \frac{GMm}{R^2}$$

$$\text{OR, } 10 = GM \cdot \frac{1}{R^2}$$

$$\text{OR, } \frac{GM}{R^2} = 10$$

For the satellite,

Mass, $m_e = 100 \text{ kg}$

Distance, $d = 2R$

We have,

$$F_1 = \frac{GMm_e}{(2R)^2}$$

$$\begin{aligned}
&= \frac{GM}{R^2} \cdot 100/4 \\
&= 25 \cdot \frac{GM}{R^2} \\
&= 25 \cdot \frac{GM}{R^2} \\
&= 25 \times 10 = 250N
\end{aligned}$$

Concept to know:

- xiii) *The gravitational pull exerted by the Earth on the Moon causes the Moon to orbit the Earth, providing the necessary centripetal force. This force is directed perpendicular to the velocity of the Moon, meaning the Earth continuously pulls the Moon toward its center. Despite this gravitational interaction, the Moon does not fall to the Earth.*
- xiv) *The speed of a satellite in a circular orbit is perpendicular to Earth's gravitational field. Therefore, the gravitational field has no effect on the satellite's motion, and the speed of the satellite remains constant.*
- xv) *The time period of a pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Inside the artificial satellite, $g = 0$ so that T becomes infinite from the expression and a pendulum will not vibrate.*
- xvi) *The gravitational interaction between the bodies doesn't depend on an intervening medium, so it isn't possible to shield a body from the gravitational effects.*

The followings are the conditions and points where our weight becomes zero.

- i) *At the centre of the Earth*
- ii) *During free fall*
- ii) *In an artificial satellite*
- iv) *At a point where gravitational pull of the Earth is equal to the gravitational pull of the Moon*
- xvii) *The force of gravitational pull between the Earth and a body is the same. However, while the body moves towards the Earth, the Earth itself does not appear to move towards the body.*

xviii) Here, we have, $F = \frac{GMm}{r^2}$

Now the acceleration on a body due to the Earth's pull, $g = \frac{GM}{r^2}$ and the acceleration on the Earth due to the body's pull, $a = \frac{F}{M} = \frac{Gm}{r^2}$

Hence, we get $\frac{a}{g} = \frac{m}{M} \Rightarrow a \ll g$ and hence the body is pulled towards the Earth but not the Earth.

7.4 Inertial and Gravitational Mass

Inertial mass is the resistance to changing velocity (or acceleration) when force is applied. If the applied force and resulting acceleration are known, mass can be evaluated using Newton's second law of motion, $F = ma$ where $m = \frac{F}{a}$ is the measure of inertial mass.

Gravitational mass is the mass in the gravitational field that is measured under the gravitational pull.

Inertial Mass and Gravitational Mass

Inertial mass is measured when objects are in motion whereas gravitational mass when it is under gravity but no motion

For a body of mass m , in a gravitational field,

$$F = \frac{GMm}{d^2} \Rightarrow m = \frac{Fd^2}{GM}$$

The concept of mass measured in either case is the same; only the method differs—one is derived from Newton's second law of motion, and the other from Newton's law of gravitation.

7.5 Gravitational Field Strength

Gravitational Field:

Observe the phenomena carefully:

- xix) During the time your physics teacher is in the class room, the whole class room is under the influence of your teacher's presence.
- xx) The student at the front desk experiences a clear understanding of the words from the teacher and a distinct view of the board. The students in the middle desks have a lower level of listening comprehension and a reduced vision of the board, while the students at the back have the least understanding of the words and the poorest vision of the board. In this scenario, the students essentially serve as nominal entities with no grasp of the subject matter, akin to test masses in gravity.
- xxi) By analogy, the region around a body within which its gravitational pull is perceptible is the gravitational field. The gravitational field intensity (or strength), E , of a body at a point inside the field is the force experienced by a unit test mass placed at that point. Consequently, the unit test mass does not alter the field configuration of the body.
To evaluate the gravitational field strength at point P at a distance r due to the body of mass m , let's suppose that a body of mass m is placed at the point.

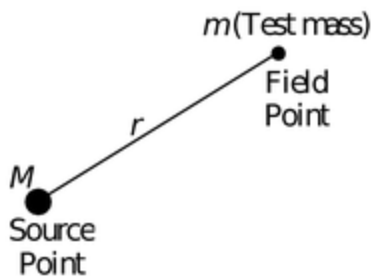


Fig 7.3: Test mass inside the field

Then, $E = \frac{F}{m}$

$$= G \frac{Mm}{r^2} \cdot \frac{1}{m}$$



$$= \frac{GM}{r^2}$$

Fig7.4: Gravitational field strength is the force per unit test mass

$\therefore E = \frac{GM}{r^2}$, directed towards the center of mass m and is a vector quantity.

For the body on the surface of the Earth,

$$\begin{aligned} E &= \frac{F}{m} \\ &= \frac{GMm}{R^2 \cdot m} \\ &= \frac{GM}{R^2} \end{aligned}$$

$\therefore E = g$

And hence the gravitational field strength nearby the Earth's surface is equal to the acceleration due to gravity 9.8 ms^{-2} (or 9.8 N/kg)

Moreover, $E = -g$

From the graph it can be seen that the gravitational field strength g at a radius r is equal to minus the value of the gradient of the gravitational potential U . Note that E is positive because the value of dV/dr itself is negative. Multiplying a negative by a negative gives a positive.

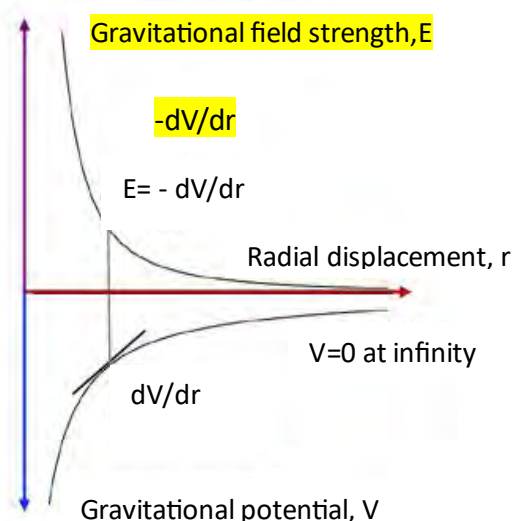


Fig 7.5: Showing variation gravitational field strength and potential with distance

The gravitational field strength is a vector quantity whose direction is the direction of the force that a unit mass experiences at a point of interest.

Q: 1. What is the difference between gravitational force and gravitational field strength?

Ans: Gravitational field strength is the force experienced by a unit mass at that point of interest. Gravitational force is the amount of force acting on a body. It is the product of field strength times the mass under consideration. Gravitational pull is just a more colloquial name for gravitational force.

Q 2. Why is gravitational field strength different?

The more the mass of an object, the greater the gravitational field it has. The Earth hence has a greater gravitational field than the Moon. The Moon is attracted to the Earth because it is within the Earth's gravitational field.

Sample Problems

1. What is the gravitational field strength of the Earth at 6.37×10^6 m above the surface of the Earth?

Solution: Here, mass of the Earth, $M = 5.98 \times 10^{24}$ kg
 $r = 2 \times 6.37 \times 10^6$ m as the height above the surface is equal to the radius of the Earth.

Now the gravitational field strength, $E = 6.67 \times 10^{-11} (5.98 \times 10^{24} / (2 \times 6.37 \times 10^6)^2)$
 $= 2.46 \text{ N/Kg}$

7.6 Gravitational Potential and Gravitational Potential Energy

The gravitational potential, U , at a point in the gravitational field is defined as the amount of work required to bring a unit mass from infinity to that point.

Let's consider a body of unit mass lying at a point of infinity from the center of the Earth of radius R and mass M that is supposed to be concentrated at its center.

Let P be the point at ($OP=r$) where gravitational potential has to be determined.

In the course of moving the body from infinity to the point P , let it lie at point A such that $OA = x$.

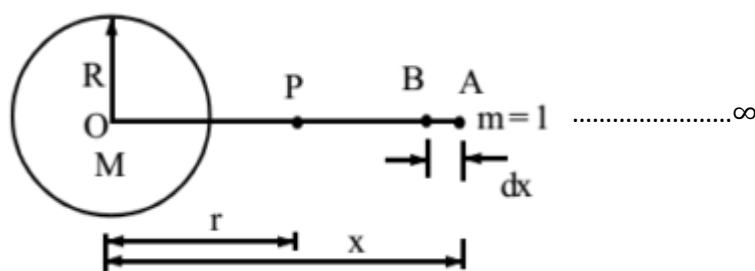


Fig 7.6: Calculation of gravitational potential

The gravitational force hence is written as,

$$F = \frac{GM}{x^2}$$

To displace the unit mass to the point B through an infinitesimal increment dx such that $AB = dx \rightarrow 0$, the work done is

$$\begin{aligned} dW &= F \cdot dx \\ &= Gm/x^2 \cdot dx \end{aligned}$$

The total work done in bringing the body from infinity to the point P hence is written as,

$$\begin{aligned}
 W &= \int_{\infty}^A dw = \int_{\infty}^A x^{-2} dx \\
 &= (G+x) \cdot \left| \frac{x^{-2+1}}{-2+1} \right|_{\infty}^r \\
 &= - (Gm) \left| \frac{1}{x} \right|_{\infty}^r \\
 &= - \frac{Gm}{r}
 \end{aligned}$$

Clearly, this much amount of work done is the gravitational potential, V and hence

$$V = - \frac{Gm}{R}$$

* Its SI unit is joule per kilogram

7.7 Gravitational potential Energy

Activity 7.4

xxii) Take a ball and throw it in the air and watch the motion of the ball

Take a ball and throw it into the air, then observe its motion. Consider what happens to the kinetic energy (K.E.) you imparted to the ball upon throwing it, and why its speed gradually decreases. The kinetic energy converts into gravitational potential energy (P.E.), and the decrease in speed of the ball is due to the influence of Earth's gravitational field. When the ball is thrown, it possesses kinetic energy and experiences acceleration, resulting in a force acting on it.

One can express the change in energy (ΔE) as $\Delta E = mg\Delta x$, where Δx represents the change in distance (or height), and for a large distance, we use Δx , while for a small range, we use h . When considering the change in energy between a distance r and infinity, we get $\Delta E = f.dr$.

*Therefore, one can define gravitational potential energy as follows: **The gravitational potential energy of a body at any point within a gravitational field is the amount of work required to bring the body from infinity to that point without causing acceleration.***

As described in the previous article, let us consider a body of mass ' m ' lying at a point of infinity from the center of the Earth at radius R and mass m supposed to be concentrated at its center.

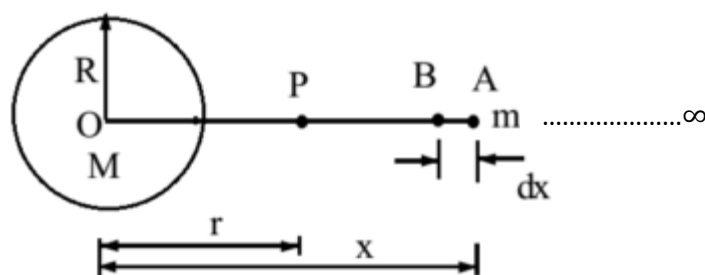


Fig 7.7: Method to find the gravitational energy

In bringing the body from infinity to the point P ($OP=r$) let at any instant it lies at A ($OA=x$) inside the field.

Therefore, we write;

$$F = \frac{GMm}{x^2}$$

The small work done in shifting the body to the point B towards the Earth through an infinite increment dx ($AB = dx \rightarrow 0$), $dw = F \cdot dx$

$$= \frac{GMm}{x^2} dx$$

The total work to bring the body from infinity to the point A is obtained as;

$$W = \int_{\infty}^A dw = GMm \int_{\infty}^A x^{-2} dx$$

$$= -GMm \left[\frac{1}{x} \right]_{\infty}^A$$

$$= - \frac{GMm}{r}$$

$$= - \frac{GMm}{r} \cdot m$$

$$= - V \cdot m, V \rightarrow \text{gravitational P.E at } r$$

The gravitational P.E on the Earth's surface hence is written as;

$$U_R = - \frac{GMm}{R}$$

Gravitational potential energy is negative at the surface of the Earth, because work is done by the gravitational field in bringing a mass from infinity to the surface.

Sample Example:

- Two stars each of the mass equal to the solar mass 2×10^{30} kg are approaching each other and initially are at a distance of 10^9 km, their initial speeds being negligible. What is the speed with which they collide? The radius of the star is 10^4 km. ($G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$)

Solution:

Here, Mass of a star, $M = 2 \times 10^{30}$ kg

Distance between the stars, $r = 10^9 \text{ km} = 10^{12} \text{ m}$

Radius of the star, $R = 10^4 \text{ km} = 10^7 \text{ m}$

The total energy of the stars initially = P.E + K.E = $\frac{-GMM}{r} + \frac{1}{2}Mv^2 = \frac{-GMM}{r}$, as K.E equals to zero due to initial negligible speed.

When they collide, total energy = P.E + K.E = $\frac{-GMM}{2R} + \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = \frac{-GMM}{2R} + Mv^2$

Now, from conservation of energy, $\frac{-GMM}{2R} + Mv^2 = \frac{-GMM}{r}$

$$\Rightarrow v^2 = GM \left(\frac{1}{2R} - \frac{1}{r} \right) = 6.67 \times 10^{-11} \times 2 \times 10^{30} \left(\frac{1}{10^7} - \frac{1}{10^{12}} \right)$$

$$\therefore v = 2.58 \times 10^6 \text{ ms}^{-1}.$$

7.8 Variation of Acceleration due to Gravity

Observe the following activities:

- xxiii) A body that is difficult to lift at the bottom of a high hill is easier to lift at the top.
- xxiv) The same body weighs more near the pole than near the equator

- xxv) A body thrown upwards decreases its velocity with each passing second, and the final velocity becomes zero at its maximum height.
- xxvi) It is difficult to stop while running downhill, and our velocity increases with every passing second.

Similar phenomena claim that there is a variation in the value of acceleration due to gravity based on points on the Earth, which is fully illustrated below.

7.8.1 Variation of g with altitude.

Consider a body lying at an altitude 'h' above the Earth's surface. The radius of the Earth is R and its mass M is supposed to be concentrated at its center O.

For the body on the surface of the Earth,

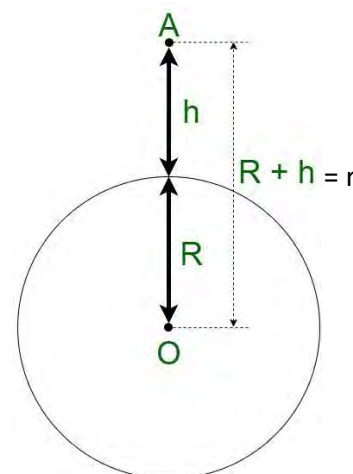
$$g = \frac{GM}{R^2} \text{ ----- i)}$$

For the body lying at the given altitude,

$$g' = \frac{GM}{(R+h)^2} = \frac{GM}{r^2} = \text{----- ii)}$$

From the equations above, if $R + h > R$, then $g' < g$, meaning that the increased value in the denominator decreases the value of g. This relationship is represented as $g' < g$.

Hence the acceleration due to gravity goes on decreasing as the altitude from the surface of the Earth increases.



7.8.2 Variation of g with depth

When a body lies within the Earth's surface, the part of the Earth above it doesn't contribute to the mass of the Earth. Only the part beneath and including the body contributes to it. Hence, the mass of the Earth decreases, which in turn tries to decrease the value of acceleration. Similarly, there is a decrease in the radius, which tries to increase the value of 'g', as per the relation $g \propto \frac{1}{R^2}$. Let's see what happens to the net value of 'g' with an increase in depth.

Consider a body of mass 'm' lying inside the Earth's surface at a depth x.

The radius of the Earth is R and its mass is M that is supposed to be concentrated at its center. Here ρ be the uniform density of the Earth elsewhere.

For the body inside the surface of the Earth,

$$g = \frac{GM}{R^2} \text{ ----- i)}$$

For the body inside the Earth's surface,

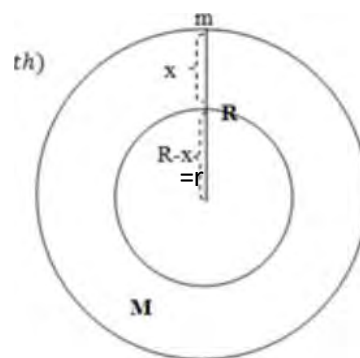


Fig7.9: Variation of g with depth

$$g' = \frac{GM'}{(R-x)^2} \text{ -----ii)}$$

Here M' is the mass of the Earth equivalent to a radius $R-x$.

Let $R - x = r$ and $M' = v' \times \rho$, where v' is the volume of the Earth equivalent to a radius $R-x$.

$$\text{Therefore, } g' = \frac{G \times \frac{4}{3}\pi(R-x)^3}{(R-x)^2}$$

$$= \frac{4}{3}\pi G(R-x) = \frac{4}{3}\pi G r$$

$$\text{Hence } g' = \frac{4}{3}\pi G r.$$

As the depth (x) increases from the surface of the Earth, the value of r decreases, and hence the acceleration due to gravity decreases with increasing depth. It can be expressed as $g' < g$. From the obtained relation, the value of g decreases as the depth increases.

Note: At the center of the Earth, when $x = R$ and $r = 0$, $g' = 0$, thus the weight of a body at the center, mg' , is zero.

Moreover, g goes on decreasing in the either case and it becomes maximum on the surface of the Earth.

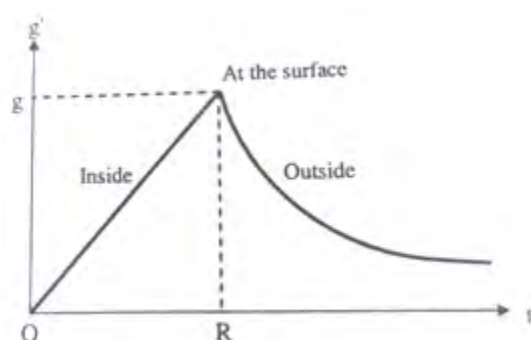


Fig 7.10: Variation of g with distance from the centre of the Earth

7.9 Center of Mass and center of Gravity

7.9.1 Center of Mass

The center of mass is a representative point in a rigid body where whole of the mass of the body is supposed to be concentrated.

A rigid body consists of large number of tiny particles of individual masses $m_1, m_2, \dots, m_n$, and so on. The sum of the masses of all these particles constitutes the mass of the body. The center of mass is a point where a force can be applied to produce the same linear motion as the motion of the entire body. The force acting upon this point produces linear acceleration but no angular motion.

Let a rigid body consisting of a particle of masses be $m_1, m_2, \dots, m_n$ at positions $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ respectively. Consider $F_1, F_2, \dots, F_n$ be the forces applied to the individual particles $m_1, m_2, \dots, m_n$ so that they produce acceleration $a_1, a_2, \dots, a_n$.

One writes, $F_1 + F_2 + \dots + F_n = m_1 a_1 + m_2 a_2 + \dots + m_n a_n$

$$\text{Or, } \sum F = m_1 \frac{d^2 x_1}{dt^2} + m_2 \frac{d^2 x_2}{dt^2} + \dots + m_n \frac{d^2 x_n}{dt^2}$$

$$= \frac{d^2}{dt^2} (m_1 x_1 + m_2 x_2 + \dots + m_n x_n)$$

$$\text{Or, } \frac{\sum F}{\sum m} = \frac{d^2}{dt^2} \left[\frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{(m_1 + m_2 + \dots + m_n)} \right]$$

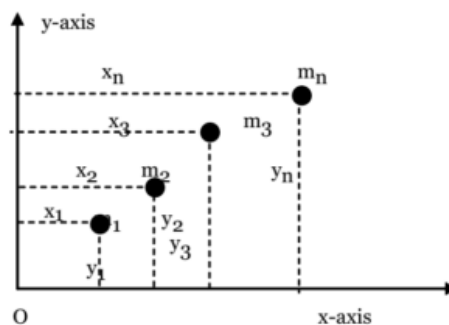


Fig 7.12: Showing position of the particles

For (X, Y) to be the point of center of mass, we concluded from above ;

$$X = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{(m_1 + m_2 + \dots + m_n)}$$

And,

$$Y = \frac{m_1 y_1 + m_2 y_2 + \dots + m_n y_n}{(m_1 + m_2 + \dots + m_n)}$$

Therefore, the point of center,

$$(X, Y) = \left(\frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{(m_1 + m_2 + \dots + m_n)}, \frac{m_1 y_1 + m_2 y_2 + \dots + m_n y_n}{(m_1 + m_2 + \dots + m_n)} \right)$$

For a two-body system of masses m_1 and m_2 as shown, let P be the point of center of mass at positions x_1 and x_2 respectively from the bodies.

Then, we write, $m_1 x_1 = m_2 x_2$

$$\therefore x_1 = \frac{m_2 x_2}{m_1}$$

$$\text{and } x_2 = \frac{m_1 x_1}{m_2}$$

The position of the centre of mass determines the position of the centre of gravity.

Hence the greater the mass, the nearer is the position of the center of gravity.

Worked Out Examples:

Q: Two particles of masses 1kg and 2kg are located at $x=0$ and $x=3$ m. Find the position of the centre of mass.

Solution:

The centre of mass clearly lies between the particles on the positive X-axis.

Let $r_1 = x$ be the position of the centre of mass from the particle of 1 kg mass and hence $r_2 = 3-x$ be the position of the centre of mass from the particle of 2kg mass.

Now from $m_1 r_1 = m_2 r_2$,

$$1. x = 2(3-x) \Rightarrow x = 2\text{cm}$$

Hence the centre of mass lies at 2cm from the origin.

Center of Gravity:

It is the point through which the resultant weight of the body is supposed to be acting vertically downwards. The center of gravity of an object is a geometric property representing the average location of the body's weight. The center of gravity (C.G) may or may not lie within the body. For a spherical or disk-like body, it lies at the center within the material of

the body. However, for a ring or spherical enclosure, the C.G also lies at the center but not within the material of the body.

The center of gravity is used in designing static structures such as buildings, bridges, flying airplanes, kites, etc. *If the center of gravity lies in the middle and at a lower position, it provides more stability to the structure.*

Activity 7.1

Do the activities that follows

- i) Take a long, uniform bar and support it horizontally at the tips of two fingers to keep the bar level.
- ii) Take a circular disc, square, rectangular, or triangular lamina. Support it on a tip of a needle towards the center point of the body.
- iii) Find the position of the center of gravity for each item taken. Clearly, the center of gravity is the point where, when the body is supported on it, the body remains in equilibrium.

The shapes of the bodies and position of C.G for the various bodies are listed below:

- i) The uniform bar (rod) → Center point
- ii) Disc or Circular ring → Center point
- iii) Sphere body and Angular disc → At the Center
- iv) Triangular lamina → Point of interstion of the medians
- v) Rectangle or Cubical block → Point of interstion of the diagonals
- vi) Square, Rectangular or Parallelogram like lamina → Point of interstion of the diagonals
- vii) Cylinder → Middle point of axis
- viii) Cone of Pyramid → on the line joining the center to the base at a distance equal to $\frac{1}{4}$ of the length of the line from the base.

Conditions of Equilibrium.

Activity 7.2

Do the activities that follows

- xxvii) Take a funnel, and place it on a ground as shown in the following positions.

Fig:



Fig 7.13; Showing the equilibrium condition

xxviii) Give a strong wind blow (use a table fan). Observe which object will be disturbed first and which one will remain the most stable.

In the above cases, position second of the funnel is highly unstable, while position first is highly stable. *Hence for the stability (equilibrium of a body), the base of the body should be as wide as possible.*

- i.) The bottom of the ship is made heavier for this reason. The addition of cargo enforces the center of equilibrium to lie at the lowest position, thereby producing more stability.
- ii) A double - decker bus may easily topple over as there is shift of center of gravity upwards due to the passenger's weight.
- iii) The boat loses its balance when passengers stand on it due to the upward shift of the center of gravity.

Activity to observe

Observe the activities that follows

* Consider three cases: a man holding a water bucket in his hand, another man carrying a load on his back, and a third person carrying a load on his shoulder (or head).

* During walking, if a leg stumbles, which person has a greater chance of falling down?

The first person in activity 7.5 shifts his center of gravity downwards and leans in the opposite direction, stretching out his hand to pass the center of gravity within the base. Consequently, the person gains more stability and has the least chance of falling. In the second case, the center of gravity is shifted upwards, and in the third case, the center of gravity is shifted even higher, resulting in a greater chance of falling.

Activity to observe

Observe the activities that follow.

* Watch the motion of a man and a cow along a downhill muddy road.

* You might observe the man falling on the road due to lack of friction, while the cow (or any other four-footed animals or vehicles) doesn't easily fall down.

In the case of a four-footed animal, the center of gravity lies as low as possible and within the base of the body, which helps to maintain stability. Therefore, for stability, the center of gravity should be positioned as low as possible within the base of the body.

The conditions of getting equilibrium are listed below.

- i. The C.G should lie within the base of the body.
- ii. The C.G should lie as low as possible and,
- iii. The base of the body should be as wide as possible.

xxix) *The center of gravity is based on weight, whereas the center of mass is based on mass. So, when the gravitational field across an object is uniform, the two are identical.*

7.9 Motion of a Satellite

Observe the activities that follow.

- i) The motion of the moon in the sky with its various shapes changing day by day.
- ii) The immediate news of geographical areas experiencing natural disasters in remote corners of the Earth, beyond human access.
- iii) The Moon orbits the Earth, completing its cycle in approximately 27.3 days, causing it to appear in different phases.



Fig 7.14: Showing the motion of the satellite

→ There are numerous manmade objects orbiting the Earth, equipped with powerful cameras that focus on different territorial regions and transmit images to humans on Earth. In both cases mentioned above, these orbiting bodies are satellites. The former, the moon, is the Earth's natural satellite, while the latter are artificial satellites.

A satellite is an object that orbits a planet, such as the Earth or the Sun. These objects can be natural or artificial. Artificial satellites are utilized for various purposes including research, communication, and gathering intelligence. *The gravitational force between the Earth and the satellite provides the centripetal force required for the satellite to maintain its orbit.* The moon is Earth's natural satellite, and today, there are numerous artificial satellites in orbit around our planet.



Fig7.15: Satellite taking photos

When an object is horizontally projected from a high altitude above the Earth's surface, it follows a parabolic path resulting in a certain range. With increased velocity of projection, the range also increases, eventually enabling the object to orbit the Earth without falling, thus forming a satellite. The velocity attained by the projectile at that altitude determines the

orbital motion of the satellite. This process of projection is typically achieved using a rocket. The first artificial satellite, Sputnik I, was launched by the Soviet Union in 1957.

7.9.1 Orbital velocity of the Satellite

It is the velocity with which a satellite moves in its closed orbit.

Consider a satellite of mass (m) orbiting the Earth at an altitude h above the Earth's surface with the orbital velocity v .

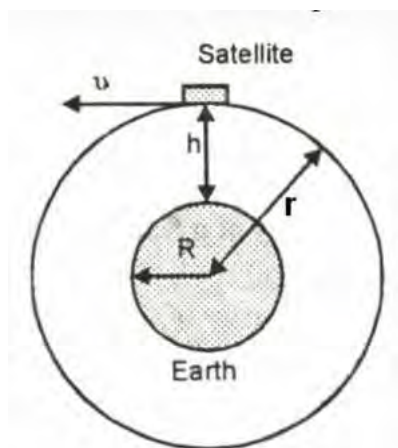


Fig7.16: Satellite revolving the Earth

Here R is the radius of the Earth and M is its mass supposed to be concentrated at its centre. For the satellite,

Since the gravitational force provides centripetal force,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\text{OR, } v^2 = \frac{GM}{r}$$

$$\therefore v = \sqrt{\frac{GM}{r}}$$

As we know, $g = \frac{GM}{R^2}$, $GM = gR^2$

$$\therefore v = \sqrt{\frac{gR^2}{r}} = R \sqrt{\frac{g}{r}}$$

$$= R \sqrt{\frac{g}{R+h}}$$

It is the orbital velocity of the satellite.

On the surface of the Earth,

$$v = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

$$= \sqrt{(9.8 \times 6400 \times 1000)}$$

7.9.2 Time Period of the Satellite

It is the time interval required for a satellite to round once the Earth.

Here we have for a satellite,

$$\frac{GMm}{r^2} = m\omega^2 r$$

$$\text{OR, } \omega = \sqrt{\frac{GM}{r^3}}$$

$$\text{OR, } 2\pi/T = \sqrt{\frac{GM}{r^3}}$$

$$\therefore T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

In the vicinity of the Earth's surface,

$$\begin{aligned} h \rightarrow 0 \Rightarrow T &= 2\pi\sqrt{\frac{R^3}{gR^2}} \\ &= 2\pi\sqrt{\frac{R}{g}} \\ &= 2\pi\sqrt{\frac{6400 \times R}{9.8}} \\ &= 5000\text{s} \\ &= 1 \text{ hour } 24 \text{ minutes.} \end{aligned}$$

$$\text{Also from } T = 2\pi\sqrt{\frac{(R+h)^3}{GM}},$$

$$\text{OR, } (R+h)^3 = T^2 \frac{GM}{4\pi^2}$$

$$\text{OR, } R+h = \left(T^2 \frac{GM}{4\pi^2}\right)^{1/3}$$

$$\therefore h = \left(T^2 \frac{GM}{4\pi^2}\right)^{1/3} - R$$

It gives the expression for the height of the satellite from the Earth's surface.

Concept to know:

The orbit that the Earth goes round the sun is elliptical, resulting in varying distances between the Sun and the Earth during different seasons. This variation in distance leads to a corresponding variation in gravitational force.

The resistive force acting on a satellite is directly proportional to its speed. Since the speed is greater during the descent phase, a satellite with small mass tends to experience more resistance during descent, causing it to burn up during this phase rather than during ascent.

Worked Out Examples

1) Find the value of g from the motion of the moon, assuming that its period of rotation round the Earth is 27 days and 8 hours and that the radius of its orbit is 60.1 times the radius of the Earth (Radius of the Earth is $6.36 \times 10^6 \text{m}$).

Solution:

Here,

$$\begin{aligned} \text{Time period, } T &= 27 \text{ days and } 8 \text{ hours.} \\ &= (27 \times 24 \times 60 \times 60) \text{ s} + (8 \times 60 \times 60) \text{ s} = 2361600\text{s} \end{aligned}$$

Radius of the Earth, $R = 6.36 \times 10^6 \text{m}$

For the moon, the radius of the orbit, $r = 60.1 R$

For the moon as a satellite system,

$$\frac{GMm}{r^2} = m\omega^2$$

$$\text{OR, } \frac{GM}{r^2} = r \left(\frac{2\pi}{T} \right)^2$$

$$\text{OR, } GM = r^3 \cdot \frac{4\pi^2}{T^2}$$

$$\text{OR, } \frac{GM}{R^2} = (60.1)^3 R$$

$$\text{OR, } g = (60.1)^3 \times 6.36 \times 10^6 \times 4\pi^2 / (2361600)^2 \\ = 9.78 \text{ms}^{-2}$$

2) A purposed communication satellite would revolve round the Earth in a circular orbit in the equatorial plane at a height of 35880 km above the Earth's surface. Find the period of revolution of the satellite in hours and comment on the result.

(Radius of the Earth is 6370km, Mass of the Earth is $5.98 \times 10^{24} \text{kg}$, $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2}$)

Solution:

Here, $r = R + h = 6370 + 35880 = 42250 \text{km}$.

Radius of the Earth, $R = 6370 \text{km}$.

Mass of the Earth, $M = 5.98 \times 10^{24} \text{kg}$

$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2}$

We have for the satellite,

$$\frac{GMm}{r^2} = m\omega^2$$

$$\frac{GM}{r^2} = r \left(\frac{2\pi}{T} \right)^2$$

$$\therefore T = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$= 2\pi \sqrt{\frac{(4250 \times 10^3)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$$

$$= 86419 \text{second}$$

$$= 24 \text{ hours.}$$

7.10 Escape Velocity

When an object is thrown upwards, it eventually returns to the Earth due to gravity. If the initial velocity of projection is increased, the object will ascend to greater heights before eventually falling back to the Earth. By repeating this process with increasing velocity, there comes a point where the projected body just escapes the gravitational pull of the Earth (or planet). This velocity is known as the escape velocity.



Fig7.17: Showing how escape velocity originates

It is hence defined as the minimum velocity with which it just goes out of the gravitational pull of the Earth.

Expression

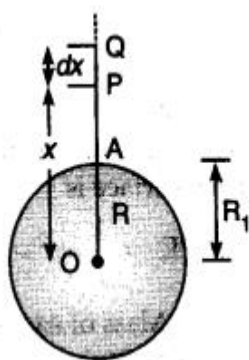


Fig7.18: Determination of escape velocity

Consider an object of mass m outgoing of the gravitational pull with an escape velocity v . During the travel, let the object is at point P at a distance x from the center of the Earth O. The radius of the Earth is R and its mass is M .

Hence for the point,

$$F = \frac{GMm}{x^2}$$

If the object reaches to the next point Q ($PQ = dx \rightarrow 0$), then the small amount of work done , $Dw = F \cdot dx$

For the escape velocity, the object is projected from the Earth's surface that reaches to a point at infinity beyond the Earth's gravitational pull.

Therefore, the total work,

$$\begin{aligned} W = dw &= GMm \int_R^\infty x^{-2} dx \\ &= GMm \left[\frac{1}{x} \right]_R^\infty \\ &= GMm \left[\frac{1}{\infty} - \frac{1}{R} \right] \\ &= -\frac{GMm}{R} = K.E \end{aligned}$$

i.e., this much amount of work done on a body imparts K.E to the body.

$$\text{Therefore, } \frac{1}{2}mv^2 = -\frac{GMm}{R} \Rightarrow v = \sqrt{\frac{2GM}{R}}$$

$$\therefore v = \sqrt{2gR}$$

It gives the required expression for the escape velocity to the object considered.

$$\begin{aligned} \text{Here, } v &= \sqrt{2 \times 9.8 \times 6400 \times 10^3} \\ &= 11.2 \text{ kms}^{-1}. \end{aligned}$$

Below this value of the velocity, the body can't be projected out of the gravitational pull of the Earth.

7.11 Potential Energy and the Kinetic Energy of the Satellite

A satellite revolving around a planet has both kinetic energy (K.E.) and potential energy (P.E.). Let a satellite goes round the Earth at a height h above the Earth's surface with an orbital velocity v .

For the satellite,

Centripetal force = Gravitational force.

$$\frac{mv^2}{R+h} = \frac{GMm}{(R+h)^2}$$

Here, $R \rightarrow$ radius of the Earth and $M \rightarrow$ mass of the Earth.

$$\text{From above, } v^2 = \frac{GM}{R+h}$$

$$\therefore K.E, K = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m (GM/(R+h)) = \frac{GMm}{2(R+h)} = \frac{GMm}{2r}$$

$$\text{The P.E, } U \text{ for the satellite} = -\frac{GMm}{(R+h)} = -\frac{GMm}{r}, r = R+h$$

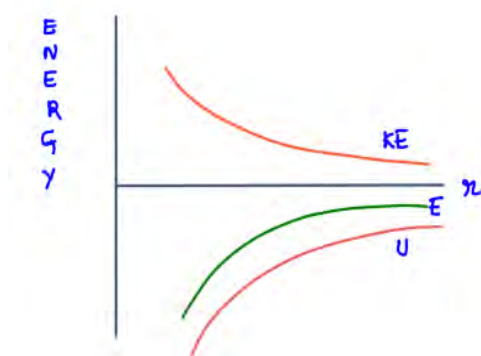
Now, total energy, $E = K.E + P.E$

$$= \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)} = -\frac{GMm}{2(R+h)} = -\frac{GMm}{2r}$$

$$\therefore E = K.E + P.E = -\frac{GMm}{2(R+h)} = -\frac{GMm}{2r} = \frac{-U}{2}, U \text{ is the P.E of the Earth.}$$

$$\text{Hence } E = -\frac{GMm}{2r} \Rightarrow E = -K$$

The variation of K.E (K) and P.E (U) and the total energy (E) with distance r has been



outlined in the fig.

Fig 7.19: Variation of K.E and P.E with distance

The -ve sign in the expression of K.E. signifies that the satellite is bound by the Earth's gravitational pull. To free from this pull, energy is to be supplied externally.

7.12 Geostationary Satellite

Activity 7.3

Observe the activities that follow:

- i) Step out of your home, gaze up at the sky, and then take a telescope to focus your attention on a star positioned just above the zenith in the evening.
- ii) Get up precisely at around midnight, and observe the same star again, noting its position as before.
- iii) Repeat the same observation in the early morning and note the position of the star.
- iv) Repeat the same observation in the early morning and note the position of the star. Do the same activity around noon (you may find it hard to see the star) and note the position of the star.

Q: i) Where will be the position of the star in each of the case?

What was the resultant motion of the star during this period of observation?

The star you observe consistently appears directly above the zenith. The Earth completes a cycle in a total time of 24 hours.

The fact that the star maintains the same position every 24 hours indicates that it follows the same motion as the Earth, with the same time period. This makes the star appear stationary when viewed from the Earth.

Therefore, in each instance, you perceive the star as if it were at rest, akin to a car in a park, and hence the name given to the orbit as a parking orbit or the geostationary orbit.

A geostationary orbit or a parking orbit has the same time period as that of the Earth and is known as a synchronous orbit with $T = 24$ hours. A satellite revolving in such a parking orbit with $T = 24$ hours is called a geostationary satellite or a synchronous satellite. Such satellites are used in telecommunications, weather forecasting, and live video transmission from one part of the Earth to another.

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow T = \frac{2\pi}{R} \sqrt{\frac{r^3}{g}}, r = R+h$$

$$\text{And } h = \left\{ \frac{T^2 R^2}{4\pi^2} g \right\}^{1/3} - R$$

With $T = 24 \text{ hours} = (24 \times 60 \times 60) \text{ s}$, $R = 6.4 \times 10^6 \text{ m}$ and $g = 9.8 \text{ ms}^{-2}$; $h = 3.6 \times 10^7 \text{ m} = 3600 \text{ km}$.

And orbital velocity, $v = \frac{2\pi r}{T}$
 $= 3.1 \times 10^3 = 3.1 \text{ kms}^{-1}$ is the speed of the satellite.

More precisely to say, the parking orbit must be circular lying on an equatorial plane and rotating from west to east.

Concept to know:

The rotation of the Earth on its axis is from west to east. When we launch a satellite from west to east, the two velocities (linear velocities) are added to each other, enhancing the launching velocity of the rocket. In the equatorial plane, there is a high value of the linear velocity of the Earth that adds to the launching velocity of the satellite, making the launch more effective.

Weightlessness and Free Fall

** When a body experiences zero weight, it is termed weightlessness. When a body falls under the effect of gravity without external resistance, it is referred to as free fall.*

** Weightlessness is the effect of the free fall while the free fall is the cause of weightlessness.*

** **Following** are the conditions our weight becomes zero:*

- i) At the centre of the Earth*
- ii) During free fall*
- iii) In an artificial satellite*
- iv) At a point where gravitational pull of the Earth is equal to the gravitational pull of the moon.*

The orbiting spacecraft is in the gravitational field of the Earth that experiences an appreciable weight and hence it is not in zero gravity though the case of weightlessness.

7.13 Global Positioning System (GPS)

Satellite-based navigation is utilized to synchronize location, velocity, and time while traveling by air, sea, or land. Similar fields that utilize GPS include military operations, surveying, aviation, road transportation, and smartphones.

The GPS, or Global Positioning System, is a global navigation satellite system consisting of a receiver and a constellation of 24 satellites distributed across six Earth-centered orbital planes,

with each plane hosting four satellites. These satellites orbit at an altitude of about 20,000 km above the Earth at a speed of 14,000 km/h.

To determine a location on the Earth's surface, GPS requires signals from three satellites. A fourth satellite is then used to validate the information obtained from the other three. This additional satellite enables GPS to provide altitude information, thus moving us into the third dimension. GPS operates using a technique known as trilateration or triangulation. Trilateration utilizes signals to calculate location, velocity, and elevation, while triangulation is employed to measure angles rather than distances.

Satellites transmit signals to GPS devices on or near the Earth's surface, and a device must be capable of receiving signals from at least four satellites to calculate its location.

Each satellite orbits the Earth twice a day, transmitting a unique signal along with orbital parameters and time information. A GPS system can receive signals from six or more satellites.

A GPS device captures microwave signals from satellites to determine the distance between the satellite and the GPS system. However, since GPS provides only distance information, it alone cannot provide precise location data.

A single satellite sends a signal, creating a circle. The second satellite creates another circle, narrowing down the location to one of two points where the circles intersect. With the third satellite, the location is determined as the device lies at the intersection point of the three circles.

However, our world is three-dimensional in nature. Satellites generate spheres instead of circles, resulting in two points of intersection for the three spheres from the three satellites. Finally, the point nearest to the Earth's surface is chosen as the location.

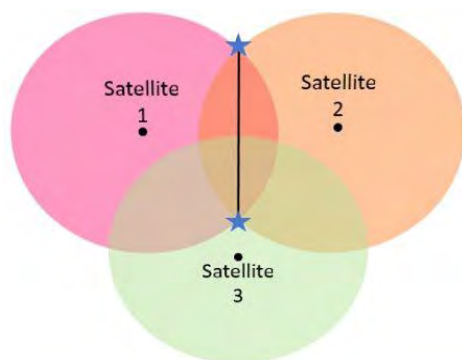
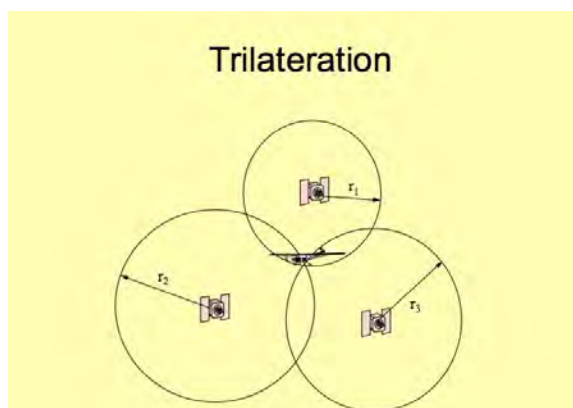


Fig7.20: Global Position of the GPS with 3 satellite Arrangement

As GPS moves, the radius, which is the distance from the two satellites, changes, thereby generating new spheres that provide updated positions.

GPS operates continuously, regardless of weather conditions. Hence, it is a powerful and dependable tool in various industries and other areas of business and organizations. Pilots, boat captains, surveyors, scientists, and workers rely on GPS information in their daily work. It is used for preparing accurate surveys and maps, conducting precise time measurements, and tracking positions or locations

Uses of GPS:

- i) Location findings
- ii) Navigation (or finding of another location from a given location)
- iii) Tracking (For an object's or a person's movement)
- iv) Mapping of the world.
- v) Timing for the precise measurement

Factors Affecting GPS Accuracy:

Following are the key factors that affect the GPS accuracy:

- i) Atmospheric conditions: solar storms, heavy storm cover and ionospheric delays disturb the GPS system.
- ii) Physical conditions: High altitude mountainous regions, trees and tall buildings disturb the signals passage time.
- iii) Calculation Error: It occurs from the device's hardware.
- iv) Ephemeris: It occurs due to the incorrect orbital model within the satellite.

Components of GPS System:

It consists of mainly three segments:

Space (satellite segment): The 24-satellite orbiting twice a day transmit signals to users on the Earth with timing.

Control Segment: It includes tracking and operating the satellite in space and monitoring transmissions. For this, 5 monitoring stations are kept almost in all the continent with North and South America combined. Qualified signals are sent to the master control stations. In case of defects in signals, they are sent back to the GPS satellite. **FIG**

User equipment (user segment); GPS receivers and transmitters collect and process signals from the GPS satellites and determine distance. The measurement accuracy increases with the number of satellites.

GPS vs GNSS; The Difference:

GPS is one of several satellite constellations, while GNSS describes all satellite constellations in orbit. GNSS is formed by combining multiple constellations. The accuracy of positioning technology depends on these various constellations.

Project Work:

Project 6.1 Measure the size of Earth's Circumference by using the Eratosthenes technique.

Hint: Materials you may require:

Two poles(sticks) known as gnomons, measuring tape, protractor, and a synchronized time-keeping clock.

Theory to execute: The difference in the angle made by the shadow with the pole in the two cases is equal to the angle subtended by the arc length between the poles at the centre. For $\Delta\theta$ as the difference in the angle and X as the arc length, the circumference

C is calculated as $C/X = 360/\Delta\theta \Rightarrow C = \frac{360}{\Delta\theta} \times X$

Procedural idea:

- i) Choose two distant locations on the Earth's surface on the same meridian (line of longitude), ideally at north-south alignment.
- ii) Measure the distance X using a map or GPS.
- iii) Keep the poles at these locations and find the angle made by their shadows with the top of the respective poles exactly at the same time in the noon. It gives $\Delta\theta$.
- iv) Finally find the value of the circumference.

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Project 6.2 Execute the project work on “Study the value of g by using microgram weighing tool.”

Exercise

1. What happens to the period of a simple pendulum with the increase in altitude?
 - a. increase
 - b. decrease
 - c. Remain constant
 - d. First increases then decreases
2. What will be the value of g on a planet with 80 times less mass and half the radius of the Earth, while the value of g on the Earth's surface is 9.8 ms^{-2} ?
 - a. 0.05 ms^{-2}
 - b. 0.5 ms^{-2}
 - c. 5.0 ms^{-2}
 - d. 5.2 ms^{-2}
3. What would be the value of the acceleration due to gravity on the Earth's surface if the mass remains constant but the radius of the Earth shrinks by 1%?
 - a. 2% increase
 - b. 1% decrease
 - c. 1% increase
 - d. $\frac{1}{2}$ % decrease
4. Which is the best alternative for the quantity to which the orbital speed of a satellite of mass m is proportional?
 - a. m^0
 - b. m
 - c. $m^{1/2}$
 - d. $m^{-1/2}$
5. At what height above the Earth's surface will the acceleration due to gravity be half the value of g at the surface, given the radius of the Earth as 6400 km?
 - a. 3050 km
 - b. 2650 km
 - c. 3250 km
 - d. 3800 km
6. How deep inside the Earth (of radius R) should a man go so that his weight becomes one-fourth of what it is on the Earth's surface?
 - a. $R/4$
 - b. $3R/4$
 - c. $5R/4$
 - d. $5R/2$
7. If the distance between the Earth and the sun were half of its present value, what would be the number of days in a year?
 - a. 129
 - b. 730
 - c. 182.5
 - d. 64.5
8. What is the period of a geostationary satellite?
 - a. 12 hour
 - b. 24 hour

- a. 18 hour
 - b. 6 hour
9. A missile misses its target and goes into an orbit of radius 4 times the radius of the geostationary orbit. What is the period of revolution?
- a. 4 days
 - b. 8 days
 - c. 12 days
 - d. 18 days
10. What is the value of g at the centre of the Earth?
- a. half the value on the surface
 - b. double the value on the surface
 - c. equal to the value on the surface
 - d. zero
11. What is the value of the period of a satellite in a parking orbit?
- a. 1 year
 - b. 365 days
 - c. 8760 hour
 - d. All of above
12. For a body to feel weightlessness in a falling lift, what must be the value of acceleration?
- a. equal to g
 - b. greater than g
 - c. less than g
 - d. double the g
13. What is the force on a body due to the Earth called?
- a. gravitation
 - b. gravity
 - c. reaction force
 - d. none of above
14. Where is the value of g maximum?
- a. on the surface of the Earth
 - b. below the Earth's surface
 - c. above the Earth's surface
 - d. Midway between the Earth and the moon
15. What is the value of g at the centre of the Earth?
- a. infinite,
 - b. zero,
 - c. greater than at the equator,
 - d. less than at the equator.
16. (a) Why is G known as the universal gravitational constant?
- (b) Assuming the Earth as a uniform sphere of mass M and radius R , show that the acceleration of a free fall on the Earth's surface is $g = GM/R^2$.
- (c) What is the acceleration of a satellite moving in a circular orbit around the Earth of radius $2R$?
17. (a) Define escape velocity.
- (b) Describe how you would deduce the value of the escape velocity on the Earth's surface.

- (c) Why does this value differ according to the planet?
18. (a) Define the term weightlessness.
 (b) Find the apparent weight of a body of a mass of 50 kg falling down with the acceleration of 5 ms^{-2} . (250N)
 (c) Do the same activity on a body going up with the same acceleration (750N)
19. (a) The gravitational potential is the work done in bringing a body of unit mass from a point at infinity to a point under consideration without acceleration.

Answer the following questions on the basis of it:

- i. What happens when the body of unit mass is replaced by a body of some other higher mass?
 ii. How does it differ from gravitational potential energy?
 (a) iii. How is it related to the value of acceleration due to gravity on the surface of the Earth?
 b. (a) What are the concepts of Centre of Mass and Centre of Gravity?
 (b) A uniform rod of length 200 cm and mass 500g is balanced on a wedge placed at the mark of 40 cm. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass 'm' is suspended from the rod at the mark of 160 cm as shown in the fig. Find the value of 'm' such that the rod is in equilibrium ($g = 10 \text{ ms}^{-1} \text{ }^2$). (Ans: $\frac{1}{12} \text{ kg}$)

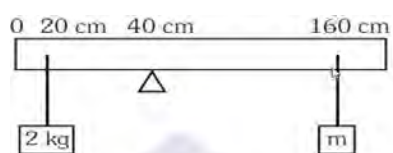


Fig 7.21

- c. (a) How is positioning done in GPS?
 (b) What is the minimum number of satellites required to determine one's position using GPS?
 (c) Name the Global Navigation Satellite Systems that can be used as alternatives to GPS.
22. (a) Define acceleration due to gravity. Derive an expression showing how it varies with altitude.
 (b) At what height from the surface of the Earth would the value of g be reduced by 36% from the value at the surface? The radius of the Earth is 6400 km (Ans: 1600 km)
23. (a) Estimate your weight at the centre of the Earth.
 (b) Find an expression to show how the value of acceleration due to gravity changes with the increment in the depth from the surface of the Earth.
 (c) Calculate the force due to the Earth's gravity on a 75 kg astronaut who is repairing the Hubble Space Telescope 600 km above the Earth's surface. The radius of the Earth = 6400 km. (Ans: 606N)
 (c) Compare this value with the weight on the Earth's surface. (ratio of the two $\Rightarrow 7:8$)
24. If an object is launched from the Earth's surface at a high enough velocity so that it can ascend into space and never return, this is known as the Earth's escape velocity. Now write the answer to the following questions:
 (a) Why does this velocity require a high value?
 (b) Derive an expression for this high velocity on the surface of the Earth.

- (c) If the Earth has a mass 9 times and the radius twice that of the planet Mars, calculate the minimum velocity required by a rocket to pull out of the gravitational force of Mars. Take the escape velocity on the surface of the Earth to be 11.2 km s^{-1} . (5.3 km s^{-1})
25. A satellite in a parking orbit has the time period 24 hours, equal to that of the Earth.
- Why is the satellite in this orbit called a geostationary satellite?
 - Derive an expression for the height of the parking orbit from the Earth's surface.
 - An artificial satellite revolves around the Earth in a circular orbit and is at a height of 300 km above the surface of the Earth. Find the orbital speed and the time period of the satellite. The radius of the Earth = 6400 km, $g = 980 \text{ cm s}^{-2}$ (Ans: 7.74 km s^{-1} , 5439s)
26. (a) Differentiate gravitational potential and gravitational P.E.
(b) Obtain the formula for the gravitational P.E. on the surface of the Earth.
- (c) In an otherwise empty universe, two planets each with the mass of $2 \times 10^{25} \text{ kg}$ are 10^{12} m apart with their negligible speed. Both the planets have a radius of 10^6 m . What are their speeds when they collide? ($2.58 \times 10^4 \text{ m s}^{-1}$)
27. (a) What is the orbital velocity of a satellite?
(b) Find the expression for the orbital velocity and the time period of a satellite.
(c) An Earth satellite moves in a circular orbit with an orbital speed of 6200 m s^{-1} . (a) Find the time of one revolution. (b) Find the radial acceleration of the satellite in its orbit. (Ans: 175 min, 3.7 m s^{-2})
28. Find the period of revolution of a satellite moving in a circular orbit around the Earth at a height of $3.6 \times 10^6 \text{ m}$ above the Earth's surface. Assume the Earth is a uniform sphere of radius $6.4 \times 10^6 \text{ m}$, the Earth's mass is $6 \times 10^{24} \text{ kg}$ and G is $6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. (Ans: 9911.13 s)
29. Assuming the mean density of the Earth to be 5500 kg m^{-3} , that G is $6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ and that the Earth's radius is 6400 km, find a value for the acceleration of free fall at the Earth's surface. (Ans: 9.9 m s^{-2})
30. Two binary stars, masses 10^{20} kg and $2 \times 10^{20} \text{ kg}$ respectively, rotate about their common centre of mass with an angular velocity ω . Assuming that the only force on a star is the mutual gravitational force between them, calculate ω . Assume that the distance between the stars is 10^6 m and that G is $(6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})$. (Ans: $1.4 \times 10^{-4} \text{ rad s}^{-1}$.)
31. A preliminary stage of spacecraft Apollo 11's journey to the moon was to place it in an Earth parking orbit. This orbit was circular, maintaining an almost constant distance of 189 km from the Earth's surface. Assuming the gravitational field strength in this orbit is 9.4 N kg^{-1} , calculate (i) the speed of the spacecraft in this orbit and (ii) the time to complete one orbit. (Radius of the Earth = 6370 km.). (Ans: (i) 7852 m s^{-1} (ii) 5249 s)
32. Explorer 38, a radio-astronomy research satellite of mass 200 kg, circles the Earth in an orbit of average radius $3R/2$ where R is the radius of the Earth. Assuming the gravitational pull on a mass of 1 kg at the Earth's surface to be 10 N, calculate the pull on the satellite. (Ans: 889N)
33. The Earth is elliptical with polar and equatorial radii equal to $6.357 \times 10^6 \text{ m}$ and $3.678 \times 10^6 \text{ m}$ respectively. Determine the difference, Δg , in the values of the acceleration of free fall at a pole and at the equator due to this difference in radii. What other factors

affect Δg ? (Mass of the Earth = 5.957×10^{24} kg. Gravitational constant, $G = 6.670 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$). (Ans: 0.0646 ms^{-2})

34. Obtain the value of g from the motion of the moon, assuming that its period of rotation around the Earth is 27 days 8 hours and that the radius of its orbit is 60.1 times the radius of the Earth. (Radius of Earth = $6.36 \times 10^6 \text{ m}$). (Ans: 9.8 m s^{-2})

35. The orbit of the moon is approximately a circle of radius 60 times the equatorial radius of the Earth. Calculate the time taken for the moon to complete one orbit, neglecting the rotation of the Earth.

Acceleration of free fall at the poles of the Earth = 9.8 ms^{-2} . Equatorial radius of the Earth = $6.4 \times 10^6 \text{ m}$. 1 day = 8.6×10^4 seconds. (Ans: 27.4 days)

36. Two particles of masses 1kg and 2kg are located at $x=0$ and $x=3 \text{ m}$. Find the position of the centre of mass. (Ans: $0.02 \text{ m from the origin}$)

37. With what velocity should a satellite be launched at a height of 600 km above the surface of the Earth so as to move in a circular path? Given: $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$; radius of Earth = $6.4 \times 10^6 \text{ m}$, Mass of the Earth = $5.98 \times 10^{24} \text{ kg}$. (Ans: 7.545 kms^{-1})

Answer to the multiple choice questions

Q.N	1.	2.	3.	4.	5.
Ans	b	b	a	a	b
Q.N	6.	7.	8.	9.	10.
Ans	b	a	b	b	d
Q.N	11.	12.	13.	14.	15.
Ans	d	a	b	a	b

Chapter 8

Elasticity

Robert Hooke [Born on July 18 (July 28, New Style), 1635, Freshwater, Isle England], is an English physicist who discovered the law of elasticity, known as Hooke's law. In 1655 Hooke was employed by Robert Boyle to construct the Boylean air pump. Five years later, Hooke discovered his law of elasticity, which states that the stretching of a solid body (e.g., metal wood) is proportional to the force applied to it. The law laid the basis for studies of stress and strain and for understanding of elastic materials. He applied these studies in his designs for the balance springs of watches; his interest in timekeeping was further reflected in his effort to improve the pendulum for clock regulation. Robert Hooke has (1660) developed a mathematical relationship between tension and extension. In 1660, he discovered the law of elasticity which states the stretching of a solid is proportional to the force applied. Poisson, Cauchy and G Stoke showed the equation of the general theory of elasticity. In 1662 he was appointed curator of experiments to the Royal Society of London and was elected a fellow the following year.



Fig: Robert Hooke

8.1 Introduction

Activity 8.1

Do the activities that follows:

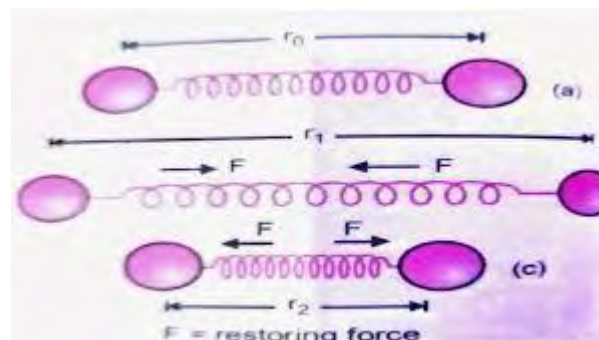


Fig 8.1: Showing the elastic behavior

- i) Take a metallic (or a hard rubber) spring with a small marble attached at the free ends and, place it on a table.
- ii) Bring the marbles close to each other and then release them. Observe what happens next.
- iii) Pull the marbles away from each other and release them again. Observe what happens.

Think for yourself and collect more ideas on what's going on there.

The force given in either case is the deforming force that changes the configuration of the body. When the deforming force in either case is removed, the body regains its original configuration. In doing so, there is a force setting up internally, called the restoring force.

When the deforming force employed to extend the spring is large enough, the body can't regain its original configuration. Such a stage is the stage of plasticity, and the body is said to be in a plastic stage (or in a damaged state in some context of engineering and structural analysis). When a body regains its original configuration, it is in the elastic stage, and the phenomenon is **elasticity**.

Hence *elasticity is the property of a matter by virtue of which it regains its original shape and size after the removal of the deforming force.*

In physics, the more the rigidity, the more is the elasticity.

Hence this is why iron is said to be more elastic than rubber.

In the same sense, a solid is more elastic than a liquid and the liquid is more elastic than gases. The origin of elasticity is due to the presence of short-range molecular interaction inside the matter.

Every matter is made of atoms and molecules. The separation of such particles in a normal state is r_0 . If the material is elongated using a deforming force, the separation r_0 increases, and an attractive force between the particles comes into play, returning it to its original configuration. If the body is contracted, r_0 decreases, and a repulsive force establishes itself, capable of returning it to its original configuration.

If the deforming force is so large that the distance between the particles is much greater than r_0 , the two particles cannot attract each other, and the body is in the plastic stage. Hence, the point of the deforming force at which the body regains its original configuration is the elastic limit. Beyond this point, the body is in the plastic stage (or damaged in the case considered).

Concept to know:

- i) *Springs are generally made of steel rather than copper because springs are used in spring balance for which the material of the spring should have high value of elasticity. Steel is more elastic than copper, and requires greater force for the same extension. Hence, we favor steel for the spring.*
- ii) *RCC rods are used in the construction of buildings. In physics, the rods have a high degree of elasticity, allowing buildings to regain their shape even after a heavy earthquake. The design and materials used in these rods contribute to the overall resilience of the structure.*
- iii) *High-tension cable is commonly found in a twisted form. If it has already been twisted, it indicates that some deforming force (stress) has been applied, making it easier to fold and*

extend the cable. Moreover, the twisted form is more flexible, kink-resistant, and break-resistant, with higher resistance to metal fatigue compared to solid cable.

8.2 Hooke's law:

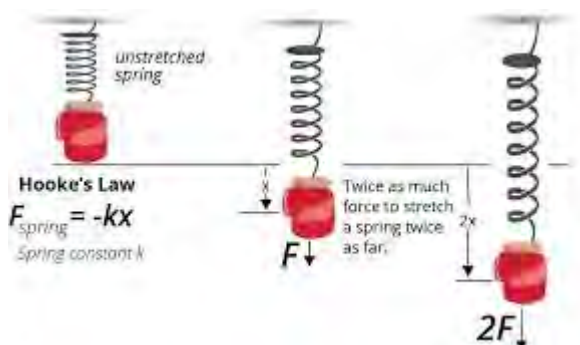


Fig 8.2: Showing Hooke's law

In the context of the activities studied and based on the figure above, Robert Hooke investigated the elastic behavior of materials, and developed a mathematical relationship between the applied tension and the resulting extension.

In the course of his research, he established *that within the elastic region, extension produced is directly proportional to the load applied to a body, meaning extension is proportional to the load i.e., extension $\propto$ load*

$$\text{Or, } x \propto F$$

$$\text{Or, } F \propto x$$

$$\therefore F = -kx \dots\dots\dots(8.1)$$

Here, $k \longrightarrow$ spring constant (or force constant)

If A is the cross-sectional area of a wire and L is its length,

$$\text{Then } \frac{F}{A} = -K \frac{x}{L}$$

$$\text{Or, } \frac{F}{A} = -KL/A \cdot \frac{x}{L} \times$$

Since K , L and A are constants,

$$\frac{F}{A} \propto \frac{x}{L}$$

I.e. Stress $\propto$ Strain

$$\text{Or, stress} = E \text{ strain}$$

$$= E \dots\dots\dots (8.2) \text{ is the Hooke's law}$$

Hence Hooke's Law can also be stated as stress is directly proportional to the strain produced within the elastic region.

$$\text{Or, stress} = E \text{ strain}$$

$$\therefore \frac{\text{Stress}}{\text{Strain}} = E \dots\dots\dots (8.2) \text{ is the Hooke's law}$$

Here, $E \longrightarrow$ Modulus of elasticity

Experimental verification

The experimental arrangement to verify Hooke's law has been shown in the figure below. Two wires, one being a test wire and the other as a reference wire, are taken and suspended from a rigid support, hanging vertically.

A Vernier apparatus is set up on the wires so that the extension on the test wire with respect to the reference wire is easily readable.

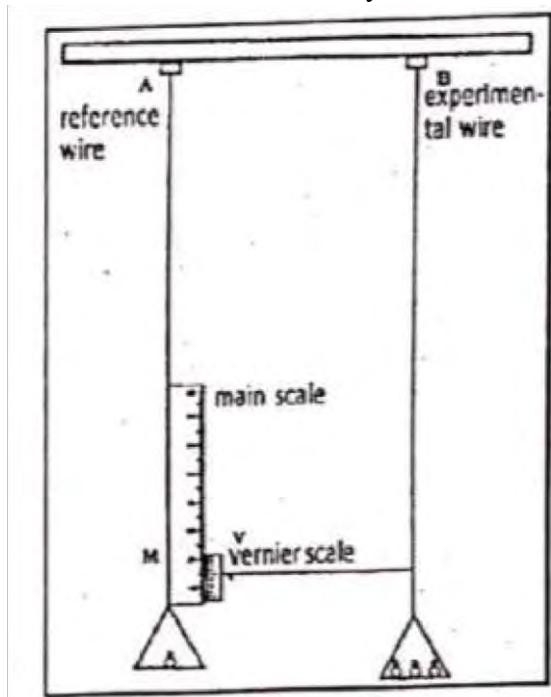


Fig 8.3: Hook's Law verification

Loads are added in steps to a test wire, and the resulting extensions are noted at regular intervals. This process is conducted within the elastic limit. After removing the weights, contractions are noted. Finally, the extensions produced by the loads are calculated.

Experimentally, it is found that the extension produced is proportional to the applied load.

i.e, $x \propto F \Rightarrow F \propto x = F = -kx$

$$\text{Or, } \frac{F}{A} = -\frac{K}{A} \cdot L \frac{x}{L}$$

$$\text{Or, } \frac{F}{A} = \text{constant} \cdot \frac{x}{L}$$

Or, stress = constant. Strain

$\therefore$ Stress $\propto$ strain

8. Strain, Elasticity and Plasticity:

Stress:

The restoring force per unit area of a deformed body is called stress.

$$\text{i.e, stress} = \frac{\text{restoring force}}{\text{area}}$$

$$\text{Or, } \frac{F}{A} = \text{Nm}^{-2} \text{ and } (\text{ML}^{-1}\text{T}^{-2})$$

Stress and pressure are both terms used in physics and engineering to describe the distribution of force within a material, but they refer to slightly different concepts:

Stress is a measure of the internal force per unit area acting within a material that arises from external forces applied to the material, such as stretching, compressing, shearing, or bending, and is a vector quantity.

Pressure is a measure of the force per unit area applied on a surface, typically exerted by a fluid (liquid or gas), and is a vector quantity.

Types of Stress:

On the basis of the nature of the applied force, the stress developed is categorized as follows:

1) Normal stress:

The force applied is normal to the surface. The normal stress is further divided as follows:

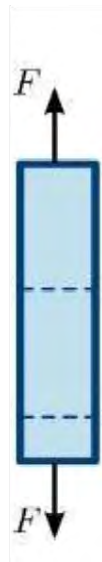


Fig 8.4: Normal stress

i) Tensile stress:

If the normal stress acts in the direction of the length and there is an increase in length, it is the **tensile stress**.

ii) Compression stress:

If the normal stress acts in the direction of the length and there is a decrease in length, it is **compression stress**.

2) Tangential (shearing) stress:

When the force applied is parallel to the surface keeping the lower surface fixed, it is called tangential stress.

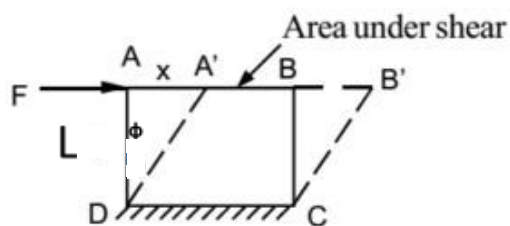


Fig 8.5: Showing tangential stress

In the figure, tangential force is parallel to the upper surface AB so that the opposite lower face CD remains fixed.

In doing so, ultimately the point A tends to A' and B tends to B' such that $AA' = BB' = x$.

Strain:

Strain is defined as the ratio of change in dimension to the original dimension.

$$\text{i.e., Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

It is the ratio of two similar quantities, and hence has no unit and dimension.

Types of Strain:

On the basis of the dimension change, there are three types of strain:

i) Longitudinal strain:

It concerns with the change in length and is defined as the ratio of change in length to the original length.

$$\text{i.e., longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = \frac{x}{L}$$

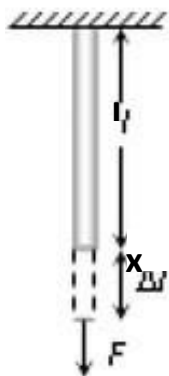


Fig 8.6: Longitudinal strain

Concept to know:

A heavy uniform rod hanging vertically from a fixed support, is stretched by its own weight. The diameter of the rod is smallest at the top and gradually increases down the rod due to the increasing weight downwards.

ii) Volumetric strain:

It concerns with volume change and is defined as the ratio of change in volume to the original volume

$$\text{i.e., volumetric strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{-v}{V}$$

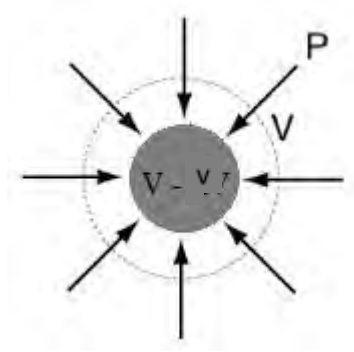


Fig 8.7: Showing volumetric strain

Shear strain:

Under the tangential stress, the upper surface shifts towards the direction of the force such that $AA' = BB' = x$.

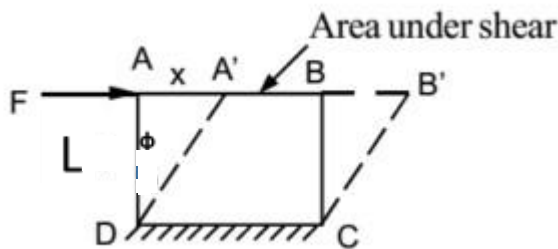


Fig 8.8: Showing shear strain

Hence the angle ϕ is the shear angle which is very small.

$$\therefore \phi \sim \tan \phi = \frac{x}{L} \text{ is the shear strain.}$$

Elasticity:

As already mentioned, elasticity is the property of matter by virtue of which it regains its original shape and size after the removal of the deforming force.

If the body completely regains its original configuration immediately after the removal of the deforming force, then the body is said to be perfectly elastic body.

This is only the ideal assumption. Quartz and phosphor bronze approach this assumption.

Plasticity:

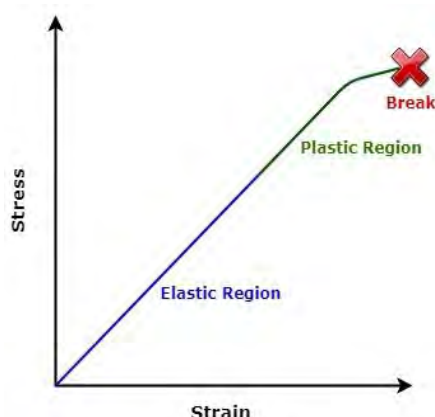


Fig 8.9: Stress vs Strain curve Showing plasticity

A perfectly plastic body is one that does not regain its original configuration at all after the removal of the deforming force, regardless of the magnitude of the force applied. Examples of such materials include paraffin wax, putty, and wet clay.

Applications of Elasticity

- i) A thick metallic rope used in a crane is employed to lift the maximum load, which should not exceed the elastic limit of the rope's material. By understanding this elastic limit and the extension per unit length of the material, one can determine the required area of the wire's cross-section.
- ii) Elasticity is used in estimating the altitude of a mountain. Elasticity principles, such as atmospheric pressure changes, can indirectly influence altitude measurements.
- iii) When designing a bridge, we must consider elasticity along with factors such as traffic load, bridge weight, wind force, and others. The bridge is designed to ensure it neither bends excessively nor breaks under various conditions.

Elastic limit:

The elastic limit, denoted as E , is the upper limit beyond which a body loses its ability to retain its original shape and size after the removal of the deforming force, undergoing plastic deformation instead. It represents the maximum load a body can withstand without undergoing plastic deformation. As a safety measure, it is common practice to prescribe only one-third of the elastic limit's weight to ensure proper elasticity and avoid permanent deformation of the material.

Modulus of Elasticity:

From Hooke's law,

$$\text{Stress} \propto \text{Strain}$$

$$\therefore \text{Stress} = E \text{ Strain}$$

Here, $E = \frac{\text{Stress}}{\text{Strain}}$ is called the modulus of elasticity

Moduli of Elasticity

On the basis of the stress applied and the strain produced, there are three types of moduli of elasticity:

a. Young's Modulus of Elasticity (Y):

It causes the deformation in the length of a body.

Therefore, the ratio of normal stress to the longitudinal strain produced is the Young's modulus of elasticity.

$$\text{i.e, } Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F}{A} / \frac{x}{L}$$

$$\therefore Y = \frac{FL}{Ax} \dots\dots\dots (8.3)$$

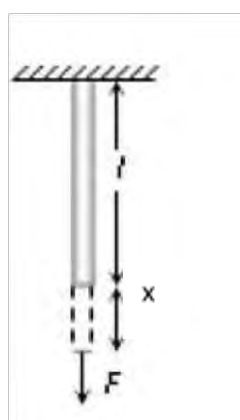


Fig 8.10: Showing Young's modulus of elasticity

Concept to know:

- i) With a rise in temperature, the Young's modulus decreases.
It's because Young's modulus is directly proportional to the stiffness and stiffness decreases with the rise in temperature.
- ii) A uniform wire of mass m , length L , area of cross-section A and Young's modulus Y hangs from the ceiling. Its elongation under its own weight is $mgL/2AY$

Worked Out Examples:

1. A steel cable with cross-sectional area of 3.00 cm^2 has an elastic limit of $2.40 \times 10^8 \text{ Pa}$. Find the maximum upwards acceleration that can be given a 1200 kg elevator supported by the cable if the stress is not to exceed one-third of the elastic limit.

Solution: Here,

$$\text{Area, } A = 3.00 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2$$

$$\text{Elastic limit, } E = 2.40 \times 10^8 \text{ Pa}$$

$$\text{Mass, } m = 1200 \text{ kg}$$

$$\text{Acceleration, } a = ?$$

$$\text{Stress, } \frac{F}{A} = \frac{1}{3} E \Rightarrow F = \frac{1}{3} E A = \frac{1}{3} \times 2.40 \times 10^8 \times 3 \times 10^{-4} = 2.4 \times 10^4 N$$

$$\text{For the upwards acceleration, } F - mg = ma \Rightarrow a = 10.0 \text{ m/s}^2$$

2. A woman distributes her 500 N weight equally over the heels of her high – heeled shoes. Each heel has an area of 0.750 cm². (a) What is the pressure exerted on the floor by each heel?

Solution:

Here, weight, $w = 500 \text{ N}$

Area of the heel, $A = 0.750 \text{ cm}^2 = 0.75 \times 10^{-4} \text{ m}^2$

Now (a) the pressure on each heel, $P = \frac{w/2}{A} = 500/2 \times 0.75 \times 10^{-4} = 3.33 \times 10^6 \text{ N/m}^2$

b. Bulk Modulus of Elasticity(B):

It concerns with the change in volume on the deformed body.

Therefore, the ratio of normal stress to the longitudinal strain is called the Bulk modulus of elasticity.

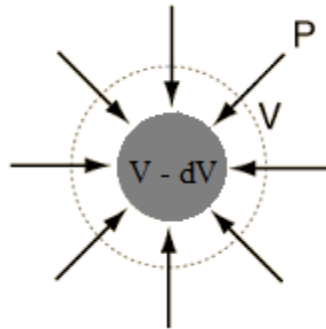


Fig8.11: Showing Bulk Elasticity

$$\begin{aligned} \text{i.e, } B &= \frac{\text{Normal stress}}{\text{Volumetric strain}} \\ &= \frac{F}{A} / \frac{-\Delta V}{V} \\ \therefore B &= \frac{F/A}{-\Delta V/V} \\ &= P / \frac{-\Delta V}{V} \dots\dots\dots (8.4) \end{aligned}$$

Concept to know:

- iv) Bulk modulus is the reciprocal of compressibility. Therefore, water has a higher bulk modulus, making it more elastic.
- v) When concrete is mixed with cement, the compressibility decreases, which in turn increases the elasticity of the mixture.

Worked Out Examples

1. A specimen of oil having an initial volume of 600 cm³ is subjected to a pressure increase of $3.6 \times 10^6 \text{ Pa}$ and the volume is found to decrease by 0.45 cm³. What is the Bulk modulus of the material and the compressibility, the reciprocal of the Bulk modulus of elasticity?

Solution: Here,

Initial volume, $V = 600 \text{ cm}^3 = 600 \times 10^{-6} \text{ m}^3$

Pressure increase, $P = 3.6 \times 10^6 \text{ Pa}$

Volume decrease, $-v = 0.45 \text{ cm}^3 = 0.45 \times 10^{-6} \text{ m}^3$

Bulk modulus, $B = ?$

Compressibility, $K = ?$

We have, $B = P / \left(\frac{-v}{v} \right) = 3.6 \times 10^6 / \frac{0.45}{600} = 4.8 \times 10^9 \text{ Pa}$

$K = 1/B = 1/4.8 \times 10^9 = 2.1 \times 10^{-10} \text{ Pa}^{-1}$

Modulus of Rigidity or shear Modulus (η) :

It causes the change in the shape of the deformed body.

Hence it is defined as the ratio of tangential stress to the shear strain.

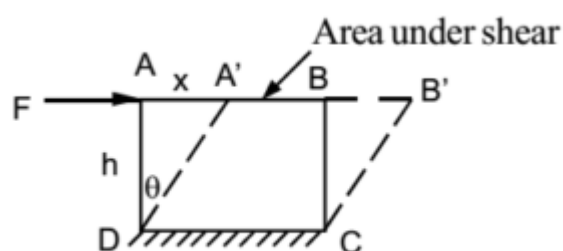


Fig 8.12: Showing Shear modulus

i.e, $\eta = \text{Tangential stress/shear strain}$

$$= \frac{F/A}{\phi}, \phi = x/L.$$

$$\therefore \eta = \frac{F/A}{\phi} \dots\dots\dots(8.5)$$

Worked Out Examples:

1. How much force is required to punch a hole in 1 cm diameter in a 5 mm thick steel sheet whose shearing strength is $2.76 \times 10^8 \text{ N/m}^2$?

Solution:

Here,

Diameter, $d = 1 \text{ cm} = 0.01 \text{ m}$

Thickness, $t = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$

Shear strength = $2.76 \times 10^8 \text{ N/m}^2$

The hole is cylindrical in nature.

$$\therefore A = 2\pi r \times \text{thickness} = 2\pi r t = 2\pi d/2 \times t = \pi \times 0.01 \times 5 \times 10^{-3} \\ = 0.05\pi \times 10^{-3}$$

Now, Shear strength = F/A

Or, $F = \text{Shear strength} \times A$

$$= 2.76 \times 10^8 \times 0.05\pi \times 10^{-3}$$

$$= 4.34 \times 10^4 \text{ N}$$

Experimental Determination of Young's Modulus of Elasticity

The experimental arrangement is shown in the fig. below. It consists of two vertical wires, one is experimental wire and the other is reference wire, both suspended from a rigid support. The free ends are attached with some reference loads just to make the wires straight without any kinks. A scale M is attached on the reference wire A

and a Vernier scale is attached on the experimental wire B just above M. The length L and the cross-sectional area A of the wire B are measured.

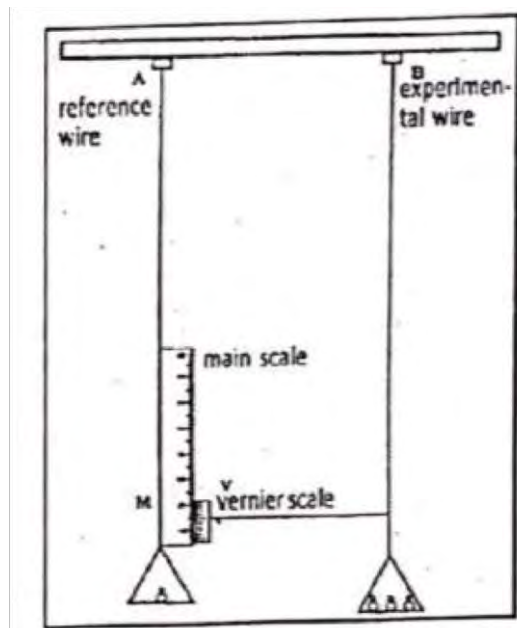


Fig 8.13a: Experimental set up to Young's modulus of elasticity

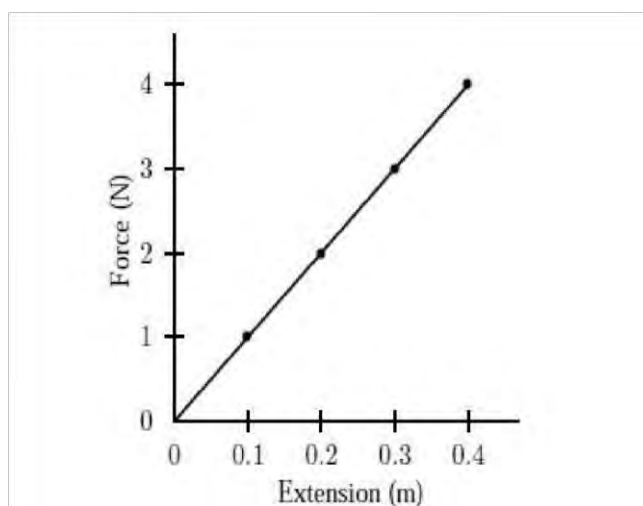


Fig 8.13b: load vs strain curve.

Weights of 1kg are hung on wire B in increments within the elastic limit, and the corresponding extensions produced by each 1kg weight are recorded. Subsequently, a plot of load (F) versus extension (x) is obtained, which yields a straight line passing through the origin. By measuring the slope of this line, denoted as F/x , the value of the Young's modulus can be determined as;

Tensile stress of the wire = F/A

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

And the Young's modulus, $Y =$

$$= \frac{F/A}{x/L} \Rightarrow Y = \frac{F L}{x A} = \text{slope} \times \frac{L}{A} \dots\dots\dots (8.6)$$

Hence the value of Y is determined.

Elastic Potential Energy

When a wire is stretched using a deforming force, some work has been done on it. According to the work-energy theorem, an equivalent amount of energy has been stored there, known as elastic potential energy.

Expression:

Let us consider a wire of length L and cross sectional area A under the deforming force F at its free end. Let the extension produced on the wire be x .

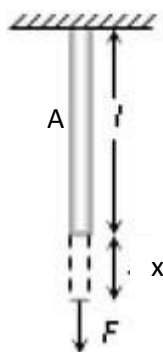


Fig 8.14a: Wire under deforming force

Therefore, tensile stress = F/A ,

And the longitudinal strain = $\frac{x}{L}$

$$\text{Now, } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{x/L}$$

$$\therefore Y = \frac{FL}{Ax} \text{ and } F = \frac{YA}{L} \cdot x$$

For a further small increment in length dx , the small work done, $dw = F \cdot dx$

$$\text{Or, } dw = \frac{YA}{L} \cdot x \cdot dx$$

The total work done in stretching the wire from 0 to x is obtained on integrating-

$$\text{i.e., } \int dw = \frac{YA}{L} \int_0^x x \cdot dx = \frac{YA}{L} \cdot \frac{x^2}{2} = \frac{1}{2} \cdot \frac{YA}{L} \cdot x \cdot x = \frac{1}{2} Fx = \text{elastic P.E as per the work energy theorem.}$$

$$\text{Hence the elastic P.E = Energy stored, } E = \frac{1}{2} Fx.$$

Graphical Method:

As the spring is stretched more and more, the force also increases. If the force exerted by the spring (F) is plotted as a function of the extension (x), the graph forms a straight line, and the slope of the line is equal to the spring constant, k . And, hence the graph follows Hooke's law, and one writes-

$F = kx$ and for the restoring force, $F = -kx$. Since the area beneath a force-extension graph for the elastic region of a material gives the work required to stretch (or compress) a material to a particular extension (x). This work done is equal to the area of a triangle. This is shown clearly in the fig 8.14c.

$$\text{Hence the area} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\begin{aligned}
 &= \frac{1}{2} \text{ extension} \times \text{force} = \frac{1}{2} F \cdot x = \frac{1}{2} kx \cdot x = \frac{1}{2} kx^2 \\
 &= \frac{1}{2} F \cdot x = \frac{1}{2} kx \cdot x \\
 &= \frac{1}{2} kx^2 = \text{work done} = \text{elastic P.E.}
 \end{aligned}$$

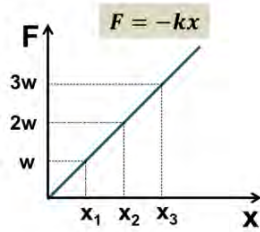


Fig 8.14b: Showing how extension varies with load

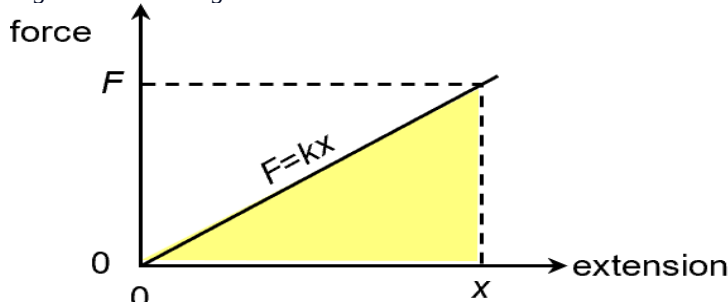


Fig 8.14c: Force-extension graph with area under the graph highlighted

Energy Density:

It is the energy stored per unit volume of a deformed body.

$$\begin{aligned}
 \text{i.e, Energy density, } E_D &= \frac{E}{V} = \frac{1}{2} \frac{Fx}{V} \\
 &= \frac{1}{2} \frac{Fx}{AL} = \frac{1}{2} \frac{F}{A} \cdot \frac{x}{L} \\
 &= \frac{1}{2} \text{ stress} \times \text{strain}
 \end{aligned}$$

$$\text{Further, Energy density} = \frac{1}{2} Y \cdot \text{strain} \cdot \text{strain} = \frac{1}{2} Y (\text{strain})^2$$

Hence the energy density is directly proportional to the square of strain.

Concept to know:

- A compressed spring has stored potential energy. When it is dissolved in acid, the P.E. stored is imparted to acid molecules thereby gaining vibratory motion with the rise of temperature.
- The extended catapult before firing the bullet has PE stored. When a bullet is released, the KE it gains is from the PE stored on the catapult.

Worked Out Examples:

- What force must be applied to a steel wire of 6 m length and the diameter of 1.6 mm to produce an extension of 1mm? ($Y = 2.0 \times 10^{11} \text{ N/m}^2$)

Solution:

Here, length of the wire, $L = 6 \text{ m}$

Diameter of the wire, $d = 1.6 \text{ mm} = 1.6 \times 10^{-3} \text{ m}$

$\therefore$ Area of the wire, $A = \pi d^2/4 = \pi/4 \times 1.6 \times 1.6 \times 10^{-6} \text{ m}^2$

Extension, $x = 1 \text{ mm} = 10^{-3} \text{ m}$

Force, $F = ?$

We have,

$$F = YA x/L = 2.0 \times 10^{11} \times \pi/4 \times 1.6 \times 1.6 \times 10^{-6} \times 10^{-3}/6$$

$$= 67 \text{ N}$$

2. What stress would cause a wire to increase in length by one-tenth of 1% if the Young's modulus for the wire is $12 \times 10^{10} \text{ N/m}^2$? What force would produce this stress if the diameter of the wire is 0.56 mm ?

Solution:

Increase in length, $x = \left(\frac{1}{10}\right) \times 1\% \text{ of } L$, L is the original length of the wire.

$$\Rightarrow x = \frac{1}{10} \times \frac{1}{100} \times L = \frac{L}{1000}$$

$Y = 12.0 \times 10^{10} \text{ N/m}^2$

We have, $Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \text{Stress} = Y \times \text{Strain}$

$$= 12.0 \times 10^{10} \times \frac{x}{L}$$

$$= 12 \times 10^{10} \cdot \frac{1}{1000} = 1.2 \times 10^8 \text{ N/m}^2$$

Now, $\frac{F}{A} = \text{stress} \Rightarrow F = \text{stress} \times \text{area}$

$$= 1.2 \times 10^8 \times \frac{\pi}{4} \times (0.56 \times 10^{-3})^2$$

$$= 29.6 \text{ N}$$

3. The fig. below shows the force -extension graph for a spring. Find the elastic P.E. stored on the spring when it is extended from its original length to 0.50 m .

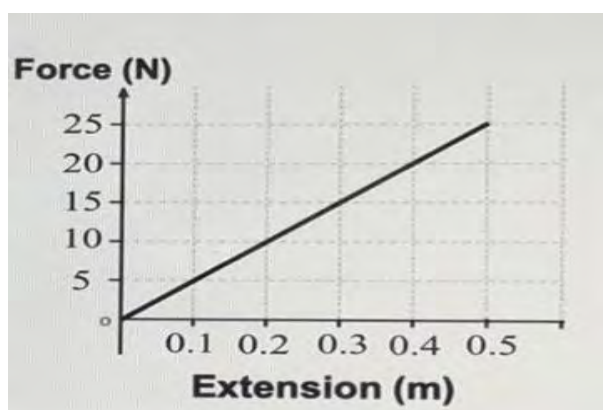


Fig 8.15

Solution:

Here, extension, $x = 0.50 \text{ m}$,

Force, $f = 25 \text{ N}$

$$\text{Therefore, elastic P.E} = \frac{1}{2} F \times x = \frac{1}{2} \times 25 \times 0.50 = 6.3 \text{ J}$$

Sample Questions:

1. The rubber cord of a catapult has cross-section area of 1mm^2 and a total unstretched length of 10 cm. It is stretched to 12 cm and then released to project a particle of mass 5 g. Calculate the velocity of projection (Y for rubber is $5 \times 10^8 \text{N/m}^2$) (Ans: 20ms^{-1})
2. Wires A and B are made from the same materials. A has twice the diameter and three times the length of B. If the elastic limits are not reached, when each stretched by the same tension, what will be the ratio of energy stored in A to that in B? (Ans: 3:4)

Poisson's Ratio:

When a wire or a rod is deformed using a force, it either lengthens or contracts, resulting in a decrease or an increase in diameter. The change in diameter compared to the original diameter is called lateral strain, while the change in length compared to the original length is called longitudinal strain. Hence, the ratio of the lateral strain to the longitudinal strain within the elastic limit is called Poisson's ratio, σ .

Sample Example:

1. What force must be applied to a steel wire 6 m long and diameter 1.6 mm to produce an extension of 1mm? ($Y = 2.0 \times 10^{11} \text{N/m}^2$)

Solution:

Here, length of the wire, $L = 6 \text{ m}$

Diameter of the wire, $d = 1.6 \text{ mm} = 1.6 \times 10^{-3} \text{ m}$

$\therefore$ Area of the wire, $A = \pi d^2/4 = \pi/4 \times 1.6 \times 10^{-3} \times 10^{-3} \text{ m}^2$

Extension, $x = 1 \text{ mm} = 10^{-3} \text{ m}$

Force, $F = ?$

We have,

$$F = YAx/L = 2.0 \times 10^{11} \times \pi/4 \times 1.6 \times 1.6 \times 10^{-6} \times 10^{-3} / 6$$

$$= 67 \text{ N}$$

2. What stress would cause a wire to increase in length by one-tenth of 1% if the Young's modulus for the wire is $12 \times 10^{10} \text{N/m}^2$? What force would produce this stress if the diameter of the wire is 0.56 mm?

Solution:

Increase in length, $x = \left(\frac{1}{10}\right) \times 1\% \text{ of } L$, L is the original length of the wire.

$$\Rightarrow x = \frac{1}{10} \times \frac{1}{100} \times L = \frac{L}{1000}$$

$Y = 12.0 \times 10^{10} \text{N/m}^2$

We have, $Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \text{Stress} = Y \times \text{Strain}$

$$= 12.0 \times 10^{10} \times \frac{x}{L}$$

$$= 12 \times 10^{10} \times \frac{1}{1000} = 1.2 \times 10^8 \text{N/m}^2$$

Now, $\frac{F}{A} = \text{stress} \Rightarrow F = \text{stress} \times \text{area}$

$$= 1.2 \times 10^8 \times \frac{\pi}{4} \times (0.56 \times 10^{-3})^2$$

$$= 29.6 \text{ N}$$

3. The fig. aside shows the force-extension graph for a spring. Find the elastic P.E. stored on the spring when it is extended from its original length to 0.50m.

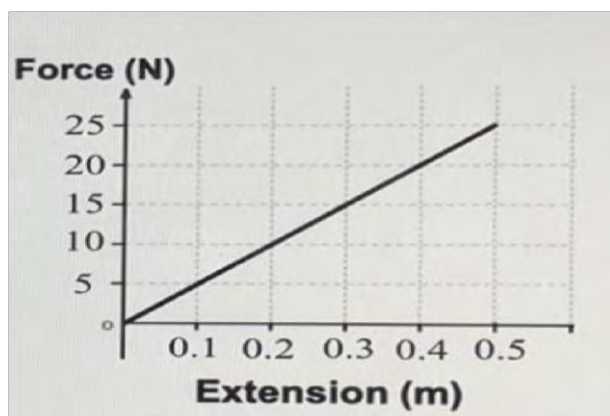


Fig 8.15

Solution: Here, extension, $x = 0.50$ m,

Force, $F = 25$ N

Therefore, elastic P.E. $= \frac{1}{2} F \times x = \frac{1}{2} \times 25 \times 0.50 = 6.3$ J

Sample Questions:

1. The rubber cord of a catapult has cross-section area of 1mm^2 and a total upstretched length is 10 cm. It is stretched to 12 cm and then released to project a particle of mass of 0.005 kg. Calculate the velocity of projection (Y for rubber is $5 \times 10^8 \text{N/m}^2$) (Ans: 20ms^{-1})
2. Wires A and B are made from the same materials. A has twice the diameter and three times the length of B. If the elastic limits are not reached, when each stretched by the same tension, the ratio of energy stored in A to that in B is: (Ans: 3:4)

Poisson's Ratio:

When a wire or a rod is deformed using a force, it either lengthens or contracts, resulting in a decrease or an increase in diameter. The change in diameter compared to the original diameter is called lateral strain, while the change in length compared to the original length is called longitudinal strain. Hence, the ratio of the lateral strain to the longitudinal strain within the elastic limit is called Poisson's ratio, σ .

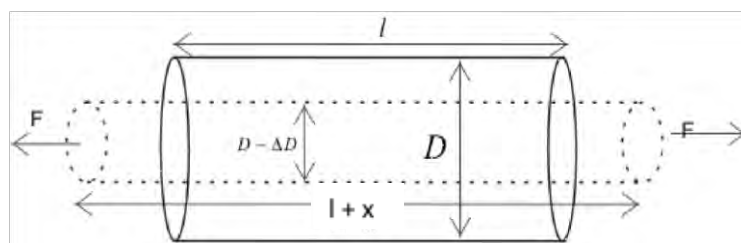


Fig 8.15: Poisson's ratio

i.e Poisson's ratio, $\sigma = \text{lateral strain} / \text{longitudinal strain} = \frac{\beta}{\alpha}$

If the diameter decrease is d and increase in its length is x , then lateral strain $= \frac{\Delta D}{D}$ and the longitudinal strain $= \frac{x}{L}$

Therefore, Poisson's ratio, $\sigma = \frac{-\Delta D/D}{\frac{x}{L}} = \frac{-\Delta D \cdot L}{Dx}$ (8.7)

Some Terms

Stress – strain curve:

The stress strain curve has been shown in the fig.

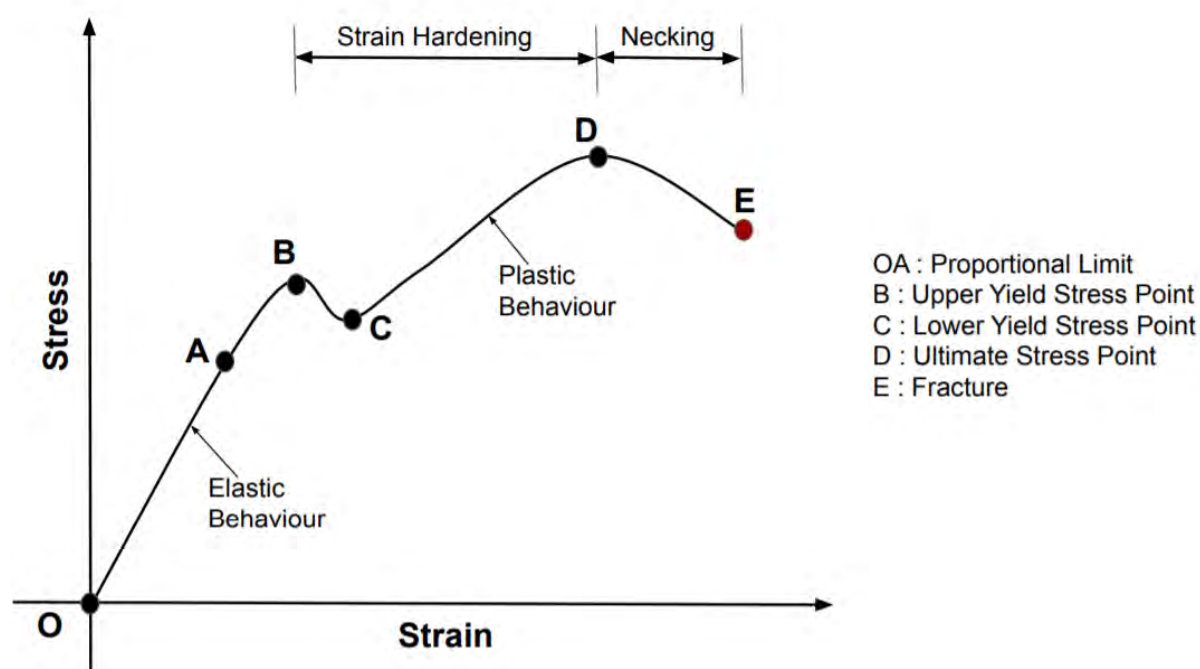


Fig 8.16: Stress- Strain curve

From the point of origin to point A, stress is proportional to the strain, following Hooke's law, and hence this point is called the proportional limit point. From point A to B, a large strain is observed; however, the body remains within the elastic limit. Upon applying stress beyond point B to point C, the wire does not return to its original state, resulting in a permanent set. The material exhibits a dual nature of elasticity and plasticity between these two points, and point C is therefore called the yield point. Moving from point C to point D, there is a large strain, and point D is known as the breaking stress point. Beyond point D, even a small stress causes continuous stretching until the material breaks.

Ductile and Brittle Substance:

Substances that undergo continuous elongation until they break, experiencing plastic deformation, are called ductile substances. Metals are prime examples of ductile substances.

On the other hand, substances that break after surpassing the elastic limit are called brittle substances. Glass is a well-known example of a brittle substance.

The plastic region between the elastic limit (point B) and the fracture point (point E) determines the ductile nature of materials. A longer plastic region indicates greater ductility, while a shorter region signifies brittleness. Ductile materials, such as those used for bending wires, are preferred.

Materials that do not follow Hooke's law even within the elastic limit are called elastomers. Rubber is a prime example of this type.

A Few Terms:

The elasticity of a body refers to its tendency to regain its original configuration after the removal of a deforming force. While some bodies immediately regain their configuration, others may take some time. This delay in regaining the original configuration is called the elastic after-effect.

For materials like quartz and phosphor bronze, the elastic after-effect is negligible, whereas for materials like glass fiber, it is significant. This difference explains why suspension coils in moving coil galvanometers are made of quartz and phosphor bronze.

1. *Elastic after-effect for a perfectly elastic body is zero.*
2. *Elastic after-effect for a perfectly inelastic body is infinite.*

Elastic Hysteresis

When a material wire is stretched several times to its original length using a deforming force, the stress applied may not be proportional to the strain produced. However, the material regains its original configuration through a different path after the removal of the deforming force.

Hence, the phenomenon of following different paths of increasing and decreasing stress is called hysteresis. The loop formed in these two cases is known as the hysteresis loop.

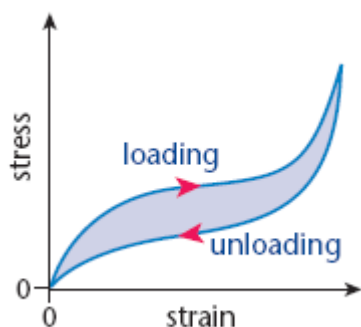


Fig 8.17: Elastic Hysteresis

A high amount of work is required to stretch the wire compared to contracting it, leading to the presence of a non-conservative force. The difference in the work done represents the energy loss, which is balanced by the area of the hysteresis loop.

This phenomenon is the reason for using rubber as a vibration absorber, as rubber exhibits a large hysteresis loop.

Elastic Fatigue

When a material is subjected to an alternating deforming force, it undergoes elongation and contraction accordingly. After several cycles of elongation and contraction, the strain of the material becomes large compared to the applied stress. Consequently, the material becomes

less elastic, leading to elastic fatigue or fatigue failure, similar to human beings experiencing fatigue due to repeated exertion.

The effects of elastic fatigue are:

3. Within the elastic limit, a wire can be broken.
4. Without the use of equipment, a wire can be sliced into pieces.
5. After a long period of service, railway tracks and bridges are deemed unsafe.
6. After a long period of usage, spring balances give incorrect readings.

Project Work:

Project 7.1 Measure and compare the modulus of elasticity of different types of wires commonly available in society.

Hint: Materials you require: Different material wires such as iron, steel, copper, brass, aluminum, etc; micrometer and Vernier apparatus, suitable weights, clamps and supporter.

Theory: $\frac{\text{stress}}{\text{strain}} = Y$ (Young's modulus of elasticity), $\text{stress} = \frac{F}{A}$ and $\text{strain} = \frac{x}{L}$. The slope of strain- stress plot hence gives Y.

Procedural idea:

- i) **Different types of wires** such as Copper, Aluminum, Steel, Iron, etc.
- ii) Equal lengths wires are suspended from a rigid support, cross-sectional area of each is determined by measuring diameter, and loads in steps are attached on the free ends.
- iii) Extension produced is measured with the Vernier apparatus attached.
- iv) Finally *Stress vs Strain* plot is obtained and Y is measured. Repeat it with several other wires and finally *compare Ys of all which will be helpful to the society*.

Exercise

1. Which is the correct alternative for the followings?
 - a. Shear modulus
 - b. Bulk modulus of elasticity
 - c. Young's modulus of elasticity
 - d. All of above
2. What is the elongation produced in a steel wire with a spring constant k when a mass m is attached to one end, and the other end is fixed to a ceiling, and the mass is suddenly released?
 - a. mg/k
 - b. $2mg/k$
 - c. $mg/2k$
 - d. $3mg/k$(Hint: $mgx = 1/2kx^2$)
3. Which one is the correct expression for Poisson's ratio?
 - a. $\Delta R/L \times R/\Delta L$
 - b. $\Delta R/R \times L/\Delta L$
 - c. $\Delta R/\Delta R \times L/\Delta L$
 - d. $\Delta R/R \times \Delta L \times L$
4. A rod is placed between two fixed supports, and it is heated. What type of stress is developed on a rod?

- a. compressive stress
 - b. tensile stress
 - c. shear stress
 - d. relative stress
5. Which of the following substances has the highest elasticity?
 - a. rubber
 - b. sponge
 - c. copper
 - d. steel
 6. How does the value of the Young's modulus change with a rise in temperature?
 - a. increases
 - b. decreases
 - c. remains unchanged
 - d. changes erratically
 7. What are the substances called that can be stretched to cause large strains?
 - (a) brittle
 - (b) ductile
 - (c) plastic
 - (d) elastomer
 8. Which is the correct alternative for the shearing stress applied on a body?
 - (a) length
 - (b) breadth
 - (c) shape
 - (d) volume
 9. A uniform wire of mass m , length L , area of cross-section A and Young's modulus Y hangs from the ceiling. What is its elongation under its own weight?
 - a. zero
 - b. $mgL/2AY$
 - c. $2mgL/AY$
 - d. mgL/AY
 10. How is shearing strain expressed?
 - (a) angle of shear
 - (b) angle of twist
 - (c) decrease in volume
 - (d) increase in volume
 11. A spherical ball contracts in volume by 0.01% when subjected to a normal pressure of 100 atmospheres. What is the bulk modulus of its material in N/m^2 ?
 - a. 1×10^{10}
 - b. 2×10^{11}
 - c. 1×10^{13}
 - d. 1×10^{11}
 12. What is the phenomenon of temporary delay in regaining the original configuration by an elastic body, after the removal of a deforming force?
 - (a) elastic fatigue
 - (b) elasticity
 - (c) plasticity
 - (d) elastic after-effect

13. (a) State Hooke's law.
 (b) Is Hooke's law applicable to all materials?
 (c) The elastic limit of steel is $8 \times 10^8 \text{ Nm}^{-2}$ and Young's modulus is $2 \times 10^{11} \text{ Nm}^{-2}$.
 What will be the maximum elongation of a half-meter steel wire that can be given without exceeding the elastic limit? (Ans: $2 \times 10^{-3} \text{ m}$)
14. (a) Name the three types of moduli of elasticity.
 (b) Which type of elasticity does water bear?
 (c) An 80 cm long and 1.0 mm diameter guitar steel string is to be tightened to a tension of 2000N by turning a screw. How far is the string stretched? (Young's modulus for the steel is $2.0 \times 10^{11} \text{ Nm}^{-2}$) (Ans: $1.02 \times 10^{-2} \text{ m}$)
15. (a) State what you mean by Poisson's ratio.
 (b) Define the term elastic after-effect.
 (c) Calculate the increase in energy of a brass bar of length 0.5 m and cross-sectional area $1 \times 10^{-4} \text{ m}^2$ when compressed with a load of 5 kg along its length. (Y for brass = $1.0 \times 10^{11} \text{ N/m}^2$, $g = 10 \text{ ms}^{-2}$) (Ans: $6.25 \times 10^{-5} \text{ J}$)
16. (a) What is elastic PE? Derive an expression for it on a stretched wire.
 (b) If a material with a Poisson's ratio of 0.2 is applied to a uniform rod, and experiences a longitudinal strain of 4×10^{-3} , calculate the percentage change in its volume. (Ans: 0.24%)
 (c) A copper wire with a cross sectional area of 0.01 cm^2 is under a tension of 25N. Find the decreases in the cross-sectional area (Y for copper = $1.1 \times 10^{11} \text{ Nm}^{-2}$ and $\sigma = 0.32$). (Ans: $1.45 \times 10^{-10} \text{ m}^2$)
17. a) Discuss the different types of moduli of elasticity.
 b) What force is required to stretch a steel wire of cross-sectional area $1 \times 10^{-4} \text{ m}^2$ to double its length? Given Young's modulus for the wire is $2.0 \times 10^{11} \text{ N/m}^2$. (Ans: $1.90 \times 10^7 \text{ N}$)
 c) Two wires one of steel and another of copper, both having equal cross-sectional area are joined end to end. When the combination is kept under tension, the elongation on the two wires is found to be equal. Given Young's modulus for steel and copper are $2.0 \times 10^{11} \text{ N/m}^2$ and $1.1 \times 10^{11} \text{ N/m}^2$. Find the ratio between the lengths of steel and copper wires. (Ans: 20/11)
18. a) What are elastic limit and breaking stress?
 b) Define elastic Hysteresis.
 c) A sample of fishing line is 1.0 m long and its radius of circular cross-section is 0.25 mm. When a weight is hung on the line, the extension is 50 mm and the stress is $2.0 \times 10^8 \text{ Pa}$. Calculate (i) the cross-sectional area of the line, (ii) the weight added, (iii) the strain on the line, and (iv) Young's modulus. (Ans: (i) $2.0 \times 10^{-7} \text{ m}^2$, (ii) 39N, (iii) 0.05, (iv) $4.0 \times 10^9 \text{ Pa}$)

19. Young modulus for steel is $2.0 \times 10^{11} \text{ Nm}^{-2}$, calculate the work done in stretching a steel wire of 100 cm in length and of cross-sectional area 0.030 cm^2 when a load of 100 N is slowly applied without the elastic limit being reached. (Ans: $8.3 \times 10^{-3} \text{ J}$)
20. A uniform steel wire of density 7800 kg m^{-3} weighs 16 g and is 2.50 m long. It lengthens by 1.2 mm when stretched by a force of 80N. Calculate (a) the value of the Young modulus for the steel, (b) the energy stored in the wire. (Ans: $2.0 \times 10^{11} \text{ N/m}^2$, $4.8 \times 10^{-2} \text{ J}$)
21. The rubber cord of a catapult has a cross-sectional area of 1.0 mm^2 and the total unstretched length of 10.0 cm. It is stretched to 12.0 cm and then released to project a missile of mass 5.0 g. From energy consideration, or otherwise, calculate the velocity of projection ($Y = 5.0 \times 10^8 \text{ N/m}^2$) (Ans: 20 ms^{-1})
22. A copper wire and a steel wire of the same cross-sectional area and of length 1 m and 2 m respectively are connected end to end. A force is applied, which stretches their combined length by 1 cm. Find how much each wire is elongated. Y of copper $= 1.2 \times 10^{10} \text{ Nm}^{-2}$ and Y of steel $= 2.0 \times 10^{10} \text{ Nm}^{-2}$
- (Ans: copper = 0.0045m, steel = 0.0055m)
23. A uniform wire of density 8000 kg m^{-3} weighs 20 gm and is 2.5 m long. It lengthens by 1 mm when stretched by a force of 80N. Calculate the value of Young's modulus of steel and the energy stored in the wire. (Ans: $Y = 2.0 \times 10^{11} \text{ N/m}^2$, $E = 40 \times 10^{-3} \text{ J}$)
24. A wire of length 3.0 m and cross-sectional area $1.0 \times 10^{-6} \text{ m}^2$ has a mass of 15 kg hung on it. What is the stress produced in the wire? If, $g = 9.8 \text{ ms}^{-2}$. If Young's modulus for the material is $2.0 \times 10^{11} \text{ Nm}^{-2}$, what is the extension x produced? When extended, how much energy is stored in the wire? (Ans: $1.47 \times 10^8 \text{ Nm}^{-2}$, $2.2 \times 10^{-3} \text{ m}$, 0.162 J)
25. A circular steel wire of 2.00m length must stretch no more than 0.25 cm when a tensile force of 400N is applied to each end of the wire. What minimum diameter is required for the wire? $Y = 20 \times 10^{10} \text{ N/m}^2$. (Ans: $1.43 \times 10^{-3} \text{ m}$)
26. A vertical brass rod of circular section is loaded by placing a 5 kg weight on top of it. If its length is 50 cm and its radius of cross-section is 1 cm, and the Young modulus of the material $3.5 \times 10^{10} \text{ Nm}^{-2}$, find (a) the contraction of the rod, (b) the energy stored in it. (Ans: a) $2.27 \times 10^{-6} \text{ m}$ (b) $5.7 \times 10^{-5} \text{ J}$)
27. In a material testing laboratory, a metal wire made from a new alloy is found to be broken when a tensile force of 90.8N is applied perpendicular to each end. If the diameter of the wire is 1.84 mm, what is the breaking stress of the alloy? (Ans: $3.41 \times 10^7 \text{ Pascal}$)
28. A metal rod that is 4.00 m long and 0.50 cm^2 in the cross-sectional area is found to be stretched to 0.20 cm under a tension of 5000N. What is Young's modulus for this metal? (Ans: $2.0 \times 10^{11} \text{ N m}^{-2}$)
29. A specimen of oil having an initial volume of 600 cm^3 is subjected to a pressure increase of $3.6 \times 10^6 \text{ Pa}$, and the volume is found to decrease by 0.45 cm^3 . What is the bulk modulus of the material? The compressibility? (Ans: $4.8 \times 10^9 \text{ Pa}$, $2.1 \times 10^{-10} \text{ Pa}^{-1}$)

Answer to the Multiple Choice Questions:

Q.N	1.	2.	3.	4.	5.	6.
Ans	b	B	b	a	d	b
Q.N	7.	8.	9.	10.	11.	12.
Ans	d	c	b	a	d	d

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ii. *Text Book by Albert Lewis Ford, Hugh D. Young and Roger A. Freedman* iii.

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It concerns with volume change and is defined as the ratio of change in volume to the original volume

Chapter 9

Heat and temperature



Anders Celsius (1701-1744)

Anders Celsius (27 November 1701 – 25 April 1744) was a Swedish astronomer, physicist and mathematician. He was professor of astronomy at Uppsala University from 1730 to 1744, but traveled from 1732 to 1735 visiting notable observatories in Germany, Italy and France. He founded the Uppsala Astronomical Observatory in 1741, and in 1742 proposed (an inverted form of) the Centigrade temperature scale which was later renamed Celsius in his honour.

Think?

1. What do we feel cold or warm, when we pick up ice blocks in our hands?
2. Do you have any idea why it happens?
3. What do we feel cold or hot if we accidentally touch the laundry iron base during plug-in?
4. Do you have any idea why it happens?



[Fig. 9.1]



[Fig.9.2]

9.1 Introduction

Heat and temperature have a great influence on the behavior of matter and the world around us. They have a fundamental role in all events from the everyday experience of feeling warmth from the sun to the complex processes of thermal equilibrium and heat transfer in various systems. Heat and temperature are closely related. Heat refers to the transfer of thermal energy between systems as a result of temperature difference whereas temperature is a measure of the average kinetic energy of molecules within a system. Thermal energy is the result of molecular

randomness (kinetic energy) within a system. In this chapter, we study the definitions, properties, and applications of thermal energy, heat, and temperature to dig out their mysteries.

9.2 Thermal Energy

Thermal energy is the most basic form of energy contained within a system or an atom. Thermal energy is not in transit but remains stored in the system as its kinetic energy. Thermal energy is responsible for random movements of the molecules, or atoms, and that makes the atoms and molecules move faster. Thermal energy gives the total kinetic energy possessed by a system. Therefore, the thermal energy of a body is responsible for its temperature. In a system, it increases with an increase in temperature and hence it is used to determine the internal energy of such a system. Internal energy includes the kinetic energy and potential energy of molecules within a substance. The kinetic energy component of internal energy is directly related to the thermal energy of the system.

Thermal energy has been used for heating, boiling water and cooking food, etc. since the beginning of the human race. From the beginning, the primary sources of the thermal energy were the sun and fire, but technological evolution over the years has expanded the scope to new man-made energy sources such as geothermal energy, nuclear heat, waste heat, and electrical energy. Chemical energy and wind energy are other sources of thermal energy used throughout.

Observations reveal that a hot object possesses a considerable amount of thermal energy, while a cold object has a smaller amount. This concept can be illustrated by a common experience: rubbing our hands together on a freezing winter day. In this scenario, the kinetic energy of our hands is transformed into thermal energy through the friction generated between them.

When a substance is heated, its temperature increases because the average kinetic energy of the molecules increases. At a higher temperature, molecules have greater kinetic energy and move more rapidly. This increased motion leads to more frequent and energetic collisions between molecules. Consequently, thermal energy is transferred from the higher temperature region to the lower temperature. This transfer of thermal energy is known as heat.

9.3 Heat

When we place a hot object in thermal contact with a cold object, the thermal energy gets transferred from the hot object into the cold one until they attain thermal equilibrium. This thermal energy in transit is commonly referred to as heat. Hence, heat energy is the transfer of thermal energy between two objects at different temperatures. Heat, being a form of energy, is not a physical object; therefore, we cannot sense it by merely looking at an object, but rather feel it when we touch it. The Sun stands out as a significant and natural source of heat energy. In contemporary times, the heat energy, we primarily use is derived from natural sources like gas and oil.

When we are exposed to sun light, we feel warm. Similarly, when we go near the snow covered hill we feel cold. Also, if we touch the snow we feel further cold. In all cases energy is

exchanged with our body. In the former case, our body has gained some energy from sunlight but in the later cases, our body has lost some energy. This exchange of energy is carried out by the difference in hotness between our body and sunlight in the first case and between our body and snow in the later cases. In all above cases, the exchanged amount of energy or energy in transit is heat. Hence, heat is the energy which is transferred on the account of temperature difference between one body and another body. The natural transfer of heat is a non-mechanical process.

It is not necessary for the heat to flow from a body which has more heat energy to a body which has less heat energy. For example; A full bucket of warm water contains more heat energy than a red, hot iron nail. But if we immerse the nail in warm water heat will flow from the nail to the warm water to attain thermal equilibrium.

In the molecular concept, all substances are composed of large number of molecules. These molecules are in random motion except at the absolute zero temperature. Therefore, they possess kinetic energy. “The sum of the kinetic energy of all molecules of a substance is the measure of heat energy contained in that substance”.

Heat is the energy in transit or exchange amount of energy and can be measured by using calorimeter. SI unit of heat is joule (J) but commonly heat is measured in calorie (cal). Such that $1 \text{ cal} = 4.18 \text{ J}$.

Thermal energy is the energy contained in a system but heat is the flow of thermal energy.

Comparisons between thermal energy and heat

S.N.	Thermal energy	Heat
Cause	It is due to the movement of molecules or atoms in a system.	It is due to difference in the degree of hotness or coldness of a system
Change	It increases as the temperature of a system increases due to an increase in the movement of particles.	It always flows from the hot body towards the cold body until both attain equal temperature.
Transfer	It remains in a system as a part of kinetic energy.	It is the transfer of thermal energy.
Source	Sources of thermal energy are geothermal energy, solar energy, friction, etc.	Natural sources of heat are coal, gas, oil, etc.

9.4 Temperature

Temperature is the degree of hotness or coldness of a body which determines the direction of heat flow. When two bodies at different temperatures are brought in thermal contact then the cold body warms up and the hot body cools down. Due to the difference in the thermal state (hotness), heat is lost by hot body which is transferred towards cold body. Hence heat of cold body increases. *Thus, the temperature is the thermal state of a body which determines the direction of heat flow between two bodies in thermal contact.*

In thermal contact, the temperature of a cold body goes on increasing and the temperature of a hot body goes on decreasing until both acquire the same value of temperature. Hence

temperature of a system is the property which finally acquires the same value as all other bodies placed in the thermal contact.

The body which loses heat in thermal contact is said to be at higher temperature whereas the body which gains heat is said to be at lower temperature. The body which gains heat goes on warming up and its temperature increases due to added heat energy. Hence temperature is the effect of heat.

A body at higher temperature gives more severe pain of hotness even if it may have low amount of heat. But a body at low temperature gives less severe pain of hotness even if it may have high amount of heat.

In molecular concept, *temperature of a body gives the measure of average kinetic energy of its molecules*. It is not defined for single particle but for a system of large number of particles. Hence temperature is macroscopic parameter.

The unit of temperature is Kelvin (K) in SI system and other commonly used units are degree Celsius ($^{\circ}\text{C}$) and degree Fahrenheit ($^{\circ}\text{F}$).

9.5 Effects of Heat

When heat is applied to a body, it may bring the following changes:

1. Thermoelectric effect: If a temperature difference is maintained between two junctions of a closed loop made from a pair of dissimilar metal wires, an emf is developed. This emf produces electric current in the circuit. This effect of heat is called thermoelectric effect.
2. Generation of light energy: When we increase the temperature of a substance sufficiently high, the substance starts emitting light.
3. Change in electrical resistance: On increasing the temperature, electrical resistance of a conductor increases but in case of semiconductor electrical resistance decreases.
4. Change in physical properties: Change in temperature, volume, state, colour, viscosity, elasticity, solubility rate, rate of chemical reaction and surface tension, etc. are the physical changes carried out by the application of heat.
5. Change in chemical properties: Ignition of paper, boiling of egg, etc. are the examples of chemical changes carried out by the application of heat.

Sample Questions:

1. Can an object be hotter than another if they are at the same temperature? Explain.
2. What is the temperature of a vacuum?
3. Why does heat flow from a body at higher temperature to a body at lower temperature?
4. Which factor determines the flow of heat between two bodies in thermal contact?

9.6 Thermal Equilibrium

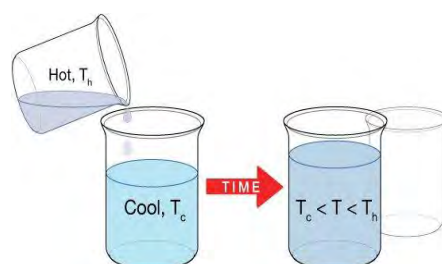


Fig. 9.3: Thermal Equilibrium

When a cup of hot tea is put on the table and after a short period of time, the temperature of the tea will also be same as that of the surroundings. This is because when two bodies at different temperatures are brought in thermal contact, heat flows from a body at higher temperature to a body at lower temperature. So, temperature of hot body goes on decreasing and temperature of cold body goes on increasing until both acquire equal value of temperature. After attaining equal temperature net flow of heat between those bodies becomes zero, and they are said to be thermal equilibrium. Actually, exchange of heat between two bodies in thermal contact does not stop at thermal equilibrium, but it occurs at same rate in opposite direction. Using this idea of thermal equilibrium temperature readings are taken otherwise a thermometer is worthless because it always registers the temperature of its own thermometric substance.

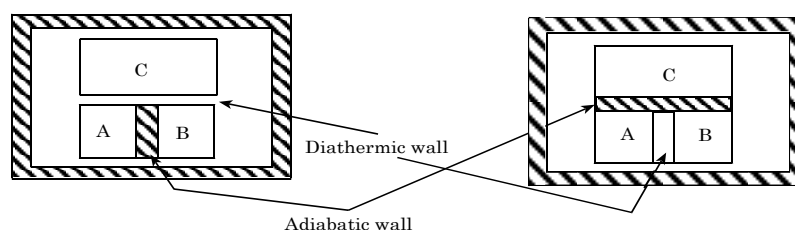
“Two bodies are said to be in thermal equilibrium if there is no net flow of heat between them even if they are in thermal contact”. In other words, two physical systems are in thermal equilibrium if there is no net flow of thermal energy between them when they are in thermal contact. Principle of thermal equilibrium is used to measure the temperature of a body from a thermometer.

9.7 Thermal Equilibrium as a Working Principle of Mercury Thermometer

To measure the temperature of a body, the bulb of the thermometer and the object whose temperature is to be measured are brought into thermal contact, and sufficient time is allowed for them to reach thermal equilibrium. At thermal equilibrium, both the thermometric substance and the object attain the same temperature. However, the temperature reading is solely indicated by the thermometer, representing the temperature of its thermometric substance. Therefore, the reading displayed by the thermometer is the temperature of its own thermometric substance. By employing the concept of thermal equilibrium, we can infer that this temperature is also the temperature of the given object. Thus, the principle of thermal equilibrium is employed in measuring the temperature of a body using a thermometer.

9.8 Zeroth Law of Thermodynamics

Statement: “When two bodies A and B are in thermal equilibrium separately with a third body C then, first two bodies A and B themselves must be in thermal equilibrium”.



[Fig. 9.4, Zeroth law of thermodynamics]

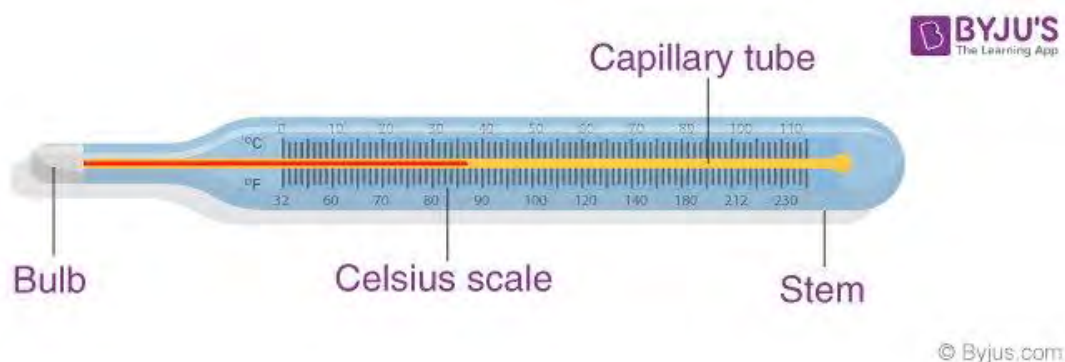
Consider two bodies A and B which are separated by non-conducting wall. Let both are separately connected with system C by conducting wall as shown in figure 9.4. If bodies are kept in such contact for sometimes, then body A and B separately exchange their heat with body C. Finally, both A and B separately attain thermal equilibrium with C. When adiabatic wall between A and B is replaced by diathermic wall and they are separated from C by adiabatic wall as in second figure, then A and B both exhibit no change in temperature. This indicates that A and B were already in thermal equilibrium. This verifies the Zeroth law of thermodynamics.

Long after the use of first and second law of thermodynamics Ralph Howard Fowler, in 1931, formulated this law for the very fundamental single valued function (temperature). Hence this law was labeled as Zeroth law of thermodynamics.

Sample Questions:

1. Why is it wise to wait for some time to measure the temperature of substance?
2. Explain why temperature readings of air are always taken with the thermometer in the shade.
3. How would a thermometer be different if glass expands more than mercury with increase in temperature?
4. A Thermometer is laid out in direct sunlight. Does it measure the temperature of the sun? Explain.
5. A student claimed that thermometers are useless because a thermometer always registers its own temperature. How would you respond?

9.9 Thermometry



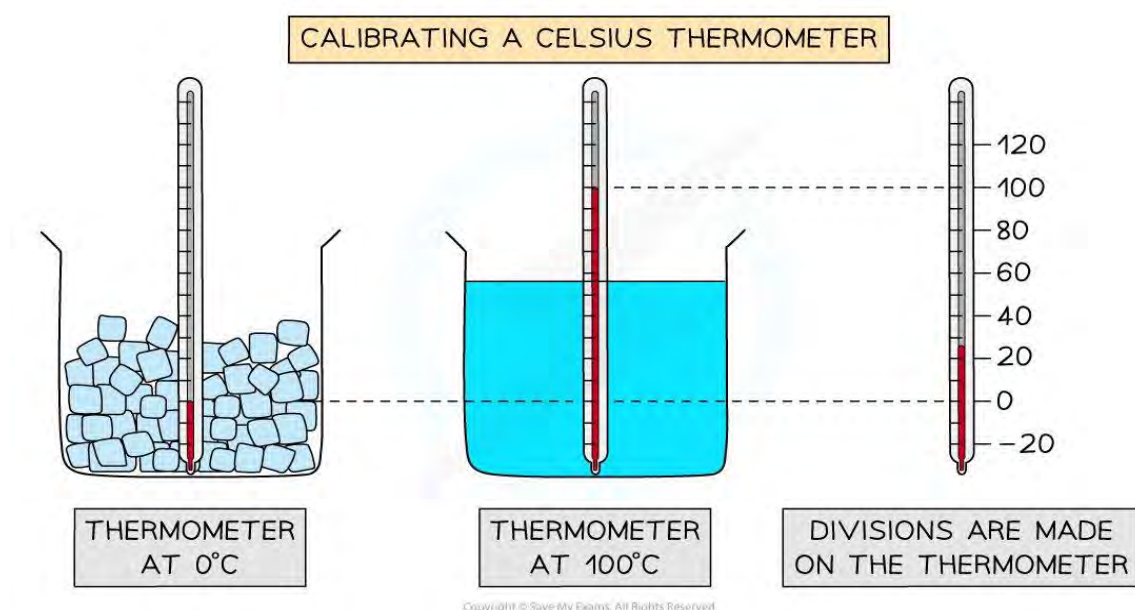
[Fig.9.5, A thermometer]

Thermometry is the science of temperature and its measurement and the device used to measure the temperature is called thermometer. Working principle of thermometer is that the change in temperature brings some changes in measurable physical properties of a body. For example; change in,

i. Length of bar	ii. Volume of liquid
iii. Volume of gas at constant pressure	iv. Pressure of gas at constant volume
v. Magnetic properties of substance	vi. Electrical resistance of a wire, etc.

To measure the temperature of a body, the bulb of thermometer and object whose temperature is to be measured are brought in thermal contact and certain time is provided to bring them in thermal equilibrium. The main three steps of thermometry are:

- i. Construction
- ii. Calibration
- iii. Sensitiveness



[Fig.9.6, Calibration of a thermometer]

Construction: A thermometer is constructed in such a way that the uniform variation in properties of thermometric substance with temperature is maintained. Thermometric substances may be solid, liquid or gas, and must be highly sensitive toward temperature change.

Calibration: While calibrating a thermometer, at first two standard degree of hotness (fixed points) are noted and then units are provided by dividing the interval between fixed points into different number of equal parts. The numbers of equal parts are different in different temperature scales.

Sensitiveness

As far as possible thermometer should be more sensitive so that;

1. It can read the temperature of body immediately.
2. It does not exchange large amount of heat with the body whose temperature is to be measured. It gives almost accurate reading.
3. It can read even a very small change in temperature.

Upper fixed point (UFP): It is the temperature at which pure water boils under standard atmospheric pressure.

Lower fixed point (LFP): It is the temperature at which pure ice melts under standard atmospheric pressure.

Activity:1

Take different laboratory thermometers calibrated in Celsius scale, Fahrenheit scale and Kelvin scale. Note temperature readings of boiling water and melting ice by different thermometers simultaneously. Calculate difference between temperature of boiling water and that of melting ice. Compare the difference of temperature expressed in different scales. Establish a relation

for difference of temperature expressed in different scales. Discuss about the result among your class friends and interact with your teacher about the conclusion.

SN	Temperature scales	Temperature of boiling water	Temperature of melting ice	Difference of temperature
1.	Celsius			
2.	Fahrenheit			
3.	Kelvin			

The values of LFP and UFP for different scales of temperature

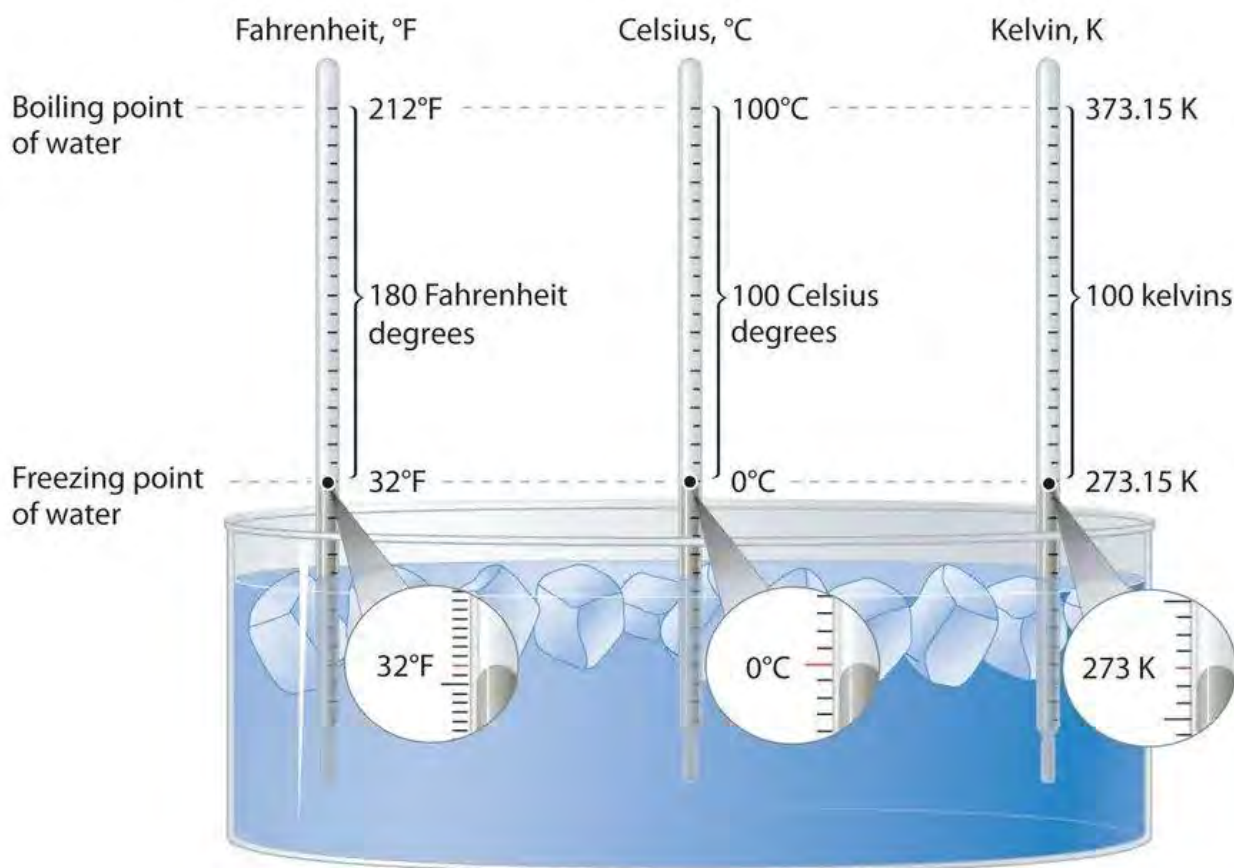
Scales	°C	°F	K
Fixed point			
LFP	0	32	273
UFP	100	212	373
UFP-LFP	100	180	100

9.10 Absolute Scale and Absolute Zero Temperature:

Temperatures expressed in Kelvin scale are absolute temperatures. The zero value of temperature in this scale is called absolute zero temperature. Absolute zero temperature is the lowest possible temperature which can exist in the universe. Temperature which is 273 divisions below the lower fixed point of Kelvin scale, and has value of -273.15°C is absolute zero temperature. At this temperature molecular motion stops and total kinetic energy of molecules becomes zero. It is highly condensed state of matter.

Importance of Absolute Zero Temperature: Temperature of a body is directly proportional to the kinetic energy of its molecules. Absolute temperature becomes zero only for the zero kinetic energy of molecules. For zero kinetic energy, velocity of all molecules must be zero. This means at absolute zero temperature molecules of a body are completely at rest.

9.11 Relationship between Different Temperature Scales



[Fig. 9.7; Relationship between different temperature scales]

Fractional change in temperature reading of a substance taken simultaneously by different thermometers, constructed identically but calibrated in different scales, are equal.

$$\frac{\text{Temperature reading} - \text{L.F.P.}}{\text{U.F.P.} - \text{L.F.P.}} = \frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{K - 273}{373 - 273}$$

$$\text{or, } \frac{C}{100} = \frac{F - 32}{180} = \frac{K - 273}{100}$$

$$\text{or, } \frac{C}{5} = \frac{F - 32}{9} = \frac{K - 273}{5}$$

Workout Examples:

Q. A faulty thermometer has its fixed points marked as 5° and 95°. What is the correct temperature in Celsius when this thermometer reads 92°?

Solution: Here,

Given, Lower fixed point (L) = 5°

Upper fixed point (U) = 95°

Temperature value (θ) = 92°

We can relate this faulty scale with Celsius scale;

$$\frac{\theta - L}{U - L} = \frac{C - 0}{100 - 0}$$

$$\text{or, } \frac{92 - 5}{95 - 5} = \frac{C}{100}$$

$$\text{or, } \frac{87}{90} = \frac{C}{100}$$

$$\text{or, } 870 = 9C$$

$$\text{or, } C = \frac{870}{9} = 96.67^{\circ}\text{C} \approx 97^{\circ}\text{C}$$

Hence, the correct temperature in Celsius scale is 97°C .

Q. Fahrenheit thermometer reads 99.0° when a standard centigrade thermometer reads 37.0° . Find error in the Fahrenheit scale.

Solution: Here,

Fahrenheit thermometer reading = 99.0°F

Standard centigrade thermometer reading = 37.0°C

Temperature equivalent to 37°C when measured in Fahrenheit scale is given by

$$\frac{C - 0}{100} = \frac{F - 32.0}{180}$$

$$\text{or, } \frac{37.0}{100} = \frac{F - 32.0}{180}$$

$$\text{or, } F = \frac{9}{5}(37.0) + 32.0 = 98.6^{\circ}\text{F}$$

Therefore,

$$\text{Error in Fahrenheit scale} = (99.0 - 98.6)^{\circ}\text{F} = 0.4^{\circ}\text{F}$$

Q. You work in a materials testing lab of Mahavir Pun's innovative centre and your boss tells you to increase the temperature of a sample by 40°C . The only thermometer you can find at your work bench reads $^{\circ}\text{F}$. If the initial temperature of the sample is 68.2°F , what is its temperature in $^{\circ}\text{F}$ when the desired temperature increase has been achieved?

Solution: Here,

The required change in temperature = 40.0°C

The change in temperature in Fahrenheit = ?

100 divisions in Celsius scale = 180 divisions in Fahrenheit scale

$$1 \text{ divisions in Celsius scale} = \frac{180}{100} \text{ divisions in Fahrenheit scale}$$

$$40 \text{ divisions in Celsius scale} = \frac{180}{100} \times 40 \text{ divisions in Fahrenheit scale}$$

$$= 72 \text{ divisions in Fahrenheit scale}$$

Since initial reading in Fahrenheit scale is 68.2°F ,

Final reading on the scale = Initial reading + increase in temperature

$$= 68.2^{\circ}\text{F} + 72.0^{\circ}\text{F} = 140.2^{\circ}\text{F}$$

Q. The distance between the upper and lower fixed points of a thermometer is 0.800 m. Find the temperature on the Celsius scale if the mercury level rises to a height of 0.104 m above the lower fixed point.

Solution:

Here,

Distance between UFP and LFP = 0.800 m

Total divisions between UFP and LFP = 100

Distance between given temperature and lower fixed point = 0.104 m

We know, $0.800 \text{ m} = 100^{\circ}\text{C}$

$$\text{or, } 0.001 = \frac{100}{0.800}^{\circ}\text{C}$$

$$\text{or, } 0.104\text{m} = \frac{100}{0.800} \times 0.104^{\circ}\text{C} = 13.0^{\circ}\text{C}$$

So the temperature is $(13.0 + 0)^{\circ}\text{C} = 13.0^{\circ}\text{C}$

Exercise

- Which of the following must have the same value for two bodies to be in thermal equilibrium?
 - specific heat capacity
 - heat energy
 - temperature
 - thermal conductivity
- Which of the following pairs of temperature scales can never give the equal numerical value of the temperature of a body?
 - Kelvin and Fahrenheit
 - Fahrenheit and Celsius
 - Kelvin and Celsius
 - Reaumur and Rankine
- One thermometer has a spherical bulb and another has a cylindrical bulb. Which thermometer responds more quickly to the temperature change?
 - having cylindrical bulb
 - having spherical bulb
 - Both
 - depends on temperature of the body
- A graph showing the temperature in $^{\circ}\text{C}$ along the X-axis and temperature in $^{\circ}\text{F}$ along the Y-axis is a straight line. Which of the following statements is correct?
 - Line intercepts the positive X-axis.
 - Line intercepts the positive Y-axis.
 - Line passes through origin.
 - Line intercepts on the negative of both axes.
- What is the temperature at which the reading of the Fahrenheit scale becomes three times the reading on the Celsius scale?
 - 60°C
 - 80°C
 - 80°F
 - $\frac{80}{3}^{\circ}\text{F}$
- Upper and lower fixed points on a thermometer are separated by 0.160m. The length of mercury thread above the lower fixed point is 0.040m. What is the value of temperature reading?
 - 25°C
 - 32°C
 - 40°C
 - 120°C
- If the temperature of a body measured by the Celsius scale is 15°C , then what is the temperature value of the same body measured by the Fahrenheit scale?
 - 15°F
 - 27°F
 - 32°F
 - 59°F

8. A faulty thermometer has its upper fixed point marked at 60°C . This thermometer reads the temperature of a body as 40°C . If the correct temperature on the Celsius scale is 50°C , then what is the ice point of the faulty thermometer?
- a. 15°C b. 20°C c. 25°C d. 30°C
9. The lower fixed points of two scales X and Y are marked at 14°C and 25°C , and upper fixed points are marked at 74°C and 120°C respectively. If the reading of body on the X- scale is 50°C then what is the reading of the same body on the Y-scale?
- a. 75°C b. 85°C c. 95°C d. 125°C
10. The temperature of a substance increases by 27°C on the Celsius scale. What is the value of this increment on the Kelvin scale?
- a. 0 K b. 7 K c. 27 K d. 300K

Question	1	2	3	4	5	6	7	8	9	10
Answer	c	c	a	b	c	a	d	b	b	c

11. (a) The so-called ‘zeroth law of thermodynamics’ states that if the temperature of body A is equal to the temperature of body B, and the temperature of body B is the same as body C, then the temperature of body C equals the temperature of body A. Explain, in terms of energy flow, why the concept of temperature would be meaningless if this law is not obeyed.
- (b) What is meant by the absolute zero of temperature?
- (c) The temperature of a water bath increases from 30.0°C to 50.0°C . Determine the temperature change of the water bath in the Kelvin scale.
12. (a) What is thermal equilibrium?
- (b) Prove the relation $C + 40 = \frac{5}{9}(F + 40)$ where the letters have their usual meanings.
- (c) A thermometer has its lower and upper fixed points marked as 10°C and 80°C . When it reads 40°C , what is the corresponding temperature on the centigrade scale? [43°C]
13. (a) How can you convert a temperature from one scale to another?
- (b) Two beakers of water, A and B, initially are at the same temperature. The temperature of the water in beaker A is increased by 10°F and the temperature of water in beaker B is increased by 10 K. After this temperature change, which beaker of water has the higher temperature? Explain.

14. A centigrade and a Fahrenheit thermometer are placed in hot water. The water is then cooled. What fall of temperature will the Fahrenheit thermometer register, when the centigrade thermometer records the fall in temperature of 45°C . [81°F]
15. A faulty thermometer has its fixed points marked at 2 and 98. What is the correct temperature on the Celsius scale when the thermometer reads 20° ? [22°C]
16. Find the value of absolute zero on Celsius and Fahrenheit scales. [-273°C , -495.97°F]
17. You feel sick and are told that you have a temperature of 40°C . What is your temperature in $^{\circ}\text{F}$? [104°F]
18. You put a bottle of drinking water in a refrigerator and leave it until its temperature has dropped by 5K. What is its temperature change in (a) $^{\circ}\text{F}$ (b) $^{\circ}\text{C}$? [(a) 9°F (b) 5°C]

Chapter 10

Thermal Expansion

William Thomson, also known as Lord Kelvin, was one of the most eminent scientists (a British physicist and mathematician) of the nineteenth century, and is best known today for inventing the international system of absolute temperature that bears his name. Thomson held a position as Professor of Natural Philosophy at the University of Glasgow.

Kelvin's main achievements include the establishment of an absolute heat scale, publishing some studies on the systems of measurement units and patenting measuring devices such as the galvanometer, helping to perfect transmissions through submarine cables. He led to the establishment of absolute zero. Thomson also became the first scientist to serve in the House of Lords.



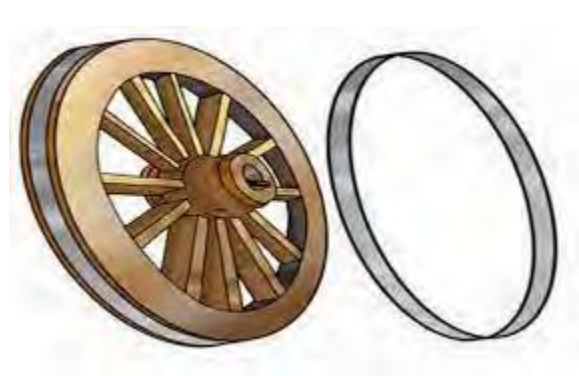
Lord Kelvin (1824-1907)

Do you know?

1. A space is left between the girders for supporting long suspension bridges.
2. When iron rim is to be put on wooden wheel, the rim is kept slightly smaller in diameter than that of the wooden wheel.



[Fig.10.1 Girders of suspension bridge]



[Fig. 20.2 Iron rim on wooden wheel]

10.1 Introduction

Thermal expansion is a fundamental concept in physics, which describes the change in size of a substance under the application of heat. When the temperature of a material increases under the application of heat, the atoms or molecules within the material begin to vibrate more, causing the average distance between them to increase. This results in the expansion of the material's length, area and volume.

Thermal expansion is a phenomenon that affects a wide range of materials from solids and liquids to gases. The degree of thermal expansion varies depending on the material's

composition, molecular structure, and the strength of intermolecular forces. The water exhibits the unique behavior that it contracts as the temperature increases from 0°C to 4°C before expanding upon further heating.

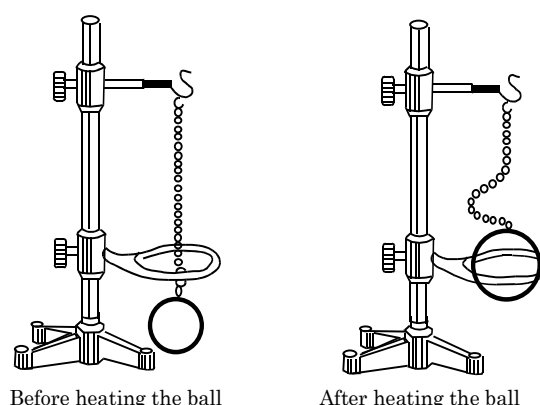
The knowledge of thermal expansion is very crucial in the fields of engineering, architecture material science, etc. It plays a significant role in the designing and construction of structures, the development of precision instruments, and the study of thermal behavior of the materials. This chapter will explore the principles of thermal expansion, different types of thermal expansion, and their applications in various contexts.

Demonstration1:

Let's do an activity in class with the help of Gravesend's ring and an iron ball.

1. Take an iron ball and Gravesend's ring.
2. Try to pass the iron ball through the ring as shown in the first figure.
3. Provide heat to the iron ball for a certain time.
4. Again, try to pass the iron ball through Gravesend's ring as in the second figure.

Result1: The iron ball easily passes through the Gravesend's ring.



[Fig:10.3. Gravesend's ball and ring

demonstration]

Explanation; The diameter of the ball is smaller than that of the ring, and it passes easily through the ring.

Result2: The iron ball could not pass through the Gravesend's ring.

Explanation; diameter (size) of ball has been increased due to **application of heat** but size of hole in the ring is same. Hence the ball could not pass through it.

Demonstration2:

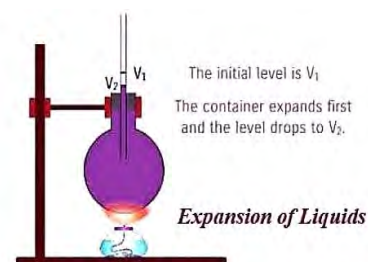
Let's do an activity with the help of a round bottom flask, a clamp with a stand, a capillary tube.

1. Fill a round bottom flask with colorful liquid; insert a glass capillary tube with the support of a cork as shown in the figure.
2. Provide heat to the flask with the support of a heat source (Bunsen's burner).
3. Observe the level of liquid in the capillary tube.

Results: Firstly, the liquid level falls slightly and then only it rises and finally goes above the initial level.

Explanation: The initial fall in the liquid level is due to an increase in the size of the flask caused **by the application of heat**.

Rise in the liquid level is due to an increase in the size of the liquid



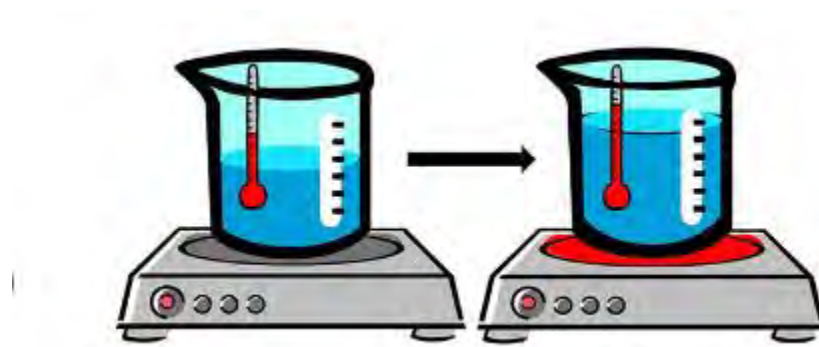
[Fig:10.4 Thermal expansion of liquid]

caused **by the application of heat**. Final level of liquid is above the initial level because **the change in the size of the liquid is greater than that of the glass flask**. The above observed changes in the size of objects are termed as thermal expansion.

10.2 Thermal expansion

In the first demonstration, the size of the ball has been changed due to the application of heat. Also, in the second demonstration, the size of the vessel as well as that of the liquid has been changed due to the application of heat. These two demonstrations prove that when heat is given to a body, not only its temperature rises but its size also increases.

When heat is applied to a body, the kinetic energy of its molecules increases. An increase in kinetic energy causes an increase in the amplitude of molecular vibration as well as an increase in molecular velocity. That is molecules move far apart giving greater intermolecular gap. As a result, the size of the body increases and it occupies more space than before. This ***increase in the size of a body due to the application of heat is called thermal expansion.***



Test yourself:

1. The bulging out of the wall of buildings can be reduced by using hot iron rods over bulged portion, how?
2. Telephone and electric wires are always kept sagging between two poles, why?
3. Why does a solid expand on heating?

10.3 Linear Expansion

Under the application of heat, the length of a linear body, like a metallic wire or a rod, increases. This phenomenon is referred to as the linear expansion of the body. Thus, *the change in linear dimension when subjected to heat is called the linear expansion of the material*. Linear expansion can be observed by measuring the change in diameter of a circular plate or the change in length of a rod or wire, etc.

Suppose a metallic rod of length l_1 at temperature θ_1 is heated up to the temperature of θ_2 so that its length becomes l_2 due to thermal expansion as shown in figure 10.5.

It is observed that change in length ($l_2 - l_1$) is;

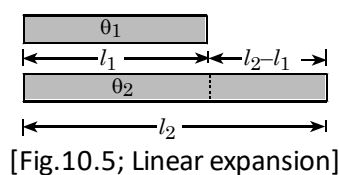
- i. directly proportional to the original length of the rod. i.e. $(l_2 - l_1) \propto l_1$... (1)
- ii. directly proportional to the difference in temperature i.e. $(l_2 - l_1) \propto (\theta_2 - \theta_1)$... (2)

Combining equations (1) and (2) we get,

$$(l_2 - l_1) \propto l_1 (\theta_2 - \theta_1)$$

$$\text{or, } (l_2 - l_1) = \alpha l_1 (\theta_2 - \theta_1)$$

$$\text{or, } \alpha = \frac{l_2 - l_1}{l_1 (\theta_2 - \theta_1)} \quad \dots (3)$$



[Fig.10.5; Linear expansion]

$$\text{or, } \alpha = \frac{\text{Change in length}}{\text{original length} \times \text{change in temperature}}$$

Where, α is the proportionality constant, and it is known as the linear expansivity of the rod.

If $l_1 = 1$ unit of length and $(\theta_2 - \theta_1) = 1$ unit of temperature then, from equation (3), $\alpha = l_2 - l_1$.

Therefore, the coefficient of linear expansion or linear expansivity (α) of a solid is defined as the change in its length per unit original length per unit change in temperature. The SI unit of α is K^{-1} . The value of α depends on the nature of the material of a body, but it does not depend on the initial length and geometrical shapes of cross-section of the rod.

From equation (3),

$$l_2 - l_1 = \alpha l_1 (\theta_2 - \theta_1)$$

$$\text{or, } l_2 = l_1 + l_1 \alpha (\theta_2 - \theta_1)$$

$$\text{or, } l_2 = l_1 [1 + \alpha (\theta_2 - \theta_1)]$$

$$\text{or, } l_2 = l_1 (1 + \alpha \Delta\theta) \quad \dots (4) \quad [\Delta\theta = \theta_2 - \theta_1]$$

This is an expression for the expanded length of the body after heating.

The coefficient of linear expansions or linear expansivities (α) of some common substances are given below in the table.

SN	Substances	α ($^{\circ}\text{C}^{-1}$ or K^{-1})
1	Invar (nickel – iron alloy)	0.05×10^{-5}
2	Quartz	0.09×10^{-5}
3	Glass	$(0.3 - 0.9) \times 10^{-5}$
4	Steel	1.1×10^{-5}
5	Iron	1.2×10^{-5}
6	Gold	1.4×10^{-5}
7	Copper	1.7×10^{-5}
8	Brass	1.9×10^{-5}
9	Silver	1.9×10^{-5}
10	Aluminum	2.6×10^{-5}

Application of Thermal Expansion in Pendulum Clock

Pendulum clocks are made of metal and its period is given by

$$t = \sqrt{\frac{l}{g}}$$

Where g is the acceleration due to gravity and l is the length of the pendulum which is temperature dependent.

When temperature of surroundings changes, the length of pendulum changes (increases or decreases) accordingly. The increase in temperature causes an increase in the period and a decrease in temperature causes a decrease in the period of the pendulum clock.

The loss or gain in time due to change in temperature of the pendulum clock is given by;

$$\text{Loss or gain in time} = \left(\frac{1}{2} \alpha \Delta \theta \right) \times \text{given duration of time}$$

When the temperature of the pendulum clock increases, there will be an increase in the period and hence it loses the time. Similarly, when there is a decrease in temperature, there will be a decrease in period and hence it will gain time.

Workout examples:

Q. Mugling Bridge on Prithivi highway has a single span of 125 m in length. Calculate the change in length of the steel deck of the span when the temperature increases from 10°C to 33°C. [α for steel = $1.2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$]

Solution: Here,

Initial length of steel span (l_1) = 125 m

Change in temperature ($\Delta\theta$) = $33 - 10 = 23^\circ\text{C} = 23 \text{ K}$

Linear expansivity for steel (α) = $1.2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$

Change in length (Δl) = ?

$\therefore$ Change in length (Δl) = $l_1 \alpha \Delta \theta = 125 \times 1.2 \times 10^{-5} \times 23 = 0.0345 \text{ m} = 0.034 \text{ m}$

Hence, the change in length of the steel deck of the span of Mugling Bridge is 0.034m.

Q. The length of an iron rod is measured by a brass scale. When both of them are at 10°C, the measured length is 50.0 cm. What is the length of the rod at 40°C when measured by the brass scale at 40°C? (α for brass = $24 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$, α for iron = $16 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$)

Solution: Here,

Initial temperature (θ_1) = 10°C

Final temperature (θ_2) = 40°C

Linear expansivity of brass (α_b) = $24 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

Linear expansivity of iron (α_I) = $16 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

Length of brass scale at 10°C (l_{10}^b) = Length of iron rod at 10°C (l_{10}^I) = 50.0 cm

Length of iron rod at 40°C (l_{40}^I) = ?

Actual length of iron rod at 40°C ,

$$l_{40}^I = l_{10}^I [1 + \alpha_I (\theta_2 - \theta_1)] = 50 [1 + 16 \times 10^{-6} (40 - 10)] = 50.024 \text{ cm}$$

Length of 1 cm division of the brass scale at 40°C

$$= 1[1 + 24 \times 10^{-6}(40 - 10)] = 1.00072 \text{ cm}$$

$$\begin{aligned} \text{Therefore, the length of the rod as measured by the brass scale at } 40^\circ\text{C} &= \frac{50.024}{1.00072} \\ &= 49.988 \text{ cm} = 49.99 \text{ cm} \end{aligned}$$

Hence, the length of the rod at 40°C is 49.99 cm.

Alternative Method

$$\text{Expansion in brass scale} = l \alpha_b \Delta\theta$$

$$\text{Expansion in iron rod} = l \alpha_s \Delta\theta$$

$$\begin{aligned} \text{The difference in expansion between brass scale and iron rod} &= l(\alpha_b - \alpha_s)\Delta\theta \\ &= 50.0(24 \times 10^{-6} - 16 \times 10^{-6})30 \\ &= 0.012 \text{ cm} \end{aligned}$$

Since the expansion of the scale is more than that of the rod, the scale will read less reading by 0.012 cm. Therefore, the observed length of the rod will be $(50.0 - 0.012) = 49.988 \text{ cm} = 49.99 \text{ cm}$.

Q. A second's pendulum made of brass keeps the correct time at 10°C . How many seconds will it lose or gain per day when the temperature of its surroundings rises to 35°C ? ($\alpha_b = 2 \times 10^{-5} \text{ K}^{-1}$)

Solution: Here,

$$\text{Initial temperature } (T_1) = 10^\circ\text{C}$$

$$\text{Final temperature } (T_2) = 35^\circ\text{C}$$

$$\text{Linear expansivity of brass } (\alpha_b) = 2 \times 10^{-5} \text{ K}^{-1}$$

Loss or gain of time per day = ?

$$\begin{aligned} \text{Loss in time} &= \frac{1}{2} \alpha_b \Delta T \times \text{duration of time} \\ &= \frac{1}{2} \alpha_b (35 - 10) \times 1 \text{ day} \\ &= 0.5 \times 2 \times 10^{-5} \times 25 \times 86400 \\ &= 21.6 = 22 \text{ s} \end{aligned}$$

Hence loss in time per day is 22 s

Sample Questions:

1. How is the coefficient of linear expansion of a substance per degree Celsius compared with its coefficient of linear expansion per degree Fahrenheit?
2. Generally wooden scales are better than metal scales for the measurement of length, why?
3. Does the coefficient of linear expansion depend on length? Explain.
4. Why are telephone wires taut in winter and sag in summer?

5. It is possible to seal a piece of platinum wire into a glass tube, but not a piece of brass or iron, why?
6. Why are small gaps left between successive rails in railway tracks?

10.4 Superficial Expansion

Under the application of heat, the area of a metallic plate increases. This phenomenon of expansion in area is referred to as the superficial expansion of the body. *Thus, the change in the area of a body when subjected to heat is called the superficial expansion of the material.* Superficial expansion can be observed by measuring the change in the area of a rectangular plate or the change in the area of a circular disc.

Suppose a metallic sheet of area A_1 at temperature θ_1 is heated to temperature θ_2 so that its area becomes A_2 due to thermal expansion as shown in figure 10.6. It is observed that the change in area ($A_2 - A_1$) is;

- i. directly proportional to the original area A_1 i.e. $(A_2 - A_1) \propto A_1 \dots (1)$
- ii. directly proportional to the change in temperature i.e. $(A_2 - A_1) \propto (\theta_2 - \theta_1) \dots (2)$

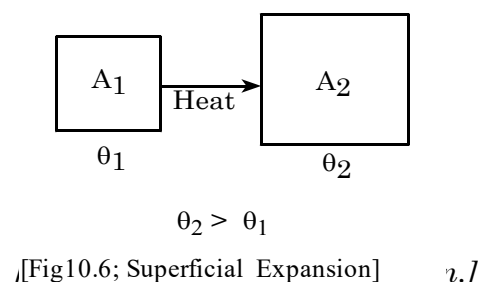
Combining both equations (1) and (2), we get;

$$(A_2 - A_1) \propto A_1 (\theta_2 - \theta_1)$$

$$\text{or, } (A_2 - A_1) = \beta A_1 (\theta_2 - \theta_1)$$

$$\text{or, } \beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)} \dots (3)$$

$$\text{or, } \beta = \frac{\text{change in area}}{\text{original area} \times \text{change in temperature}}$$



Where, β is the coefficient of superficial expansion or superficial expansivity of the given metallic sheet.

If $A_1=1$ unit of area and $(\theta_2 - \theta_1) = 1$ unit of temperature then from equation (3), $\beta = A_2 - A_1$.

Therefore, the **superficial expansivity β of a solid body is defined as the change in its area per unit original area per unit change in temperature.** Its SI unit is K^{-1} .

The value of β depends on the nature of the material of the body, but it does not depend on the original area of the body and its geometrical shape.

From equation (3), we get

$$A_2 - A_1 = \beta A_1 (\theta_2 - \theta_1)$$

$$\text{or, } A_2 = A_1 + \beta A_1 (\theta_2 - \theta_1)$$

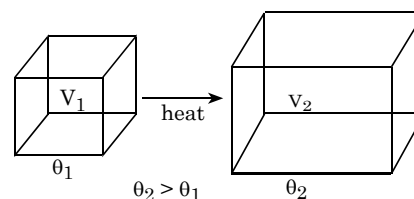
$$\text{or, } A_2 = A_1 [1 + \beta (\theta_2 - \theta_1)]$$

$$\text{or, } A_2 = A_1 (1 + \beta \Delta\theta) \dots (4)$$

This is an expression for the expanded area of the body on heating.

10.5 Cubical Expansion

Under the application of heat, the volume of a solid body increases. This phenomenon of expansion in volume is referred to as the cubical expansion of the body. *Thus, the change in volume of a body when subjected to heat is called the cubical expansion of the material.* Cubical expansion can be observed by measuring the change in the volume of a solid sphere or the change in the volume of a cuboid.



[Fig10.7; Cubical expansion

2.]

Suppose a metallic cube of volume V_1 at temperature θ_1 is heated up to temperature θ_2 so that volume increases to V_2 due to thermal expansion as shown in figure 10.7. It is observed that the change in volume ($V_2 - V_1$) of the cube is;

i. directly proportional to the original volume, i.e. $(V_2 - V_1) \propto V_1$. . . (1)

ii. directly proportional to the change in temperature, i.e. $(V_2 - V_1) \propto (\theta_2 - \theta_1)$. . . (2)

Combining equations (1) and (2), we get,

$$(V_2 - V_1) \propto (\theta_2 - \theta_1)$$

$$\text{or, } (V_2 - V_1) = \gamma V_1 (\theta_2 - \theta_1)$$

$$\text{or, } \gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} \quad \dots (3)$$

$$\text{or, } \gamma = \frac{\text{change in volume}}{\text{original volume} \times \text{change in temperature}}$$

Here, γ is the cubical expansivity or coefficient of cubical expansion of given solid.

If $V_1 = 1$ unit of volume and $(\theta_2 - \theta_1) = 1$ unit of temperature then,

$$\text{from equation (3), } \gamma = V_2 - V_1$$

Therefore, **cubical expansivity or the coefficient of cubical expansion γ is defined as the change in volume of a solid per unit initial volume per unit change in temperature.** SI unit of γ is K^{-1} . The value of γ depends on the nature of the solid but does not depend on its initial volume and the geometrical shape of the body.

From equation (3), we get;

$$\text{or, } V_2 - V_1 = \gamma V_1 (\theta_2 - \theta_1)$$

$$\text{or, } V_2 = V_1 + \gamma V_1 (\theta_2 - \theta_1)$$

$$\text{or, } V_2 = V_1 [1 + \gamma (\theta_2 - \theta_1)]$$

$$\text{or, } V_2 = V_1 (1 + \gamma \Delta\theta) \dots (4)$$

This is an expression for the expanded volume of a body on heating.

Sample Questions:

1. A cylindrical vessel of uniform internal diameter and linear expansivity α is filled with a liquid. On heating, liquid does not overflow. Estimate the cubical expansivity of liquid.

- Two bodies made of the same material have the same external dimensions and appearance, but one is solid and the other is hollow. When they are heated, is the overall volume expansion the same or different?
- Does the cubical expansivity of a body depend on its original volume? Explain.
- Cubical expansivity of mercury is $18 \times 10^{-5} \text{ K}^{-1}$. What do you understand by this statement?
- Are thermal coefficients of expansion; α , β and γ universal constants?

10.6 Relation among α , β and γ

Do you know?

1. Can the value of β and γ be determined if the value of α is given for solid?

Suppose, a square sheet of a metal of length l_1 and area $A_1 (=l_1^2)$ at temperature θ_1 . Let the sheet be heated to a higher temperature θ_2 . Due to thermal expansion, the length of the sheet becomes l_2 and the area becomes $A_2 (=l_2^2)$ as in figure 10.8(a). From the definition of linear expansivity α and superficial expansivity β , we have,

$$l_2 = l_1 (1 + \alpha \Delta\theta) \quad \dots (1)$$

[Where $\Delta\theta = \theta_2 - \theta_1$]

$$A_2 = A_1 (1 + \beta \Delta\theta) \quad \dots (2)$$

Now, using $A_1 = l_1^2$ and $A_2 = l_2^2$ in equation (2) we get,

$$l_2^2 = l_1^2 (1 + \beta \Delta\theta)$$

Again, using equation (1),

$$[l_1 (1 + \alpha \Delta\theta)]^2 = l_1^2 (1 + \beta \Delta\theta)$$

$$\text{or, } l_1^2 (1 + 2\alpha \Delta\theta + \alpha^2 \Delta\theta^2) = l_1^2 (1 + \beta \Delta\theta)$$

$$\text{or, } 1^2 + 2\alpha \Delta\theta + (\alpha \Delta\theta)^2 = 1 + \beta \Delta\theta$$

$$\text{or, } 2\alpha \Delta\theta + \alpha^2 \Delta\theta^2 = \beta \Delta\theta$$

Since, the value α is very small, the term containing α^2 can be neglected. Then,

$$2\alpha \Delta\theta + 0 = \beta \Delta\theta$$

$$\text{or, } 2\alpha = \beta$$

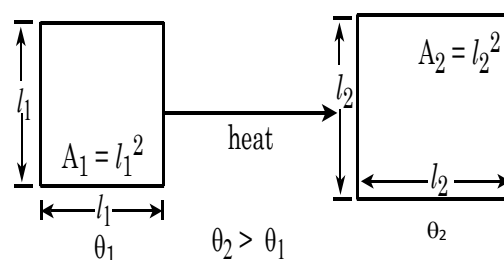
$$\text{or, } \beta = 2\alpha \quad \dots (3)$$

Again suppose, a metallic cube of length l_1 and volume $V_1 (=l_1^3)$ at temperature θ_1 . Let the cube be heated to a higher temperature θ_2 . Due to thermal expansion, length of cube becomes l_2 and the volume becomes $V_2 (=l_2^3)$ as shown in figure 10.8 (b). From the definition of cubical expansivity γ .

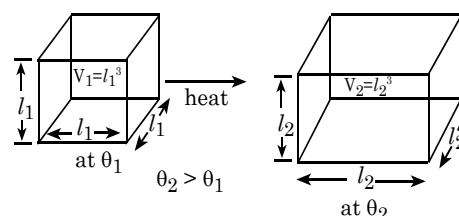
$$V_2 = V_1 (1 + \gamma \Delta\theta) \quad \dots (4)$$

Using relation $V_2 = l_2^3$ and $V_1 = l_1^3$ for the cube in equation (4) we get,

$$l_2^3 = l_1^3 (1 + \gamma \Delta\theta)$$



[Fig10.8(a): Superficial expansion]
[Fig.10.5 Superficial expansion]



[Fig10.8(b): Cubical expansion]
[Fig.10.6, Cubical expansion]

From equation (1), $[l_1 (1 + \alpha\Delta\theta)]^3 = l_1^3(1 + \gamma\Delta\theta)$

or, $l_1^3 (1 + \alpha\Delta\theta)^3 = l_1^3(1 + \gamma\Delta\theta)$

or, $l_1^3 [1^3 + 3.1 (\alpha\Delta\theta)^2 + 3.1^2 \alpha\Delta\theta + (\alpha\Delta\theta)^3] = l_1^3 (1 + \gamma\Delta\theta)$

or, $1 + 3\alpha^2\Delta\theta^2 + 3\alpha\Delta\theta + \alpha^3\Delta\theta^3 = 1 + \gamma\Delta\theta$

or, $3\alpha^2\Delta\theta^2 + 3\alpha\Delta\theta + \alpha^3\Delta\theta^3 = \gamma\Delta\theta \quad \dots (5)$

Since the value of α is very small, the terms containing α^2 and α^3 can be neglected.

Therefore, $3\alpha^2\Delta\theta^2 + 3\alpha\Delta\theta + \alpha^3\Delta\theta^3 = \gamma\Delta\theta$

or, $0 + 3\alpha\Delta\theta + 0 = \gamma\Delta\theta$

or, $3\alpha = \gamma$

or, $\gamma = 3\alpha \quad \dots (6)$

Using relations (3) and (7), $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

$\therefore \alpha : \beta : \gamma = 1 : 2 : 3$

Workout Example:

Q. A cylindrical vessel of uniform internal diameter and having linear expansivity α is filled with a liquid. On heating, liquid does not overflow. Estimate the cubical expansivity of liquid.

Since the liquid does not overflow, the volume expansion of the vessel is equal to or more than that of the liquid.

For this, cubical expansivity of liquid $\leq$ cubical expansivity of solid $= 3 \times$ linear expansivity of solid $= 3 \times \alpha = 3\alpha$.

Sample Questions:

1. Are thermal coefficients of expansion; α , β and γ the universal constants?
2. A solid rod and a hollow rod of the same material, and of the same length are heated through the same range of temperature. Which rod expands more?
3. Which one of the following is correct for solids?

a. $\gamma = \frac{3}{2} \alpha$

b. $\gamma = \frac{2}{3} \alpha$

c. $\gamma = \frac{3}{2} \beta$

d. $\gamma = \frac{2}{3} \beta$

4. The relation between the three types of coefficients of thermal expansion of a solid is;

a. $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

b. $\alpha = 2\beta = 3\gamma$

c. $\frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\gamma}$

d. $\frac{1}{\alpha} = 2\beta = \frac{1}{3\gamma}$

5. The volume of a block increases by 0.12% on heating through 20°C then the linear expansivity of metal is;

a. $1 \times 10^{-5} ^\circ\text{C}^{-1}$

b. $1.5 \times 10^{-5} ^\circ\text{C}^{-1}$

c. $2 \times 10^{-5} ^\circ\text{C}^{-1}$

d. $2.5 \times 10^{-5} ^\circ\text{C}^{-1}$

10.7 Determination of Linear Expansivity of a Solid [Pullinger's Apparatus]

Activity: 3

Let's do an activity taking Pullinger's apparatus with a spherometer at the top, and an external electrical circuit with battery and galvanometer connected between the fixed end of the rod and a leg of the spherometer as in figure 10.9.

1. Let's take a metallic rod AB.
2. Measure the length of the rod with the help of a meter scale. Let it be l .
3. Keep the rod inside a hollow metallic cylinder having three openings on its sides as in the figure.
4. Insert a thermometer in the middle opening as in the figure and note the initial temperature of rod θ_1 .
5. Rotate the spherometer such that its central leg touches the free end A of the rod AB and the galvanometer shows the deflection.
6. Take spherometer reading S_1 .
7. Rotate the spherometer back so that the rod AB gets sufficient space to expand.
8. Allow the steam to pass into the cylinder until the steam and the rod attain thermal equilibrium and the thermometer shows steady temperature θ_2 .
9. Again, rotate the spherometer forward such that its central leg again touches the rod and the galvanometer shows the deflection.
10. Take spherometer reading S_2 .
11. Calculate the change in length of rod, $\Delta l = S_2 - S_1$
12. Calculate the change in temperature, $\Delta\theta = \theta_2 - \theta_1$

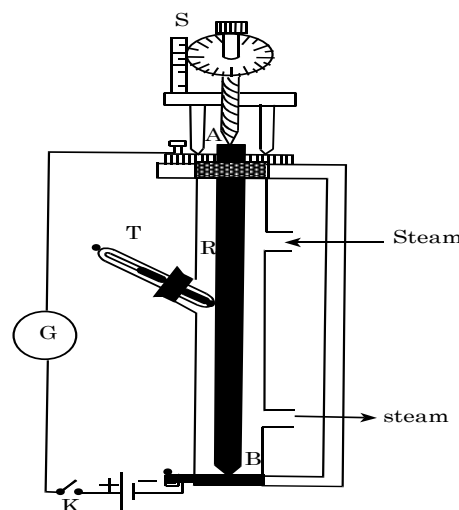
Now linear expansivity of AB is given by

$$\alpha = \frac{\text{Change in length}}{\text{Original length} \times \text{change in temperature}}$$

$$\text{or, } \alpha = \frac{S_2 - S_1}{l(\theta_2 - \theta_1)}$$

Here, all the parameters on right hand side are measured so the value of α can be calculated.

Again, using the approximate relation $\beta = 2\alpha$, $\gamma = 3\alpha$ values of β and γ of solid can also be calculated.

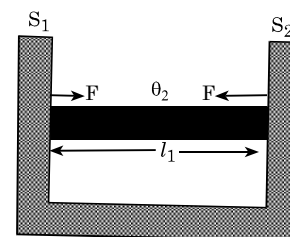


[Fig. 16.7, Pullinger's Apparatus]

Fig; 10.9 Pullinger's Apparatus

10.8 Force Set-Up Due to Expansion or Contraction and Thermal Stress

If a metal bar is heated it will expand, but if we prevent it from expanding then large forces will be set up within the bar. The force will be that which would be needed to compress the bar to its original length from the expanded position. This force is called force set up due to expansion. Similarly, if a metallic wire tied between two rigid poles is cooled, it will try to contract but will not be able to do so because of rigid poles. Instead, a large force is set up to prevent the phenomenon of contraction. This force is called force set up due to contraction.



[Fig;10.10 Force set up due to expansion]

The mechanical force that occurs in an object when it is prevented either from expansion during heating or from contraction during cooling is called force set-up due to expansion or contraction.

Mathematically, it is given by $F = YA \alpha \Delta\theta$; where Y = Young's Modulus of elasticity, A = cross-sectional area, $\alpha\Delta\theta$ = thermal strain.

The stress corresponding to this force is called **Thermal stress**.

Thermal stress $\left(\frac{F}{A}\right) = Y \alpha \Delta\theta$

The same expressions (3) and (4) are also valid for the case of contraction.

Workout Example:

Q. A 4 m long steel wire of with 2mm diameter is fixed to two rigid supports. Calculate the increase in tension when the temperature falls to 10°C . [α for steel = $1.2 \times 10^{-5} \text{K}^{-1}$ and Y for steel = $2 \times 10^{11} \text{Nm}^{-2}$]

Solution: Here,

Change in temperature $\Delta\theta = 10^\circ\text{C}$

Diameter of wire (d) = 2mm = $2 \times 10^{-3} \text{m}$,

Area of cross section (A) = $\frac{\pi d^2}{4} = 3.14 \times (2 \times 10^{-3})^2 / 4 = 3.14 \times 10^{-6} \text{m}^2$

α for steel = $1.2 \times 10^{-5} \text{K}^{-1}$, Y for steel = $2 \times 10^{11} \text{Nm}^{-2}$

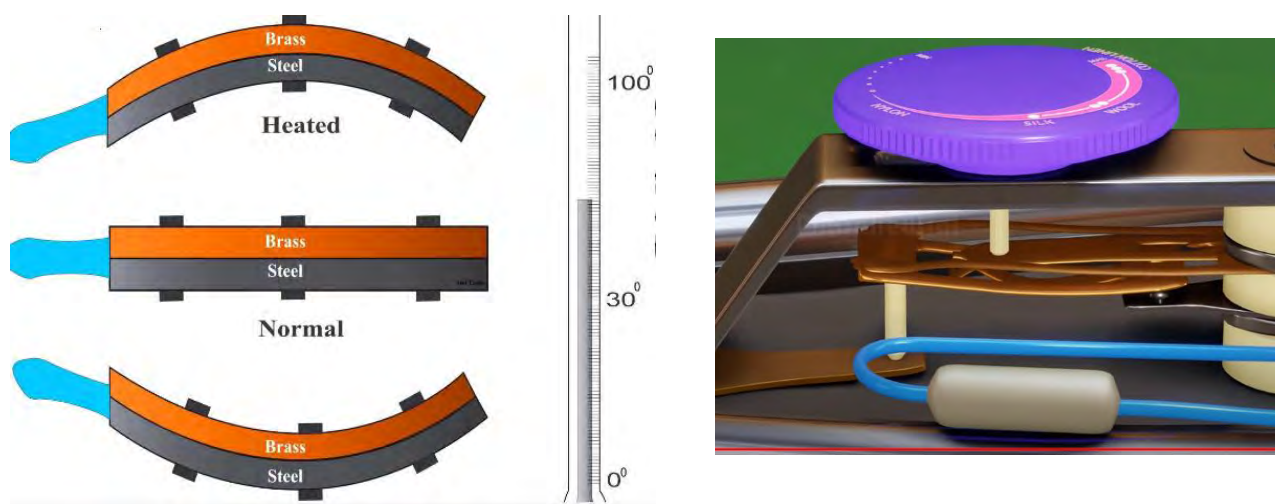
Increase in tension (F) = ?

We know increase in tension (F) = $YA\alpha\Delta\theta = 2 \times 10^{11} \times 3.14 \times 10^{-6} \times 1.2 \times 10^{-5} \times 10 = 75.36 \text{N}$

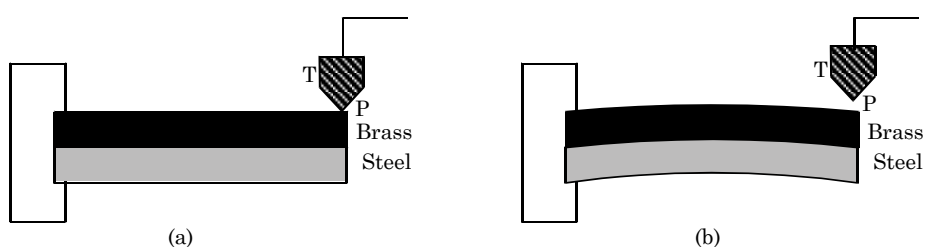
10.9 Differential Expansion

Let us think?

1. Do two bodies made of different material have the same value of coefficient of thermal expansion?
2. Do two bodies having different values of coefficient of thermal expansion exhibit the same thermal expansion?



The bimetallic thermostat is a device made up of two thin metallic strips of different linear expansivities welded together along their lengths. Brass - steel bimetallic strips are the examples of bimetallic thermostat. When brass-steel thermostat is heated, the brass having greater value of linear expansivity expands more than the steel. Since the two strips are welded together, the thermostat bends into an arc with brass in greater curvature. As a result, electric contact breaks as in figure 10.11(b). This special design can be used as safety measure to control the overflow of heat in many electrical appliances, and hence to save from accidents.



[Fig; 10.11 (a) Thermostat at normal temperature] [Fig; 10.11(b) Thermostat at higher temperature]

In the above example, the thermostat bends into an arc with brass in greater curvature due to their difference in thermal expansion. This difference in expansions of two bodies, having different values of coefficient of thermal expansion, when they are heated through the same range of temperature is called **differential expansion**. The principle of differential expansion is used in many appliances like electric irons, rice cookers, electric heating systems, aquarium, etc. as thermo switches for automatic switching of electric supply.

Using the expression for linear expansion we can write,

$$l_2 = l_1 (1 + \alpha \Delta\theta) \quad \dots (1)$$

$$l_2' = l_1' (1 + \alpha' \Delta\theta) \quad \dots (2)$$

[Where, $\Delta\theta = \theta_2 - \theta_1$]

From the figure the difference in length of the rods at temperatures θ_1 and θ_2 respectively can be written as,

$$d_1 (= l_1 - l_1') \text{ and } d_2 (= l_2 - l_2')$$

Now, the difference in expansion of the rods is given by;

$$\text{Differential expansion } (d_2 - d_1) = (l_1\alpha - l_1'\alpha') \Delta\theta$$

For zero differential expansion $d_1 = d_2$, i.e. $d_2 - d_1 = 0$.

In this condition,

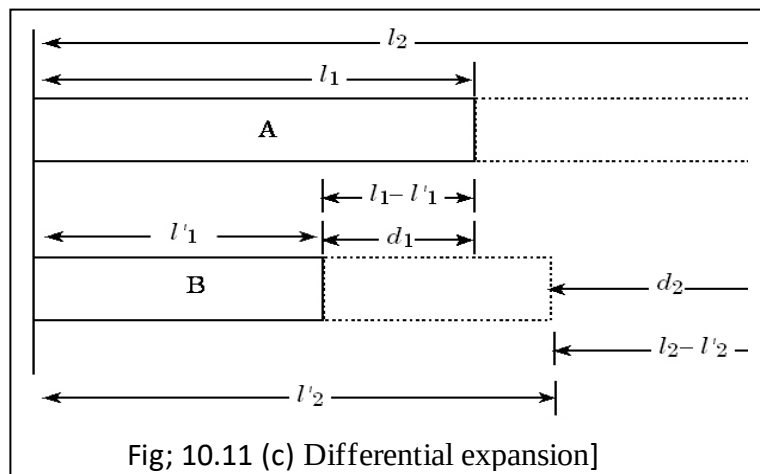
$$\text{or, } (l_1\alpha - l_1'\alpha') = 0$$

[Here, $\Delta\theta \neq 0$]

$$\text{or, } l_1\alpha = l_1'\alpha'$$

$$\text{or, } \frac{l_1}{l_1'} = \frac{\alpha'}{\alpha}$$

This principle of zero differential expansion is used in making **compensated pendulum clocks**.



Workout Example:

Q. An iron rod and a copper rod lie side by side. On heating the difference in lengths of the rods remains the same as before at a value of 10 cm. Calculate the length of the rods at 0°C . (Given $\alpha_{\text{cu}} = 1.7 \times 10^{-5}^\circ\text{C}^{-1}$, $\alpha_{\text{I}} = 1.2 \times 10^{-5}^\circ\text{C}^{-1}$)

✎ Solution: Here,

✎ Linear expansivity of copper (α_{cu}) = $1.7 \times 10^{-5}^\circ\text{C}^{-1}$

Linear expansivity of iron (α_{I}) = $1.2 \times 10^{-5}^\circ\text{C}^{-1}$

Initial difference in length = Final difference in length = 10 cm

Initial length of copper (l_1) = ?

Initial length of iron (l_1') = ?

Let l_1 and l_1' be the initial length of copper and iron rod at 0°C . Then, according to the question,

$$l_1 - l_1' = 10$$

$$\text{or, } l_1 = 10 + l_1' \quad \dots (1)$$

From the condition of zero differential expansion,

$$\frac{l_1}{l_1'} = \frac{\alpha_{\text{I}}}{\alpha_{\text{cu}}} = \frac{1.2 \times 10^{-5}}{1.7 \times 10^{-5}} = \frac{1.2}{1.7} = \frac{12}{17} \quad \dots (2)$$

Solving equations (1) and (2)

$$\frac{12}{17} = \frac{10 + l_1'}{l_1'}$$

$$12 l_1' = 170 + 17 l_1'$$

$$5 l_1' = - 170$$

$$\Rightarrow l_1' = - \frac{170}{5} = - 34 \text{ cm}$$

$$\text{Or, } l_1 = l_1' + 10 = - 34 + 10 = - 24 \text{ cm}$$

Since lengths are not negative, lengths of copper and iron rods at 0°C are 24 cm and 34 cm respectively.

Sample Questions:

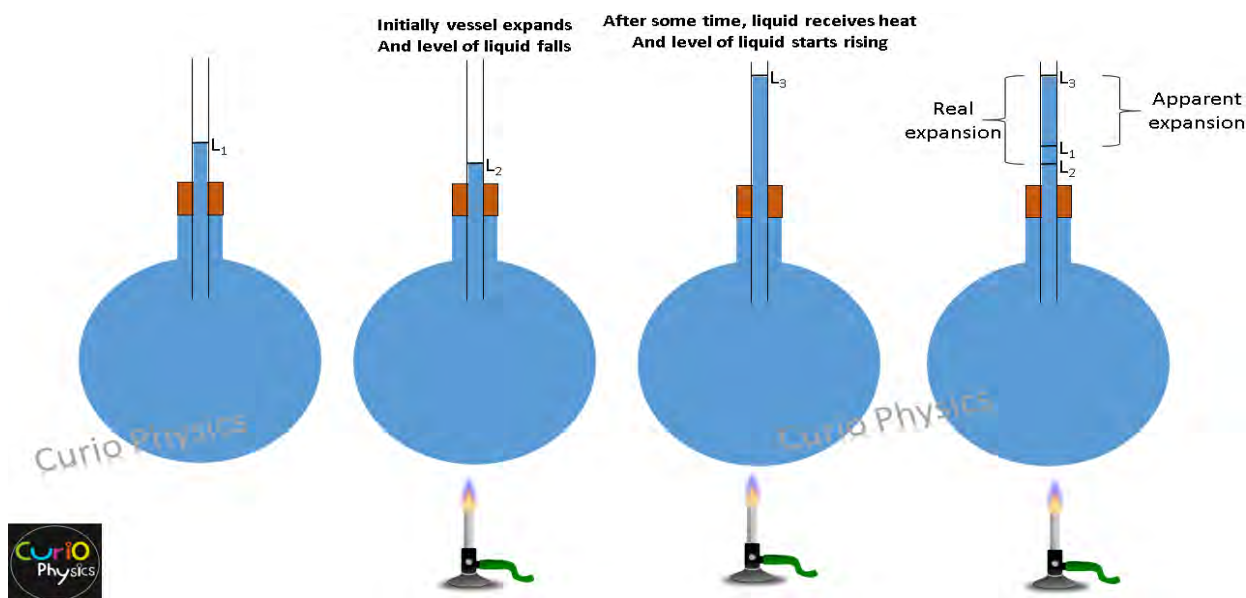
1. How thermal stress is set up?
2. A thick glass tumbler usually cracks when very hot or very cold liquids are poured into it, why?
3. Tight lid of the bottle can be loosened by running hot water over it, how?
4. A student claims that the difference between the lengths of certain brass and steel rods can be made constant at all temperatures. Is it possible?

10.10 Thermal Expansion of Liquid

Do you know?

1. Why does water level fall initially in a vessel when it is heated?
2. Sometimes milk overflows from the vessel during heating, why?

Activity: 4



[Fig; 10.12 Thermal expansion of liquid]

Let's do an activity with the help of a colourful liquid, a round bottom flask, a capillary tube, and Bunsen's burner as in figure 10.12

1. Note the level of liquid as L_1 at temperature θ_1 .
2. Heat the flask with the help of Bunsen's burner and note the lowest level attained by liquid during heating L_2 .
3. Keep on heating continuously till the liquid attains the highest level of L_3 at temperature θ_2 .

- i. Volume of liquid corresponding to level L_1 to L_2 represents the solid vessel's expansion (i.e. $\Delta V_g = L_1 L_2 \times \text{cross-section area}$)
- ii. Volume of liquid corresponding to level L_1 to L_3 represents an apparent expansion of liquid (i.e. $\Delta V_a = L_1 L_3 \times \text{cross-section area}$)
- iii. Volume of liquid corresponding to level L_2 to L_3 represents the real expansion of liquid (i.e. $\Delta V_r = L_2 L_3 \times \text{cross-section area}$)

There are different coefficients of expansion; coefficient of real expansion, coefficient of apparent expansion, and coefficient of volume expansion of glass vessel which are defined as follows.

Coefficient of real expansion of liquid: The real change in volume of liquid per unit original volume per unit rise in temperature is called the coefficient of real expansion of liquid. It is denoted as γ_r and is given by;

$$\gamma_r = \frac{\text{Real change in volume of liquid } (\Delta V_r)}{\text{Original volume of liquid } (V_0) \times \text{Change in temperature } (\Delta \theta)}$$

or,

$$\gamma_r = \frac{\Delta V_r}{V_0 \Delta \theta} \quad \dots (1) \quad [\text{Where, } \Delta \theta = \theta_2 - \theta_1]$$

Coefficient of apparent expansion of liquid: The apparent change in the volume of liquid per unit original volume per unit change in temperature is called the coefficient of apparent expansion of liquid. It is denoted by γ_a and is given by;

$$\gamma_a = \frac{\text{Apparent change in volume of liquid } (\Delta V_a)}{\text{Original volume of liquid } (V_0) \times \text{Change in temperature } (\Delta \theta)}$$

or,

$$\gamma_a = \frac{\Delta V_a}{V_0 \Delta \theta} \quad \dots (2),$$

Coefficient of volume expansion of solid: The change in volume of glass vessel per unit original volume per unit rise in temperature is called the coefficient of volume expansion of glass vessel. It is denoted by γ_g and given by;

$$\gamma_g = \frac{\text{Change in volume of solid vessel } (\Delta V_g)}{\text{Original volume of solid vessel } (V_0) \times \text{Change in temperature } (\Delta \theta)}$$

or, $\gamma_g = \frac{\Delta V_g}{V_0 \Delta \theta} \quad \dots (3)$

Relation between γ_r , γ_a , and, γ_g

From figure 10.9,

$$L_2 L_3 = L_1 L_3 + L_1 L_2$$

or, $(L_2 L_3 = L_1 L_3 + L_1 L_2) \times a$ [Here, a is cross-section area of capillary]

$$\text{or, } L_2 L_3 \times a = L_1 L_3 \times a + L_1 L_2 \times a$$

$$\text{or, } \Delta V_r = \Delta V_a + \Delta V_g \quad \dots (4)$$

$\therefore$ Real expansion of liquid = Apparent expansion of liquid + Volume expansion of vessel

From equations (1), (2) and (3) we get,

$$\Delta V_r = \gamma_r V_0 \Delta \theta, \Delta V_a = \gamma_a V_0 \Delta \theta \text{ and } \Delta V_g = \gamma_g V_0 \Delta \theta$$

Using these values in equation (4) we get,

$$\gamma_r V_0 \Delta \theta = \gamma_a V_0 \Delta \theta + \gamma_g V_0 \Delta \theta$$

or, $\gamma_r = \gamma_a + \gamma_g$

If α_g is the linear expansivity of glass then, $\gamma_r = \gamma_a + 3\alpha_g$

Workout Example:

A glass flask of volume 400 cm³ is just filled with mercury at 0°C. How much mercury overflows when the temperature of the system is raised to 80°C? (The coefficient of cubical expansivity of glass is $2.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ and that of mercury is $18 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.)

✎ Solution: Here,

✎ Initial volume of mercury = Initial volume of flask = (V_0) = 400 cm³

Initial temperature of system (θ_i) = 0°C

Final temperature of system (θ_f) = 80°C

Cubical expansivity of glass (θ_g) = $2.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$,

Cubical expansivity of mercury (θ_{Hg}) = $18 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$

Final volume of mercury = V (say)

Final volume of glass flask = V' (say)

Amount of mercury which overflows (ΔV) = Final volume of Hg (V) – Final volume of flask (V')

$$\begin{aligned} &= V_0 [1 + \gamma_{Hg} (\theta_f - \theta_i)] - V_0 [1 + \gamma_g (\theta_f - \theta_i)] \\ &= 400 [1 + 18 \times 10^{-5} (80 - 0)] - 400 [1 + 2.7 \times 10^{-5} (80 - 0)] \end{aligned}$$

0)]

$$= 4.896 \text{ cm}^3 = 5 \text{ cm}^3$$

Hence, 5 cm³ mercury overflows.

Sample Questions:

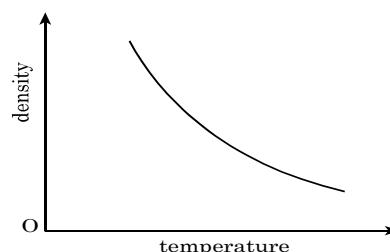
1. Why does a liquid have two coefficients of expansion?
2. The liquid of cubical expansivity γ is heated in a vessel of linear expansivity α so that no liquid overflows. Is this possible?
3. Why is it much more difficult to make a precise determination of the coefficient of expansion of liquid than that of a solid?
4. Explain why a column of mercury in a thermometer first descends slightly and then rises when placed in hot water.

10.11 Variation of Density with Temperature

When a substance is heated, its temperature increases as well as its volume changes. But mass of the substance does not change with temperature. Hence, as the volume of a substance is inversely proportional to its density for constant mass, the density of the substance changes with temperature as shown in figure 10.13.

Mathematically, the variation of density with the change in temperature is given as;

$$\rho_2 = \frac{\rho_1}{(1 + \gamma \Delta \theta)}$$



[Fig.16.13, Variation of density of matter with temperature]
[Fig;10.13 Variation of density of matter with temperature]

Workout Example:

Q. The density of silver at 0°C is 10310 kgm^{-3} and the coefficient of linear expansion is $1.9 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$. Calculate its density at 100°C .

Solution: Here,

Initial temperature (θ_1) = 0°C Final temperature (θ_2) = 100°C

Initial density (ρ_1) = 10310 kgm^{-3} , Coefficient of linear expansion (α) = $1.9 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$

Coefficient of cubical expansion (γ) = $3\alpha = 3 \times 1.9 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1} = 5.7 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$

Final density (ρ_2) = ?

We know the relation,

$$\rho_2 = \frac{\rho_1}{(1 + \gamma \Delta\theta)} = \frac{10310}{[1 + 0.000057 \times (100 - 0)]} = 10251.57\text{ kgm}^{-3} = 10252\text{ kgm}^{-3}$$

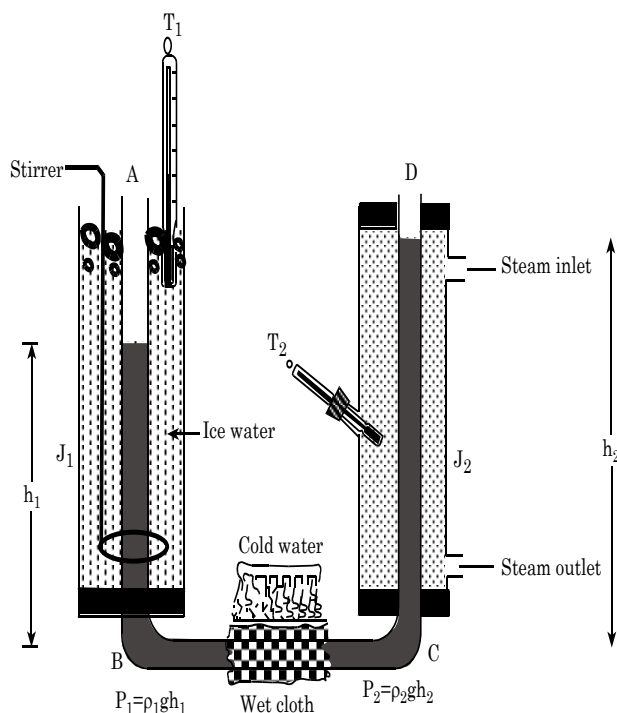
Hence, density of silver at 100°C is 10252 kg m^{-3}

Sample Question:

Determine the temperature at which wood will just sink in benzene if the density of benzene at 0°C and cubical expansivity of benzene are $9 \times 10^2\text{ kgm}^{-3}$ and $1.2 \times 10^{-3} \text{ K}^{-1}$ respectively. The density of wood at 0°C and cubical expansivity of wood are $8.8 \times 10^2\text{ kgm}^{-3}$ and $1.5 \times 10^{-4} \text{ K}^{-1}$ respectively. [295K or 22°C]

10.12 Dulong and Petit's Experiment [Balanced Column Method]

[Measurement of Absolute (Real) Expansivity of Liquid]



[Fig. 16.14. Dulong and Petit's apparatus for absolute expansivity of liquid]

[Fig.10.14 Dulong and Petit's Experiment]

1. An experimental liquid (say mercury) whose real expansivity is to be determined is filled in a U-shaped glass tube ABCD as shown in figure 10.14.
2. The limbs AB and CD are surrounded by two glass jackets J_1 and J_2 respectively.
3. Ice water is circulated through Jacket J_1 to maintain the liquid in the limb AB at a lower temperature.
4. Steam is circulated in jacket J_2 to maintain the temperature of the liquid in limb CD at a higher temperature.
5. Two thermometers T_1 and T_2 are fitted in Jackets J_1 and J_2 respectively.
6. The exchange of heat between two limbs is prevented by covering the portion BC by a wet cloth over which cold water is sprayed continuously.
7. Wait for some time to continue the process so that the liquid in the U - shaped tube attains an equilibrium state at which the height of liquid in limb CD goes above due to expansion, and that in limb A goes down due to contraction.
8. Take the measurement of the height of liquid columns and the temperatures of the liquid in cold arm AB and hot arm CD.

Let h_1 and ρ_1 be the height and density of the liquid in arm AB at temperature θ_1 whereas h_2 and ρ_2 be corresponding values of liquid in arm CD at temperature θ_2 . For the liquid to be in thermal equilibrium, the hydrostatic pressure at B is equal to the hydrostatic pressure at C.

i.e., $P_1 = P_2$

(Atmospheric pressure is the same for both arms and hence it is not considered here.)

or, $\rho_1 g h_1 = \rho_2 g h_2$

$$\text{or, } \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \quad \dots (1) \left[h \propto \frac{1}{\rho} \right]$$

$$\text{We have, } \rho_2 = \frac{\rho_1}{1 + \gamma \Delta \theta} \quad \dots (2)$$

Where $\Delta \theta = \theta_2 - \theta_1$ and γ be the absolute expansivity of liquid.

From equations (1) and (2) we get

$$\frac{h_1}{h_2} = \frac{\rho_1}{(1 + \gamma \Delta \theta) \rho_1}$$

$$\text{or, } h_1 (1 + \gamma \Delta \theta) = h_2$$

$$\text{or, } h_1 + h_1 \gamma \Delta \theta = h_2$$

$$\text{or, } h_1 \gamma \Delta \theta = h_2 - h_1$$

$$\text{or, } \gamma = \frac{h_2 - h_1}{h_1 \Delta \theta}$$

Here all the parameters on right hand side are measured. So, the value of γ can be calculated directly without considering solid vessel expansion.

Workout Example:

A U tube containing a liquid has two limbs maintained at 0°C and 100°C respectively. After the steady state is reached, the heights of the liquid columns are 60 cm and 65 cm. Find the coefficient of real expansion of the liquid.

Solution: Here,

Lower temperature (θ_1) = 0°C

Upper temperature (θ_2) = 100°C

Height of liquid at 0°C (h_1) = 60 cm

Height of liquid at 100°C (h_2) = 65 cm

Coefficient of real expansion of liquid (γ) = ?

We know the relation,

$$\gamma = \frac{h_2 - h_1}{h_1 \Delta\theta} = \frac{65 - 60}{60(100 - 0)} = 8.3 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$$

Hence, the coefficient of real expansivity of liquid is $8.3 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$.

Exercise

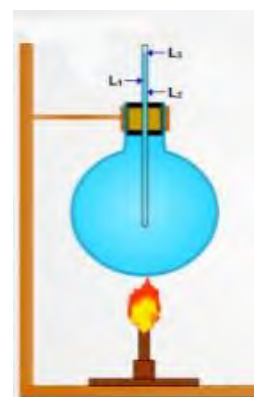
1. What will be the density of liquid with an increase in the temperature?
a. increase b. decrease c. remain the same d. cannot be predicted
2. A metal piece is being weighted in a liquid whose temperature is being raised continuously. What happens to the apparent weight of the metal piece?
a. decreases b. increases c. remains unchanged d. changes erratically
3. In which of the following cases, 5l of benzene weighs more?
a. in winter b. in summer
c. equal in summer and in winter d. cannot be predicted
4. Two rods of different materials have the same expansion over the same rise in temperature. What is the required condition?
a. $l_1\alpha_1 = l_2\alpha_2$ b. $l_1\alpha_2 = l_2\alpha_1$ c. $l_1^2\alpha_1 = l_2^2\alpha_2$ d. $l_2^2\alpha_1 = l_1^2\alpha_2$
5. Two spheres of the same size are made of the same material, but one is solid and the other is hollow. If both spheres are heated through the same range of temperature how do they expand?
a. Both spheres expand equally.
b. Solid sphere expands more.
c. Hollow sphere expands more.
d. Cannot be predicted.

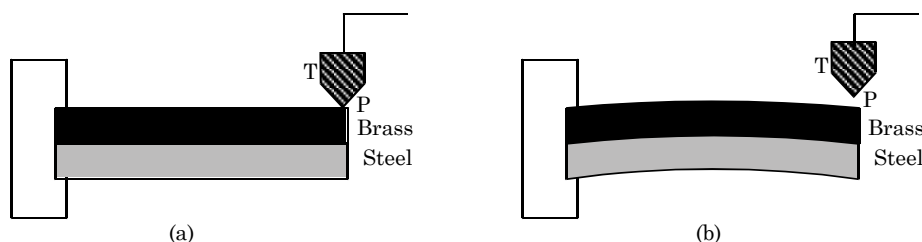
6. A bimetallic thermostat is made of two strips A and B having the coefficients of linear expansion α_A and α_B (α_A is less than α_B). Which of the following may be the result of heating the bimetallic strip?
 - a. It bends with A on the outer side.
 - b. It bends with B on the outer side.
 - c. It does not bend at all.
 - d. It twists itself.
7. A clock keeps the correct time at 20°C . It has a metallic pendulum with the coefficient of linear expansion $2 \times 10^{-5} \text{K}^{-1}$. The temperature falls to 10°C . What is the number of seconds lost or gained by the clock per day?
 - a. Loss of 8.64 sec b. Gain of 8.64 sec c. Loss of 19.28 sec d. Gain of 19.28 sec
8. A thin copper wire of length L increases in length by 1% when heated from temperature T_1 to T_2 . What is the percentage change in area when a thin copper plate of dimension $2L \times L$ is heated from T_1 to T_2 ?
 - a. 1% b. 2% c. 3% d. 4%
9. A disk ($\alpha = 1.9 \times 10^{-5} \text{K}^{-1}$) has a hole of diameter 1.5 cm at 20°C . What of the following effects will be in the diameter of the disk when it is heated to 150°C ?
 - a. Increases by 0.002 cm
 - b. Decreases by 0.002 cm
 - c. Increases by 0.004 cm
 - d. Decreases by 0.004 cm
10. The coefficient of apparent expansion of mercury in a glass vessel is $153 \times 10^{-6} \text{K}^{-1}$ and in a steel vessel is $114 \times 10^{-6} \text{K}^{-1}$. If the coefficient of linear expansion for steel is $12 \times 10^{-6} \text{C}^{-1}$, what is the value of the coefficient of linear expansion of glass?
 - a. $6 \times 10^{-6} \text{C}^{-1}$ b. $9 \times 10^{-6} \text{C}^{-1}$ c. $27 \times 10^{-6} \text{C}^{-1}$ d. $36 \times 10^{-6} \text{C}^{-1}$
11. A glass flask of volume 1 l is fully filled with mercury at 0°C . The flask and mercury are now heated up to 100°C . How much mercury will be spilt out if the coefficient of volume expansion of mercury is $1.82 \times 10^{-4} \text{C}^{-1}$ and linear expansion of glass is $0.1 \times 10^{-4} \text{C}^{-1}$ respectively?
 - a. 1.52 cc b. 2.12 cc c. 15.2 cc d. 21.2 cc
12. What will be the level of the liquid in the container, on heating a liquid with the coefficient of cubical expansion γ in a container having the coefficient of linear expansion $\gamma/3$?
 - a. rise b. fall c. will remain almost stationary d. it is difficult to say
13. The coefficient of linear expansion is $18 \times 10^{-5} \text{C}^{-1}$. What will be the value of the coefficient of linear expansion in Fahrenheit scale?

- a. $32 \times 10^{-5} \text{F}^{-1}$ b. $18 \times 10^{-5} \text{F}^{-1}$ c. $9 \times 10^{-5} \text{F}^{-1}$ d. $1 \times 10^{-4} \text{F}^{-1}$
14. An iron rod ($Y = 2 \times 10^{12} \text{dyne cm}^{-2}$, $\alpha = 1.2 \times 10^{-5} \text{C}^{-1}$) of radius 2.4cm is heated from 0°C to 50°C . What is the value of force which will be exerted if it is prevented from expanding?
- a. $4.34 \times 10^5 \text{N}$ b. $2.17 \times 10^5 \text{N}$ c. $3.12 \times 10^5 \text{N}$ d. $2.17 \times 10^4 \text{N}$
15. The length of a cube increases by 0.50% when heated to change its temperature by 100°C . What will be the increment in its volume?
- a. 0.05% b. 1.0% c. 1.5% d. 2%

Q. N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	b	b	a	a	a	b	b	b	c	b	c	c	d	b	c

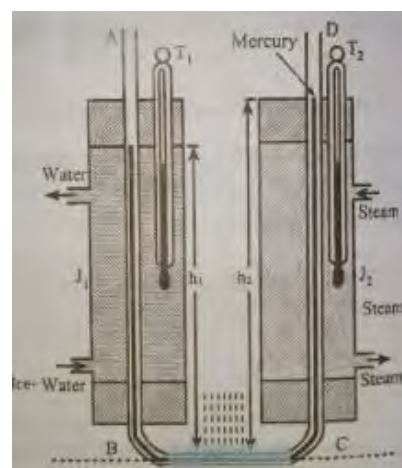
16. (a) A bimetallic strip bends to a side when heated, why?
 (b) A solid sphere is heated from temperature θ_1 to θ_2 . Establish a relationship between the final and the initial diameter of the sphere.
 (c) A hole of 2 cm diameter is drilled through a piece of steel at 30°C . What happens to the diameter of the hole as the steel is heated to 80°C ?
17. (a) Define the coefficient of linear and cubical expansion of a material.
 (b) A solid cube of metal is heated. Derive a relationship between its coefficient of linear expansion and the coefficient of cubical expansion. 2
 (c) Will the coefficient of cubical expansion change if the same solid cube is converted into a hollow cube? 1
18. (a) A liquid is heated in a conical flask as shown in the figure. Identify the real and apparent expansion of the liquid from the figure.
 (b) Establish the relation $\gamma_r = \gamma_a + \gamma_g$, where the symbols carry their usual meaning.
 (c) A glass flask of volume 200 cm^3 is just filled up to 180 cm^3 with mercury at 0°C . Calculate how much space remains above the mercury when the temperature of the system is raised to 60°C ? The coefficient of cubical expansion of glass is $2.7 \times 10^{-5} \text{C}^{-1}$ and that of mercury is $18 \times 10^{-5} \text{C}^{-1}$. [18 cm^3]
- (a) From the study of the given figure, respond to the questions given below.
 (i) Compare which one, brass or steel, expands more for the same rise in temperature.
 (ii) Explain why brass bends towards steel.
 (iii) Suppose you are provided with two rods of length 1m and cross-section 1 m^2 , one is brass and the other is steel. Compare which one needs to raise its temperature by more value for the same value of final length.





(b) A glass tube of length 133 cm and of uniform cross - section is to be filled with mercury so that the volume of tube occupied by mercury remains the same at all temperatures. If cubical expansivity for glass and mercury are respectively 0.000026°C and $0.000182^{\circ}\text{C}^{-1}$, calculate the length of mercury column. [19 cm]

19. (a) A student is tasked with measuring the real expansivity of mercury by a physics teacher. What experiment should the student conduct to measure the real expansivity without considering the expansion of the vessel?



- (b) Using the information given in figure, show that;

$$\gamma = \frac{h_2 - h_1}{h_1 \Delta \theta}$$

- (c) In the given figure the height of the liquid column is 65cm at 0°C and 66.18cm at 100°C . Calculate the real expansivity of the given liquid. [$2.8 \times 10^{-4} \text{K}^{-1}$]

20. (a) The length of a brass rod is found to be smaller on a hot summer day than on a cold winter day by the same aluminum scale, why?
- (b) A disc has a hole at its centre. When the disc is heated it expands. What will be the size of the diameter of the hole?
- (c) A rupee coin has a diameter of 2.0 cm at 20°C . The coin is made of metal alloy (mostly copper) for which the coefficient of linear expansion is $1.76 \times 10^{-5} \text{K}^{-1}$. What would its diameter be (i) on a hot day in Nepalgunj (48°C)? (ii) on a cold night in the mountains of the Annapurna range (-23°C)? [2.0009856cm , 1.9984864cm]
21. (a) Imagine you have two identical metal rods, one made of steel and the other of aluminum. When these rods are subjected to the same temperature increase, you observe that the aluminum rod expands more than the steel rod. Why does this happen?

- (b) Suppose the rods are joined along their length so that the system might bend when heated uniformly. Provide a detailed explanation and draw the necessary diagram.
- (c) Explain how the concept of differential thermal expansion can be observed and utilized in everyday life providing examples and discussing its implications.
- (d) A steel rod and an aluminum rod, each initially 1 meter long are heated uniformly to the same temperature increase. The coefficient of linear expansion for steel is $1.2 \times 10^{-5} \text{ K}^{-1}$ and for aluminum is $2.3 \times 10^{-5} \text{ K}^{-1}$. Now, the rods are allowed to cool back to their original temperature and are connected end-to-end. Calculate the final length of the combined rod. [2m]
22. The distance between the two points A and B is 625 km, find the total space that must be left between the rails to allow for a change of temperature from 36°F in winter to 117°F in summer. Given linear expansivity of iron = $12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$. [337.5 m]
23. A brass rod of length 0.40 m and a steel rod of length 0.60 m, both initially at 0°C are heated to 85°C . If the increase in length is the same for both rods, calculate the linear expansivity of brass. The linear expansivity of steel is $12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$. [$18 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$]
24. Determine the temperature at which wood will just sink in benzene if the density of benzene at 0°C and cubical expansivity of benzene are $9 \times 10^2 \text{ kgm}^{-3}$ and $1.2 \times 10^{-3} \text{ K}^{-1}$ respectively. The density of wood at 0°C and cubical expansivity of wood are $8.8 \times 10^2 \text{ kgm}^{-3}$ and $1.5 \times 10^{-4} \text{ K}^{-1}$ respectively. [294.72K or 21.72°C]
25. The markings on an aluminum ruler and a brass ruler are perfectly aligned at 0°C . How far apart will the 20.0 cm marks be on the two rulers at 100°C , if the precise alignment of the left hand ends of the rulers is maintained? [0.008cm]
26. An underground tank with a capacity of 1700l (1.70 m^3) is filled with ethanol that has an initial temperature of 19°C . After the ethanol has cooled off to the temperature of the tank and ground, which is 10°C how much air space, will there be above the ethanol in the tank? [Assume that the volume of the tank does not change and γ for ethanol is $75 \times 10^{-5} \text{ K}^{-1}$] [0.0115 m³]
27. A wire that is 1.5 m long at 20°C is found to increase in length by 1.9 cm when warmed to 420°C (a) compute its average coefficient of linear expansion for this temperature range. (b) The wire is stretched just taut (zero tension) at 420°C . Find the stress in the wire if it is cooled to 20°C without being allowed to contract. (Young's modulus for the wire is $2 \times 10^{11} \text{ Pa}$). [$3.17 \times 10^{-5} \text{ K}^{-1}$, $2.54 \times 10^9 \text{ Nm}^{-2}$]
28. The measurement of a steel tape is accurate at 20°C . It is used at 10°C to measure a kilometer of distance. What reading will the tape show? [α for steel = $12 \times 10^{-6} \text{ K}^{-1}$] [1.00012 km]

Chapter 11

Quantity of heat

Henri Victor Regnault was a French chemist and physicist best known for his careful measurements of the thermal properties of gases. He was an early thermodynamicist and was a mentor to William Thomson in the late 1840s. Regnault designed an apparatus for a large number of physical measurements and carefully determined the specific heats of many solids, liquids, and gases. He showed that no two gases have precisely the same coefficient of expansion and proved that Boyle's law of the elasticity of a "perfect gas" is only approximately true for real gases. In introducing his air thermometer, he determined the absolute expansion of mercury. He also devised a hygrometer.



Henri Victor Regnault (21 July 1810 – 19 January 1878)

Do you know?

1. A large iceberg melts at the base but not at the top.
2. Even if you add heat to an object, the temperature of the object may not change.

11.1 Introduction

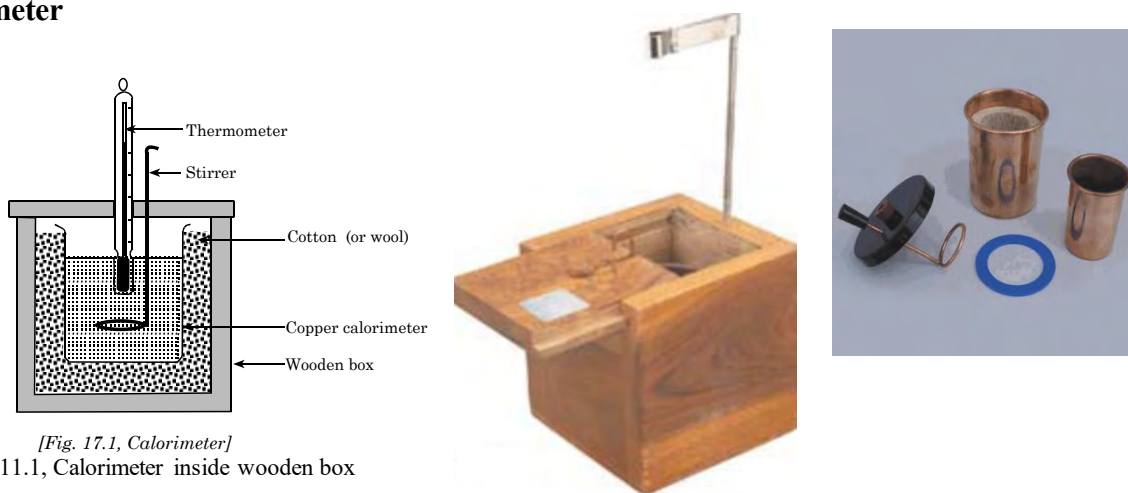
If two bodies are brought in thermal contact then the hotter body cools down and the colder body warms up until they attain equilibrium temperature. During this process, energy is transferred from the hotter body to the colder body. This energy in transit is called heat. The transfer of heat continues whenever the temperature difference exists between two bodies. Therefore, heat is a form of energy and is a scalar quantity. We cannot directly measure the quantity of heat on the body; instead, it can be measured as the quantity of heat transferred from one body to another.

The quantity of heat transferred in a process may change the temperature of the body. For a given value of heat exchange, the change in temperature will depend upon the mass of the body and the nature of the material of the body. By knowing the value of the temperature change, mass and specific heat capacity (nature of substance), we can calculate the quantity of heat exchanged by a body. When the thermos flask, which contains a liquid is shaken vigorously, the temperature of the liquid gets raised without supplying heat. A similar phenomenon can be observed in the case of gases. In both cases, work done on the system becomes a source of heat energy.

Calorimetry

We can measure the quantity of heat transfer from one body to another body. The science of measurement of the transfer of heat is calorimetry. Thus, **calorimetry is a branch of physics which deals with the experimental techniques for the quantitative measurement of heat exchange.** A device used for the measurement of heat transfer is called a calorimeter.

Calorimeter



A calorimeter is a device used for the quantitative measurement of heat exchange. Generally, calorimeters are made up of copper because copper has low specific heat capacity and its specific heat capacity is almost constant for all temperature ranges.

A calorimeter is a thin-walled, and polished cylindrical metallic vessel generally made of copper as shown in fig. 11.1. It consists of a stirrer of the same material. The calorimeter is well insulated so that the exchange of heat between the system and surroundings gets prohibited. To provide insulation, the calorimeter is wrapped in wool, cotton, or any other insulating materials and kept inside a wooden box. For the measurement of the temperature of the calorimeter and its contents, a thermometer is inserted through a hole made in the sliding lid of the wooden box.

11.2 Heat Equation

When we heat a body, it absorbs heat and gets hotter until it stays in the same state. Likewise, when a body cools down, it loses heat, and its temperature drops until it stays in the same state. The heat equation is a mathematical relationship between heat absorbed or released by a body with its mass, specific heat capacity and change in temperature.

Experimentally, it is found that the amount of heat energy absorbed or released by a body is,

- directly proportional to the mass of the body (m),
i.e., $Q \propto m$... (1)
- directly proportional to the change in temperature of the body ($\Delta\theta$),
i.e., $Q \propto \Delta\theta$... (2)

Combining equations (1) and (2) we get,

$$\begin{aligned} Q &\propto m \Delta\theta \\ \text{or, } Q &= s m \Delta\theta \\ \text{or, } Q &= m s \Delta\theta \quad \dots (3) \end{aligned}$$

[s is proportionality constant]

Here, s is known as the specific heat capacity of the material of the body.

Equation (3) is known as heat equation.

One calorie: 1 calorie is defined as the amount of heat needed to raise the temperature of 1 gram of pure water by 1 degree Celsius (generally from 14.5°C to 15.5°C). $1 \text{ cal} = 4.2 \text{ J}$

11.3 Heat Capacity or Thermal Capacity

Heat capacity or thermal capacity of a body is defined as the amount of heat required to change the temperature of given mass of the body through 1°C (or 1 K).

If Q is the amount of heat required to change the temperature of a body of mass m through $\Delta\theta$ then

$$Q = m s \Delta\theta \quad [s \text{ is specific heat capacity of the body}]$$

If the temperature changes by unit value i.e. $\Delta\theta = 1^\circ\text{C}$ then,

$$Q = ms$$

This quantity of heat is called heat capacity of given body. Hence, **heat capacity is the product of mass and specific heat capacity of the body**. Heat capacity depends on nature and mass of the body. The heat capacity is measured in J K^{-1} .

11.4 Specific Heat Capacity

Amount of heat absorbed or released by a body is given by the heat equation, $Q = m s \Delta\theta$. If we suppose unit change in temperature of unit mass, then from heat equation, $s = \frac{Q}{m \Delta\theta}$. Therefore, *specific heat capacity (s) is the amount of heat energy required to change the temperature of unit mass through 1°C or 1 K*. It is given by;

$$s = \frac{Q}{m \Delta\theta}$$

Specific heat capacity depends on nature of substance.

The specific heat capacity is measured in $\text{J kg}^{-1} \text{K}^{-1}$. Among solid and liquid, water has highest value of specific heat capacity $4200 \text{ J kg}^{-1} \text{K}^{-1}$.

Table: Specific heat capacity of some substances

SN	Substance	Specific heat in SI ($\text{J kg}^{-1} \text{K}^{-1}$)
1	Water	4200
2	Copper	380
3	Aluminum	910
4	Iron	470
5	Brass	372
6	Steel	447
7	Mercury	138
8	Sand	800
9	Glass	677
10	Kerosene	2090
11	Ice	2100
12	Alcohol	2590

Project work: 1

Take equal volume of water and cooking oil in separate but identical containers. Use thermometers to measure their initial temperatures. Apply heat to both substances simultaneously and similarly. Record the time taken for each substance to change their temperature by 5°C . Compare the results. Discuss among your friends in class about how water heats up more slowly but retains heat for long time as compared to oil. Interact with your teacher about conclusions.

SN	Temperature range ($^{\circ}\text{C}$)	Time taken by water	Time taken by oil	Conclusions
1.	25-30			
2.	30-35			
3.				
4.				
5.				

11.5 Specific Heat Capacity and Change in Temperature

Suppose an equal amount of heat is given to two different substances of equal mass m but of different specific heat capacities. Let s_1 and s_2 are respective specific heat capacities. Similarly, let $\Delta\theta_1$ and $\Delta\theta_2$ be respective changes in temperature. Then, from the given condition,

Heat given to first system = Heat given to second system

$$s_1 m \Delta\theta_1 = s_2 m \Delta\theta_2$$

or, $s_1 \Delta\theta_1 = s_2 \Delta\theta_2$

or, $\frac{s_1}{s_2} = \frac{\Delta\theta_2}{\Delta\theta_1}$

i.e., $\Delta\theta \propto \frac{1}{s}$

This indicates that the change in the temperature of a substance is inversely proportional to its specific heat capacity for constant heat supplied or absorbed.

11.6 Applications of High and Low Specific Heat Capacity

A substance which has value of specific heat capacity can absorb large amount of heat from any object brought in thermal contact. As a result of this, temperature of object in contact reduces by sufficiently large value but its own temperature changes almost negligibly. This property of substance can be used as cooling agent. Water has high specific heat capacity and shows such behavior. Therefore, water can be used,

- (i) as cooling agent in many heating appliances and in running vehicles
- (ii) to lower down temperature of a person suffering from high fever.

Also, a substance which has high value of specific heat capacity can release large amount of heat to anybody brought in thermal contact. As a result of this, temperature of object in contact increases by sufficiently large value but its own temperature changes almost negligibly. This property of substance can be used as heating agent. Water has high specific heat capacity and shows such behavior. Therefore, water can be used

- (i) as heating agent in room heating system
- (ii) as heating agent in hot water bag

Substance which has low value of specific heat capacity can absorb or lose almost negligible amount of heat from or to any other body brought in thermal contact and immediately it attains thermal equilibrium. This action does not bring any change in thermal configuration of the body in contact. Mercury has such property. Therefore, mercury can be used **as thermometric substance**.

Similarly copper also has low value of specific heat capacity so that it can be used as material of calorimeter.

Also, substance which has low value of specific heat capacity suffers sufficiently large change in temperature even from the small amount of heat applied. Cooking oil has such property. Therefore, cooking oil can be used **to make fried food items** (easier and faster cooking).

Similarly massage oil has low specific heat capacity so that it gets heated sufficiently from the heat of friction during massage.

Workout Examples:

Q. An iron ball of mass 150g is heated by 2100J of heat. Calculate the change in temperature of the iron ball whose specific heat capacity is $470 \text{ J kg}^{-1} \text{ K}^{-1}$.

Solution: Here,

Mass of iron ball (m) = 150g = $150 \times 10^{-3} \text{ kg}$

Heat supplied (Q) = 2100 J

Specific heat capacity of iron (s) = $470 \text{ J kg}^{-1} \text{ K}^{-1}$

Temperature change ($\Delta\theta$) = ?

As we know, $Q = m s \Delta\theta$

$$\Delta\theta = \frac{Q}{m s} = \frac{2100}{150 \times 10^{-3} \times 470} = 29.78 \text{ K} = 30 \text{ K}$$

Thus, change in temperature of iron is 30K or 30°C .

Q. Calculate the difference in temperature between the water at the top and bottom of a waterfall which is 200 m high. (Specific heat of water = $4200 \text{ J kg}^{-1} \text{ K}^{-1}$, use $g = 10 \text{ ms}^{-2}$)

Solution: Here,

Height (h) = 200 m

Specific heat capacity of water (s_w) = $4200 \text{ J kg}^{-1} \text{ K}^{-1}$

Temperature difference between the water at the top and bottom of a waterfall of 200 m height $\Delta\theta$ = ?

From the principle of conservation of energy,

Heat energy gained by water = loss in PE by water

or, $m_w s_w \Delta\theta = m_w g h$ (Where, m_w is mass of water)

$$\text{or, } \Delta\theta = \frac{gh}{s_w} = \frac{10 \times 200}{4200} = \frac{20}{42} = \frac{10}{21} = 0.48 \text{ K} = 0.5 \text{ K}$$

Hence, the required temperature difference is 0.5 K or 0.5°C .

Sample Questions:

1. Coastal land has higher temperature than the ocean during day but a lower temperature at night, why?

2. A calorimeter is made of metal but not of glass, why?
3. During high fever, a wet cloth is kept on the forehead of a person, why?
4. What is the value of specific heat capacity during phase change of a substance?
5. Milk boils faster than water, why?

11.7 Newton's Law of Cooling

Do you know?

- a. Animals eat more in winter than in summer.
- b. In forensic science, time of the victim's death can be effectively calculated using Newton's law of cooling.



[Fig. 11.2 (a), Soup in a bowl]

Have you ever held a warm bowl full of food in your hand, and wondered why it seems to cool down rapidly at first but stays lukewarm for much longer? Also, the hot soup will be too hot to eat for the first few minutes, but eventually, it reaches a temperature we can enjoy it. Then we can eat the soup leisurely over some more minutes, and it stays warm for much longer. We may have a query that how fast a cup of boiling water will cool down in a kitchen at room temperature. In all of above activities, there is a relationship between temperature difference of hot substance with its surroundings and the time taken by it to cool down. Newton's law of cooling describes how fast a substance cools down. Also, it establishes a relationship between the temperature of a substance and the time taken by the substance to cool down.

During cooling of hot liquid exposed to the surroundings of lower temperature, liquid loses heat continuously to the surroundings until it attains the equilibrium. The amount of heat lost by the liquid depends on different factors like nature of liquid, nature of solid vessel, surface area of liquid exposed to the surroundings, temperature difference of liquid and its surroundings, etc. The loss of heat by different processes may be explained by different laws. If the temperature difference is maintained suitable (less than 35°C) and all other factors which affect the amount of heat lost are made constant then, cooling of liquid follows the Newton's law of cooling.

Newton's law of cooling states that the rate of heat loss by a hot liquid is directly proportional to the temperature difference of the liquid and its surrounding.

Mathematically,

$$-\frac{dQ}{dt} \propto (\theta - \theta_0) \text{ [-ve sign indicates the loss of heat]}$$

Let's consider a hot liquid of mass m and specific heat capacity s at temperature θ and the temperature of the surroundings is θ_0 . If liquid loses dQ amount of heat during time interval dt , then rate of loss of heat is $-\frac{dQ}{dt}$.

According to Newton's law of cooling,

$$-\frac{dQ}{dt} \propto (\theta - \theta_0)$$

$$\text{or, } \frac{dQ}{dt} = -K(\theta - \theta_0) \quad \dots (1) \quad [K \text{ is proportionality constant}]$$

K depends on nature of liquid and area of liquid exposed to the surroundings.

If the temperature of liquid falls by $d\theta$ in time dt , then,

$$dQ = m s d\theta \quad \dots (2)$$

From equations (1) and (2) we get,

$$-m s \frac{d\theta}{dt} = K(\theta - \theta_0)$$

$$\frac{d\theta}{dt} = \left(\frac{-K}{ms} \right) (\theta - \theta_0) \dots (3)$$

$$\text{or, } \frac{d\theta}{(\theta - \theta_0)} = \left(\frac{-K}{ms} \right) dt$$

Integrating both sides we get,

$$\int \frac{d\theta}{(\theta - \theta_0)} = \int \left(\frac{-K}{ms} \right) dt$$

$$\text{or, } \log_e(\theta - \theta_0) + C_1 = \left(\frac{-K}{ms} \right) t + C_2$$

$$\text{or, } \log_e(\theta - \theta_0) = \left(\frac{-K}{ms} \right) t + C_2 - C_1$$

$$\text{or, } \log_e(\theta - \theta_0) = \left(\frac{-K}{ms} \right) t + C \quad \dots (4) \quad [C_1 \text{ and } C_2 \text{ are integration constants and } C = C_2 - C_1]$$

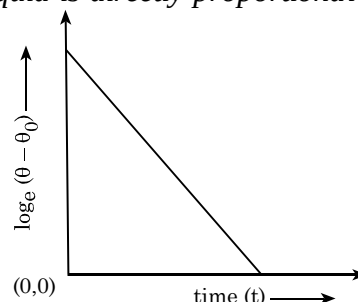
Equation (4) is in the form of $y = mx + c$, which supports the Newton's law of cooling.

The Newton's law of cooling holds true in the following conditions:

- The difference in temperature between the body and the surroundings must be small.
- The loss of heat from the body should only be caused by radiation.
- Temperature of the surroundings must remain constant during the cooling.

Cases of Newton's law of cooling

- Cooling of hot beverage: when we pour hot tea or coffee, it will cool down over time as it is exposed to the surrounding air.
- In forensic science, with information of temperature of dead body and crime scene, time of the victim's death can be effectively calculated.



[Fig. 11.2 (b) : Graphical representation of Newton's law of

3. Tea cools faster in winter than in summer.
4. Hot liquid cools faster than warm liquid, why?
5. Animals eat more in winter than in summer?

Workout Example:

Q. A substance takes 3 min in cooling from 50°C to 45°C and takes 5 min in cooling from 45°C to 40°C. What is the temperature of its surroundings? How much time it will take to cool this substance from 40°C to 35°C?

✎ Solution: Here,

✎ Case I: Time interval $(\Delta t)_1 = 3 \text{ min} = 3 \times 60 \text{ s} = 180 \text{ s}$

Temperature interval $(\Delta \theta)_1 = (50 - 45)^\circ\text{C} = 5^\circ\text{C}$

Temperature of substance $(\theta_1) = \left(\frac{50 + 45}{2}\right) = 47.5^\circ\text{C}$

Temperature of surroundings $(\theta_0) = ?$

Using Newton's law of cooling,

$$m s \frac{(\Delta \theta)_1}{(\Delta t)_1} = k (\theta_1 - \theta_0) \quad \dots (1)$$

Where m and s are mass and specific heat capacity of given substance and k is proportionality constant.

Case II: Time interval $(\Delta t)_2 = 5 \text{ min} = 5 \times 60 \text{ s} = 300 \text{ s}$

Temperature interval $(\Delta \theta)_2 = (45 - 40)^\circ\text{C} = 5^\circ\text{C}$

Temperature of substance $(\theta_2) = \left(\frac{40 + 45}{2}\right) = 42.5^\circ\text{C}$

Using Newton's law of cooling,

$$m s \frac{(\Delta \theta)_2}{(\Delta t)_2} = K (\theta_2 - \theta_0) \quad \dots (2)$$

Dividing equations (1) by (2) and using given values;

$$\frac{ms \frac{(\Delta \theta)_1}{(\Delta t)_1}}{ms \frac{(\Delta \theta)_2}{(\Delta t)_2}} = \frac{K(\theta_1 - \theta_0)}{K(\theta_2 - \theta_0)}$$

$$\text{or, } \frac{(\Delta \theta)_1}{(\Delta t)_1} \times \frac{(\Delta t)_2}{(\Delta \theta)_2} = \frac{(\theta_1 - \theta_0)}{(\theta_2 - \theta_0)}$$

$$\text{or, } \frac{5}{3} \times \frac{5}{5} = \frac{47.5 - \theta_0}{42.5 - \theta_0}$$

$$\text{or, } 5 (42.5 - \theta_0) = 3 (47.5 - \theta_0)$$

$$\text{or, } 212.5 - 5 \theta_0 = 142.5 - 3 \theta_0$$

$$\text{or, } 5 \theta_0 - 3 \theta_0 = 212.5 - 142.5 = 70$$

$$\text{or, } 2 \theta_0 = 70$$

$$\text{or, } \theta_0 = \frac{70}{2} = 35^\circ\text{C}$$

Therefore, temperature of surroundings, $\theta_0 = 35^\circ\text{C}$

Case III:

Time interval $(\Delta t)_3 = ?$

Temperature interval $(\Delta \theta)_3 = (40 - 35) = 5^\circ\text{C}$

Temperature of substance $(\theta_3) = \left(\frac{40 + 35}{2}\right) = 37.5^\circ\text{C}$

Temperature of surrounding $(\theta_0) = 35^\circ\text{C}$

Using Newton's law of cooling,

$$m s \frac{(\Delta \theta)_3}{(\Delta t)_3} = k (\theta_3 - \theta_0)$$

$$\text{or, } m s \frac{5}{(\Delta t)_3} = k (37.5 - 35) \quad \dots (3)$$

Dividing equations (2) by equation (3)

$$\frac{m s \frac{(\Delta \theta)_2}{(\Delta t)_2}}{m s \frac{(\Delta \theta)_3}{(\Delta t)_3}} = \frac{K (\theta_2 - \theta_0)}{K (\theta_3 - \theta_0)}$$

$$\text{or, } \frac{(\Delta \theta)_2}{(\Delta t)_2} \times \frac{(\Delta t)_3}{(\Delta \theta)_3} = \frac{\theta_2 - \theta_0}{\theta_3 - \theta_0}$$

$$\text{or, } \frac{5}{5} \times \frac{(\Delta t)_3}{5} = \frac{42.5 - 35}{37.5 - 35}$$

$$\text{or, } (\Delta t)_3 = 5 \left(\frac{7.5}{2.5}\right) = 5 \times 3 = 15 \text{ min}$$

Hence, the temperature of surrounding is 35°C , and time required to cool from 40°C to 35°C is 15 min.

11.8 Principle of Calorimetry

Can you answer this?

A student claims that when two bodies at different temperatures are brought in thermal contact, fall in temperature of hotter body must be equal to the rise in temperature of the colder one. Is it acceptable?

When two objects at different temperatures are brought into thermal contact, the hotter object loses the heat while the colder object gains the heat until both reach to an equal temperature. If heat loss to the surroundings is prevented, the quantity of heat lost by the hotter body is equal to the quantity of heat gained by the colder body. This is the principle of calorimetry. *It can be stated that the amount of the heat lost by a hotter body must be equal to amount of the heat gained by a colder body during the process of heat exchange between two objects in a closed system.*

That is, **heat loss = heat gain** ; for a closed system.

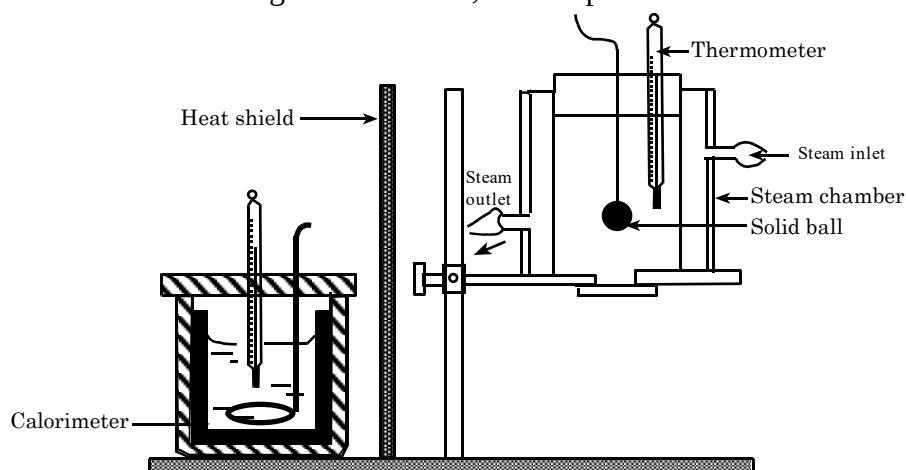
A closed system is also called an isolated system. In a closed system objects under consideration do not exchange heat with its surroundings. Principle of calorimetry is an alternative form of the law of conservation of energy. Idea of principle of calorimetry is used to calculate specific heat capacity of different substances.

11.9 Determination of Specific Heat Capacity of a Solid by the Method of Mixture

Regnault's apparatus can be used to determine the specific heat capacity of a substance by the method of mixture. In the method of mixture, we use principle of calorimetry to calculate specific heat capacity.

Procedure

At first, mass of a dry and clean calorimeter with its stirrer is measured. Then about two-third volume of calorimeter is filled with water. Again, mass of calorimeter with stirrer and water is measured. After this calorimeter is kept inside an insulated box and a thermometer is inserted into it as shown in figure 11.3. Then, the temperature is noted.



[Fig. 17.4, Determination of Specific heat capacity of solid]

[Fig. 11.3: Determination of specific heat capacity of a solid from the method of mixture]

At the meantime, a solid whose specific heat capacity (s) is to be determined is heated in steam chamber of Regnault's apparatus as shown in figure 11.3. The solid is heated continuously until it attains thermal equilibrium with steam. The steady value of the thermometer reading is taken. Now, the solid is suddenly and carefully transferred into the calorimeter by keeping it just below the steam chamber. Then, the mixture is stirred gently until the temperature of the mixture reaches to a constant value. The temperature of the mixture is noted at this steady value. Finally, the mass of the mixture is measured.

Let, mass of a dry and clean calorimeter with stirrer = m_1

Mass of a calorimeter with stirrer and water = m_2

Initial temperature of the calorimeter with stirrer and water = θ_1

Steady temperature of solid = θ_2

Final temperature of mixture = θ

Mass of mixture = m_3

Therefore,

Mass of water (m_w) = $m_2 - m_1$

Mass of solid (m_s) = $m_3 - m_2$

Decrease in temperature of solid = $\theta_2 - \theta$

Increase in temperature of water and calorimeter = $\theta - \theta_1$

Now, heat lost by solid,

$$Q_L = m_s s (\theta_2 - \theta)$$

Similarly, heat gained by water and the calorimeter,

$$Q_g = m_w s_w (\theta - \theta_1) + m_1 s_c (\theta - \theta_1) = [(m_w s_w + m_1 s_c) (\theta - \theta_1)]$$

where s_w and s_c are the specific heat capacity of water and material of the calorimeter respectively.

Therefore, from the principle of calorimetry;

$$Q_L = Q_g$$

$$\text{or, } m_s s (\theta_2 - \theta) = [m_w s_w + m_1 s_c] (\theta - \theta_1)$$

$$\text{or, } s = \frac{[m_w s_w + m_1 s_c] (\theta - \theta_1)}{m_s (\theta_2 - \theta)}$$

Hence, the value of specific heat capacity (s) is determined.

Workout Example:

Q. In an experiment on the specific heat of a metal, a 200 g block of metal at 150°C is dropped in a copper calorimeter of mass 270 g containing 150 cm^3 of water at 27°C . The final temperature is 40°C . Now, calculate the specific heat capacity of the metal. [Specific heat capacity of copper = $390\text{ J kg}^{-1}\text{K}^{-1}$]

Solution: Here,

Mass of block (m) = $200\text{ g} = 200 \times 10^{-3}\text{ kg}$

Initial temperature of block (θ_1) = 150°C

Mass of calorimeter (m_c) = $270\text{ g} = 270 \times 10^{-3}\text{ kg}$

Initial temperature of water and calorimeter (θ_2) = 27°C

Final temperature of mixture (θ_3) = 40°C

Specific heat capacity of water (s_w) = $4200\text{ J kg}^{-1}\text{K}^{-1}$

Specific heat of copper (s_c) = $390\text{ J kg}^{-1}\text{K}^{-1}$

Mass of water (m_w) = $150\text{ cm}^3 \times 1\text{ g cm}^{-3} = 150\text{ g} = 150 \times 10^{-3}\text{ kg}$

Specific heat of metal (s) = ?

Now, Heat lost by block, (Q_L) = $m s (\theta_1 - \theta_3) = 200 \times 10^{-3} \times s \times (150 - 40)$

Heat gained by calorimeter and water,

$$Q_g = m_c s_c (\theta_3 - \theta_2) + m_w s_w (\theta_3 - \theta_2) = 270 \times 10^{-3} \times 390 (40 - 27) + 0.150 \times 4200 (40 - 27)$$

$$= 1368.9 + 8190 = 9558.9\text{ J}$$

But, from the principle of calorimetry,

Heat lost (Q_L) = Heat gained (Q_g)

$$\text{or, } 200 \times 10^{-3} \times s \times 110 = 9558.9$$

$$\text{or, } s = \frac{9558.9}{110 \times 200 \times 10^{-3}} = 434.5\text{ J kg}^{-1}\text{K}^{-1} = 4.3 \times 10^2\text{ J kg}^{-1}\text{K}^{-1}$$

Hence, the specific heat of metal is $4.3 \times 10^2\text{ J kg}^{-1}\text{K}^{-1}$.

Sample Question:

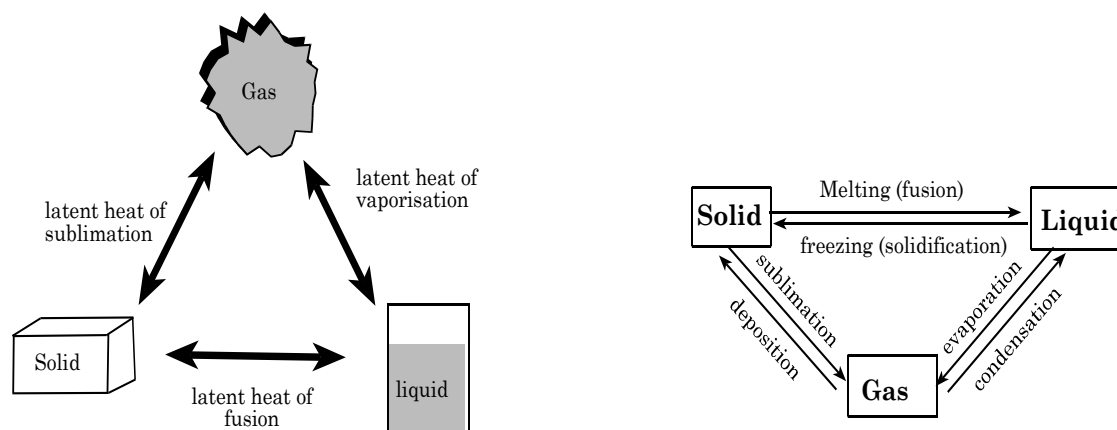
A copper calorimeter of mass 300 g contains 500 g of water at a temperature of 15°C . A 560 g block of aluminum at the temperature of 100°C is dropped into the water of the calorimeter and the temperature is observed to be increased to 22.5°C . Find the specific heat capacity of Aluminum, neglecting the heat lost to the surroundings. (sp. The heat of copper is $390\text{ J kg}^{-1}\text{K}^{-1}$, and the specific heat capacity of water is $4200\text{ J kg}^{-1}\text{K}^{-1}$). [$383.12\text{ J kg}^{-1}\text{K}^{-1}$]

11.10 Change of Phase

Do you know?

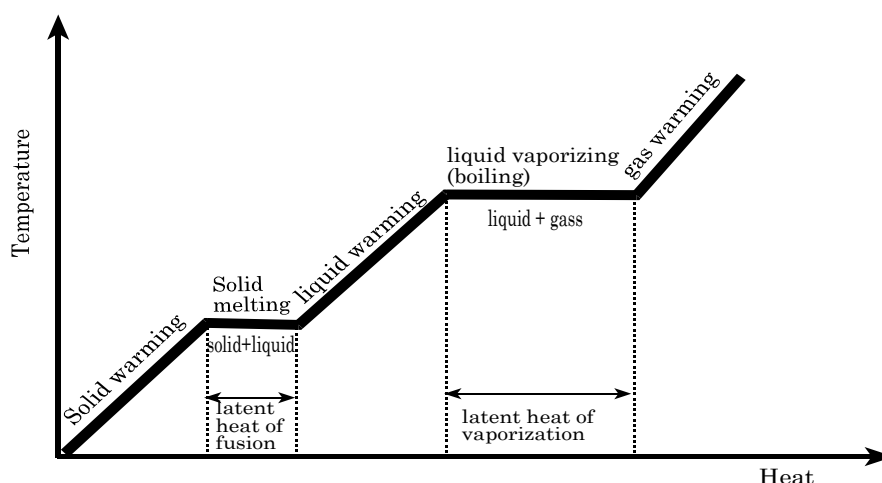
Even if you add heat to an object, the temperature of the object may not change.

Matters exist in three different states. Therefore, substances found around us may fall under either of these three states: solid, liquid, and gas. A solid state has the least intermolecular space, the liquid state has a moderate value of intermolecular space, and the gaseous state has sufficient large intermolecular space. The state of a substance is the result of heat applied, or the pressure exerted, or both to the object. Common changes of the state include melting, freezing, sublimation, deposition, condensation, and vaporization. A change of a state is a physical change in a matter.



[Fig. 11.5: Change of state of matter]

Continuous application of heat to a solid state raises the temperature of an object continuously up to a particular value called melting point. Then, the solid starts to convert into liquid, and the temperature remains constant until all the solid converts into liquid even though heat is supplied continuously. Also,



[Fig. 11.6: Variation of temperature of a body with heat]

continuous cooling of the liquid state reduces the temperature of an object continuously to a particular value called freezing point. Then the liquid starts to solidify, and the temperature remains constant until all the liquid solidifies even at continuous cooling. Hence, either the heat is supplied or it is dragged out to change the state of a matter. Fusion and solidification, vaporization and condensation, sublimation and deposition are pairs of reverse processes of change of phase.

The amount of heat required to convert the state of matter at a constant temperature is called the

latent heat of the matter. The latent heat during the change of state from solid to liquid or vice-versa is called the latent heat of fusion. The latent heat during the change of state from liquid to gas or vice-versa is called the latent heat of vaporization. Latent heat is called so because it is hidden from the thermometer reading. Latent heat is also called heat of transformation.

During the change of state of a matter, heat is used for two purposes; one to overcome the external atmospheric pressure to create a space for expansion, and the next to reduce the intermolecular force of attraction. The former is called external latent heat and latter is called internal latent heat. External latent heat is much smaller than internal latent heat. Value of both types of heat are much greater in case of conversion of state from liquid to gas than that in the case of conversion of state from solid to liquid. Therefore, the latent heat of vaporization is always greater than the latent heat of fusion.

Latent heat cannot be measured in terms of change in velocity and change in kinetic energy of molecules. But, energy can neither be created nor can it be destroyed. Hence, during the change of phase, heat is used to increase potential energy molecules but not kinetic energy. Therefore, heat can also be considered as potential energy.

11.11 Specific Latent Heat of Fusion

Experimentally, it is found that the latent heat of fusion of a substance is directly proportional to its mass, i.e.,

$$\begin{aligned} \text{Latent heat (Q)} &\propto \text{mass of substance (m)} \\ \text{or, } Q &= L m && [L \text{ is proportionality constant.}] \\ \text{or, } L &= \frac{Q}{m} \end{aligned}$$

L is the specific latent heat of fusion of a given substance, and its value depends on the nature of the substance.

If we take unit mass, i.e., $m=1$ unit then $L=Q$. Hence the specific latent heat of fusion of a substance is defined as the heat required to change the state of a unit mass of a substance from solid to liquid at its melting point. The SI unit of L is J kg^{-1} . The specific latent of fusion of ice is 3.36×10^5 in J kg^{-1} . This means $3.36 \times 10^5 \text{J}$ of heat is required to convert 1 kg of ice into water at its melting point.

11.12 Specific Latent of Vaporization

Experimentally, it is found that the latent heat of vaporization of a substance is directly proportional to its mass, i.e.,

$$\begin{aligned} \text{Latent heat (Q)} &\propto \text{mass of substance (m)} \\ \text{or, } Q &= L m && [L \text{ is proportionality constant.}] \\ \text{or, } L &= \frac{Q}{m} \end{aligned}$$

L is the specific latent heat of vaporization of a given substance, and its value depends on the nature of the substance and the system of units used.

If we take unit mass, i.e., $m=1$ unit, then $L=Q$. Hence the specific latent heat of vaporization of a substance is defined as the heat required to change the state of a unit mass of a substance from liquid to vapor at its boiling point. The SI unit of L is J kg^{-1} .

Water has a specific latent of vaporization 2.26×10^6 in J kg^{-1} . This means 2.26×10^6 in J of heat is required to convert 1 kg of water into steam at its boiling point.

Good to Know:

1. During the heating of a matter, it undergoes either change of temperature or change of state, but not both at a time. During the change of temperature, the state of matter remains unchanged and vice versa.
2. During the process of melting, the solid and the liquid states of matter co-exist in thermal equilibrium. During the process of boiling, the liquid and the gas states of matter coexist in thermal equilibrium. During the process of sublimation, the solid and the gaseous states of matter coexist in thermal equilibrium.
3. The water at 100°C is better than the water at 0°C for extinguishing fire.
4. Specific heat capacity has different values for different states of the same matter. Water has the specific heat of $4200 \text{ J kg}^{-1}\text{K}^{-1}$ but that of ice is $2100 \text{ J kg}^{-1}\text{K}^{-1}$.
5. In the heat-temperature graph, the horizontal portion of solid melting is shorter than that of liquid boiling. It is not accidental. The latent heat of vaporization is greater than the latent heat of fusion.
6. In the heat-temperature graph, the slope of the line for solid warming, liquid warming, and gas warming are different for the same matter indicating different values of specific heat for different states. Again, the slope for each line has a different value for different matter indicating that the specific heat capacity depends on the nature of a material.

Sample Questions:

1. Why is a cube of ice at 0°C more effective than water at 0°C in cooling liquor items like beer?
2. The unit of specific heat capacity is $\text{J kg}^{-1}\text{K}^{-1}$ but that of specific latent heat is J kg^{-1} . Why does the unit of sp. latent heat not include the $^{\circ}\text{C}$ to account for a temperature change?
3. Which molecules, molecules of ice at 0°C or molecules of water at 0°C , have more potential energy and why?
4. Why can you get a more severe burn from steam at 100°C than water at 100°C ?
5. Is it possible to boil water by passing steam at atmospheric pressure?
6. If you add heat to an object, does it necessarily increase its temperature?

11.13 Determination of Specific Latent Heat of Fusion of Ice by the Method of Mixture

The specific latent heat of the fusion of ice is defined as the heat required to change the state of a unit mass of ice into water at its melting point.

Using principle of calorimetry, we can determine the specific latent heat of the fusion of ice from the method of mixture.

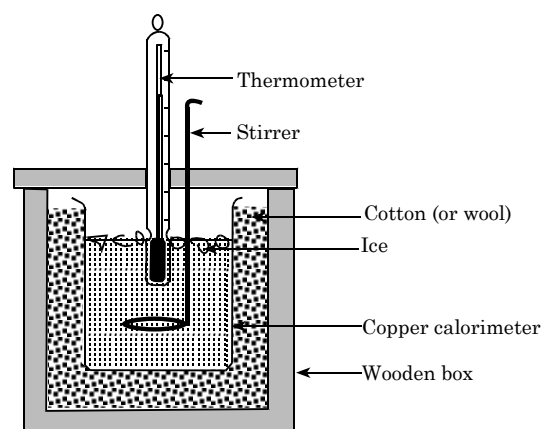


Fig. 11.7, Determination of specific latent heat of fusion of ice

Procedure

At first mass of a clean and dry calorimeter with its stirrer is measured. About two-third volume of the calorimeter is filled with water. Again, the mass of the calorimeter with stirrer and water is measured. Then, the calorimeter is placed inside insulating system of wooden box, and a thermometer is inserted into it as shown in figure 11.7. The initial temperature of the calorimeter with stirrer and water is noted.

After this, certain mass of dry pieces of melting ice are introduced in to the calorimeter. To make ice pieces dry blotting paper are used. Then the mixture is stirred well until ice melts completely. The steady temperature of mixture is noted. After complete melting of ice, reading of the thermometer remains steady for a moment. Now, the calorimeter is taken out from the wooden box, and the mass of mixture is measured.

Let, Mass of a calorimeter with stirrer = m_1

Mass of calorimeter with stirrer and water = m_2

Mass of calorimeter with stirrer, water and ice = m_3

Initial temperature of calorimeter with stirrer and water = $\theta_1^\circ\text{C}$

Temperature of ice = 0°C

Final temperature of mixture = $\theta_2^\circ\text{C}$

Therefore,

Mass of water (m_w) = $m_2 - m_1$

Mass of ice (m_i) = $m_3 - m_2$

Now using principle of Calorimetry

Heat gained by ice (Q_g) = Heat lost by calorimeter and water (Q_L)

or, $m_i L_i + m_i s_w (\theta_2 - \theta_1) = (m_s \Delta\theta)_c + (m_s \Delta\theta)_w = m_1 s_c (\theta_1 - \theta_2) + m_w s_w (\theta_1 - \theta_2)$

or, $m_i L_i = (m_1 s_c + m_w s_w) (\theta_1 - \theta_2) - m_i s_w \theta_2$

or, $L_i = \frac{(m_1 s_c + m_w s_w) (\theta_1 - \theta_2)}{m_i} - s_w \theta_2$

In the above expression, s_c and s_w are specific heat capacities of the calorimeter and water respectively which are taken from standard table. Therefore, from the given values of all parameters on right hand side, the specific latent heat of fusion of ice (L_i) which is to be determined can be calculated.

Workout Example:

Q. Evaporation of perspiration is an important mechanism for temperature control of warm-blooded animals. What mass of water must be evaporated from the surface of an 80 kg human body to cool it by 1°C ? The specific heat capacity of the human body is approximately $420 \text{ J kg}^{-1}\text{K}^{-1}$, and the latent heat of vaporization of water at the body temperature is $2.4 \times 10^6 \text{ J kg}^{-1}$.

Solution: Here,

Heat liberated from the surface of an 80 kg human body (having Sp. heat capacity $0.1 \text{ cal g}^{-1}^\circ\text{C}^{-1}$) to cool by 1°C is given by, $Q_1 = m s \Delta\theta = 80 \times 420 \times 1 = 33600 \text{ J}$

Let m_w be the mass of water that is evaporated from the human body. Heat required to evaporate is given by, $Q_2 = m_w L = m_w \times 2.4 \times 10^6 = 2.4 \times 10^6 m_w$

Here, $L = 577^\circ\text{C cal g}^{-1}$ is the latent heat of vaporization of water at the body temperature.

According to question, $Q_1 = Q_2$

$$\text{or, } 33600 = 2.4 \times 10^6 m_w$$

$$\text{or, } m_w = \frac{33600}{2.4 \times 10^6} = 1.39 \times 10^{-2} \text{ kg.}$$

Hence, 1.39×10^{-2} kg mass of water must evaporate from the surface of given human body.

Q. How much heat is required to convert 10 g ice at -10°C into steam at 100°C ? (Sp. heat capacity of ice is $2100 \text{ J kg}^{-1}\text{K}^{-1}$, Sp. heat capacity of water is $4200 \text{ J kg}^{-1}\text{K}^{-1}$, latent heat of vaporization of water is $2.26 \times 10^6 \text{ J kg}^{-1}$ and latent heat of fusion of ice is $3.36 \times 10^5 \text{ J kg}^{-1}$).

Solution: Here,

$$\text{Mass of ice } (m_i) = 10 \text{ g} = 10^{-2} \text{ kg}$$

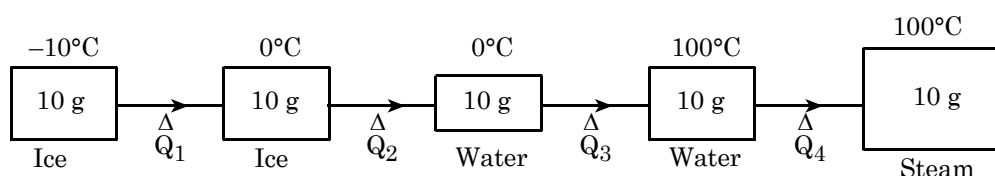
$$\text{Sp. heat of ice } (S_i) = 2100 \text{ J kg}^{-1}\text{K}^{-1}$$

$$\text{Latent heat of fusion of ice } (L_i) = 3.36 \times 10^5 \text{ J kg}^{-1}$$

$$\text{Latent heat of vaporization of water } (L_v) = 2.26 \times 10^6 \text{ J kg}^{-1}$$

$$\text{Specific heat capacity of water } (S_w) = 4200 \text{ J kg}^{-1}\text{K}^{-1}$$

Heat required (Q) = ?



Now, heat required to change temperature of 10 g ice from -10°C to 0°C is given by

$$Q_1 = m_i s_i \Delta\theta = 10^{-2} \times 2100 \times [0 - (-10)] = 210 \text{ J}$$

Amount of heat required to convert ice at 0°C into water at 0°C is given by

$$Q_2 = m_i L_i = 10^{-2} \times 3.36 \times 10^5 = 3.36 \times 10^3 \text{ J}$$

Amount of heat required to convert water at 0°C to water at 100°C is given by

$$Q_3 = m_i s_w \Delta\theta = 10^{-2} \times 4200 \times 100 = 4200 \text{ J}$$

Amount of heat required to convert water at 100°C to steam at 100°C is given by

$$Q_4 = m_i L_v = 10^{-2} \times 2.26 \times 10^6 = 2.26 \times 10^4 \text{ J}$$

Therefore, the total amount of heat required is given by

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = (210 + 3.36 \times 10^3 + 4200 + 2.26 \times 10^4) \text{ J} = 3.02 \times 10^4 \text{ J}$$

Hence, $3.02 \times 10^4 \text{ J}$ heat is required.

Q. If, 50 g of ice at -6°C is dropped into water at 0°C , how many grams of the water freeze? (Sp. heat capacity of ice = $2000 \text{ J kg}^{-1}\text{K}^{-1}$, latent heat of fusion of ice $3.36 \times 10^5 \text{ J kg}^{-1}$)

Solution: Here,

$$\text{Initial temperature of ice } (\theta_i) = -6^\circ\text{C}$$

$$\text{Final temperature of ice } (\theta_f) = 0^\circ\text{C}$$

$$\text{Mass of ice } (m_i) = 50 \text{ g}$$

$$\text{Sp. heat of ice } (s_i) = 2000 \text{ J kg}^{-1}\text{K}^{-1}$$

$$\text{Mass of water that can freeze } (m_w) = ?$$

Heat gained by ice at -6°C when it gets converted to ice at 0°C is given by

$$Q_1 = m s \Delta\theta = \left(\frac{50}{1000}\right) \times 2000 \times [0 - (-6)] = 600 \text{ J}$$

The heat lost by water during freezing is given by

$$Q_2 = m_w L = m_w \times 3.36 \times 10^5$$

According to principle of calorimetry, $Q_1 = Q_2$

$$\text{or, } 600\text{J} = m_w 3.36 \times 10^5$$

$$\text{or, } m_w = \frac{600}{3.36 \times 10^5} = 1.786 \times 10^{-3} \text{ kg} = 1.8 \times 10^{-3} \text{ kg}$$

Hence $1.786 \times 10^{-3} \text{ kg}$ water freezes.

Q. From what height a block of ice must be dropped so that it can completely melt.

It is assumed that 20% of energy of fall is retained by ice. [$L = 3.36 \times 10^5 \text{ J kg}^{-1}$, $g = 10 \text{ ms}^{-2}$]

Solution: Here,

Acceleration due to gravity (g) = 10 ms^{-2}

Latent heat of fusion of ice (L) = $3.36 \times 10^5 \text{ J kg}^{-1}$

Suppose h be the height from where a block of ice is dropped so that it can melt by taking 20% of its loss in P.E. ($= m g h$) as latent heat. Let m is mass of ice and g is acceleration due to gravity.

According to the question,

$$m L = 20\% \text{ of } m g h$$

$$\text{or, } L = \frac{20}{100} \times g \times h$$

$$\text{or, } h = \frac{L \times 100}{g \times 20} = \frac{3.36 \times 10^5 \times 100}{10 \times 20} = 168000 \text{ m} = 1.7 \times 10^5 \text{ m}$$

Hence, the block of ice should be dropped from the height of $1.7 \times 10^5 \text{ m}$.

Q. What is the result of mixing 10 g of ice at 0°C into 15 g of water at 20°C in a vessel of mass 100 g and specific heat $378 \text{ J kg}^{-1} \text{ K}^{-1}$? (Latent heat of fusion of ice (L_i) = $3.36 \times 10^5 \text{ J kg}^{-1}$, Sp. heat capacity of water $4200 \text{ J kg}^{-1} \text{ K}^{-1}$)

Solution: Here,

Mass of ice (m_i) = $10 \times 10^{-3} \text{ kg}$

Mass of water (m_w) = $15 \times 10^{-3} \text{ kg}$

Mass of vessel (m_v) = $100 \times 10^{-3} \text{ kg}$

Temperature of ice (θ_i) = 0°C

Temperature of vessel with water (θ) = 20°C

Specific heat of water (S_w) = $4200 \text{ J kg}^{-1} \text{ K}^{-1}$,

Latent heat of fusion of ice (L_i) = $3.36 \times 10^5 \text{ J kg}^{-1}$

Specific heat of vessel (S_v) = $378 \text{ J kg}^{-1} \text{ K}^{-1}$

Now, heat lost by water and vessel when the temperature decreases from 20°C to 0°C is,

$$Q_l = m_w S_w (\theta - 0) + m_v S_v (\theta - 0) = 15 \times 10^{-3} \times 4200 \times (20 - 0) + 100 \times 10^{-3} \times 378 \times (20 - 0) = 2016 \text{ J}$$

Heat required to melt 10 g of ice is,

$$Q_r = m_i L_i = 10 \times 10^{-3} \times 3.36 \times 10^5 = 3360 \text{ J}$$

Here, $Q_l < Q_r$. This suggests total ice cannot melt and whole mixture remains at 0°C . If m be the amount of ice that melts by Q_l amount of heat then,

$$Q_l = m L_i$$

$$\text{or, } m = \frac{Q_l}{L_i} = \frac{2016}{3.36 \times 10^5} = 6 \times 10^{-3} \text{ kg}$$

Hence mixture contains $(10 - 6) = 4 \text{ g}$ of ice and $(15 + 6) = 21 \text{ g}$ of water all at 0°C .

Heat gained by ice is used to change the solid phase of ice into liquid phase (water) and to raise the temperature of water so formed up to $\theta_2^\circ\text{C}$ from 0°C .

Sample Questions:

1. 10 g of steam at 100°C is passed into a mixture of 100 g of water and 10 g of ice at 0°C , find the resulting temperature of the mixture. [Specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$, latent heat of fusion of ice is $3.36 \times 10^5 \text{ J kg}^{-1}$, and latent heat of vaporization of water is $2.26 \times 10^6 \text{ J kg}^{-1}$] [46.67°C]
2. A mixture of 500 g of water and 100 g of ice at 0°C are kept in a copper calorimeter of mass 200 g. How much steam from the boiler is passed to the mixture so that temperature of the mixture reaches to 40°C ? [Specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$, latent heat of fusion of ice is $3.36 \times 10^5 \text{ J kg}^{-1}$, and latent heat of vaporization of water is $2.26 \times 10^6 \text{ J kg}^{-1}$, and specific heat capacity of copper is $386.4 \text{ J kg}^{-1} \text{ K}^{-1}$] [54.56 g]

11.14 Boiling

A fast and noisy process of conversion of liquid into vapour at a constant temperature is called boiling. During boiling, liquid takes certain amount of heat as latent heat. The temperature at which liquid changes into vapour (gas) is called the boiling point. Different objects have different values of boiling point. Boiling point of water is 100°C at 760 mm of Hg pressure.

Effect of Pressure on Boiling Point

Even for a single object, the boiling point varies according to the pressure exerted on it. The actual boiling point of a liquid is the temperature at which the saturated vapor pressure of the liquid becomes equal to the external (atmospheric) pressure on its surface. The boiling point of a liquid increases with an increase in pressure, and decreases with a decrease in pressure. Further, pressure decreases with an increase in altitude, and increases with a decrease in altitude. Therefore, the boiling point is a function of pressure, which, in turn, is a function of altitude. For example, water boils at a lower temperature in Solukhumbu than in Birgunj, and rice is cooked faster in a pressure cooker than in an open pot.

Sample Question:

Rice is cooked faster in a pressure cooker than in an open pot, why?

11.15 Evaporation

We feel cold when we spray perfume on our body. Similarly, we experience a cooling sensation in our mouth when we eat Halls. Can you explain the physics behind these phenomena? Both cases involve the cooling effect of evaporation.

A slow and silent process of conversion of liquid into vapour at all temperature below its boiling point is called evaporation. Evaporation takes place from the surface of a liquid.

During evaporation, molecules escape from the surface of a liquid, and convert into vapour. Those molecules take necessary latent heat of conversion from the neighboring molecules and hence the temperature of the remaining mass of liquid decreases. This effect is called cooling effect of evaporation.

Some examples of Evaporation

- After sweating, we feel cool due to the evaporation of the sweat from our body's heat.
- Washed or wet clothes are dried by evaporation. When the clothes are hung outside in the sun, due to the heat of the sun, the water droplets present in the clothes get dried away due to evaporation.
- Spirits are also compounds that evaporate at a very low temperature. Hence it is used to sterilize the skin where a syringe for vaccines or blood collection needs to be introduced.
- Evaporation occurs in the water cycle in which the water from any water body on the surface of the earth is evaporated into the air. This water when cools down causes rain.
- A person feels cold only while coming out in open air after having a bath in the bathroom but not inside the bathroom.

Differences between Evaporation and Boiling

SN	Evaporation	Boiling
1.	Evaporation takes place from free surface of liquid.	Boiling occurs throughout the volume of the liquid.
2.	Cooling effect is observed during evaporation.	No cooling effect is observed during boiling.
3.	Evaporation takes place at all temperatures.	Boilingt takes place only at the boiling point.

Sample Questions:

- Clothes dry faster when wind is blowing, why?
- How does evaporation of sweats from skins control the temperature of warm blooded animals?

11.16 Effect of Pressure on Melting Point of Substance

Do you Know?

- An avalanche originates when a large amount of snow collects at a location.
- A large iceberg melts at the base but not at the top.

Same physics is behind these two phenomena. In both activities, main role is of pressure. Melting point of solid is greatly affected by pressure applied on it. The effect of pressure on melting point is different for different substances.

For the substances like cast iron and ice whose volume **decreases** while melting, their

melting point **decreases** with increase in pressure.

For the substances like wax and lead whose volume **increases** while melting, their melting point **increases** with increase in pressure.

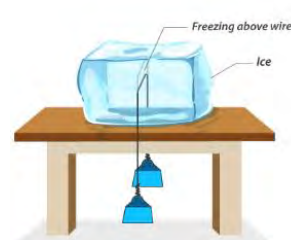
When two pieces of ice are pressed against each other, pressure on the contact portion increases and hence melting point decreases to that portion. As a result, ice melts in contact portion. If applied pressure is removed again, the melting point increases to initial value. Now water, so formed after melting of ice, freezes. During this process two pieces of ice join to form a single piece. Hence, when pressure is applied to ice, it melts and as soon as the pressure is removed, it freezes again. We refer to this phenomenon as *regelation*. *Regelation is the phenomenon of melting of ice under the application of pressure/compression and its freezing after the removal of applied pressure.*

Sample Questions:

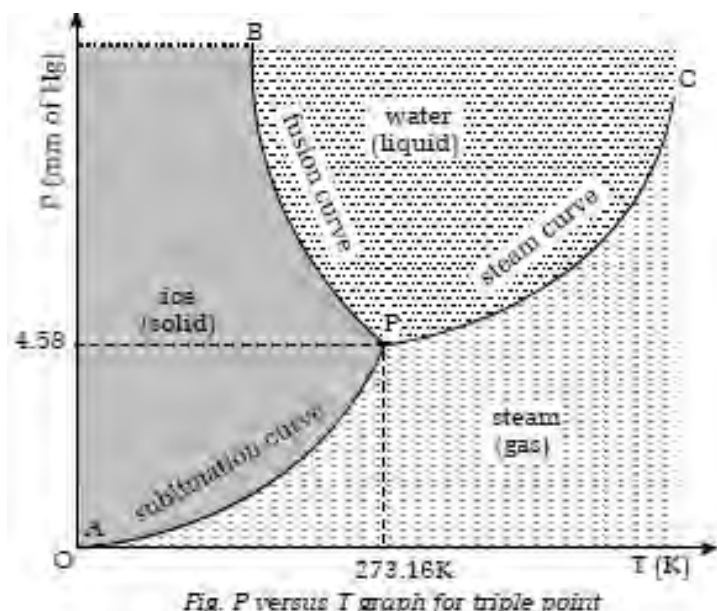
1. Are the freezing point and boiling point of water other than 0°C and 100°C ?
2. When, a fine wire supporting two heavy loads is passed over a block of ice placed across two supports, the wire finds its way through the ice block without cutting into two pieces. Explain why?

How does an avalanche occur in mountainous regions?

3. Why is it difficult to walk on the surface of a frozen lake?



11.17 Phase Diagram and Triple Point



[Fig. 11.8, phase diagram for water]

A graph between pressure and temperature which has three curves; namely fusion curve, vaporization curve and sublimation curve is called phase diagram. It is also called P-T diagram.

The phase diagram clearly explains the three different phases of a matter. The phase diagram of water is shown in figure 11.8.

- a. **Vaporization curve (steam line):** The graph showing variation of boiling point of a substance with the pressure is called vaporization curve. The substance exists in liquid state above the steam line and it exists in gaseous state below steam line.
- b. **Fusion curve (Ice line):** The graph showing the variation of the melting point of substance with pressure is called fusion curve. A substance exists in solid state above the ice line and it exists in liquid state below ice line.
- c. **Sublimation curve (Hoar-frost line):** The graph showing the variation of sublimation point of substance with pressure is called sublimation curve or hoar-frost line. At very low temperature and pressure, substances are found to sublime. For example, solid carbon dioxide (dry ice) sublimates even at atmospheric pressure. A substance exists in solid state above the hoar-frost line and it exists in gaseous state below hoar-frost line.

Triple Point

Triple point of a substance is the point in P-T diagram representing a particular value of temperature and pressure at which all three states of that substance can co-exist.

Triple point of a substance can also be defined as the point in phase diagram at which vaporization curve, fusion curve and sublimation curve of given substance intersect each other. The triple point of water is (0.006 atm., 0.01°C) and that of carbon dioxide is (5.11 atm., – 56.6°C)

Exercise

1. Which of the followings is correct about a substance having low specific heat capacity?
 - a. Gets heated quickly and stores for a long time.
 - b. Gets heated slowly and stores for a short time.
 - c. Gets heated quickly and stores for a short time.
 - d. Gets heated slowly and stores for a long time.
2. What is the value of the specific heat capacity of water during boiling?
 - a. 0
 - b. 0.5
 - c. 1
 - d. infinite
3. What must be the difference in the temperature between a body and its surroundings for Newton's law of cooling to be valid?
 - a. small
 - b. large
 - c. very large
 - d. any value
4. Which of the followings produces severe burn?
 - a. boiling water
 - b. steam
 - c. hot air
 - d. normal water
5. A 10kg iron bar (specific heat $462 \text{ J kg}^{-1}\text{K}^{-1}$) at 80°C is placed on a block of ice (the latent heat of fusion = $3.36 \times 10^5 \text{ J kg}^{-1}$). How much ice melts?
 - a. 1.1kg
 - b. 10kg
 - c. 16kg
 - d. 60kg
6. Three liquids A, B and C are at temperatures of 10°C , 25°C and 40°C respectively. When A and B are mixed, the final temperature of the mixture is 15°C . When B and C are mixed,

- the final temperature is 30°C . What is the final temperature of the mixture when A and C are mixed?
- a. 16°C b. 12°C c. 8°C d. 4°C
7. The liquids of the same volume have densities in the ratio of 1:3, and the specific heat in the ratio is 3:1. What will be the ratio of their heat capacities?
- a. 1.1 b. 1.3 c. 1.9 d. 9.1
8. Which one is correct about the melting point of ice?
- a. Decreases with a decrease in pressure.
b. Increases with an increase in pressure.
c. Is independent of pressure.
d. Decreases with an increase in pressure.
9. The latent heat of the fusion of ice is $3.36 \times 10^5 \text{ J kg}^{-1}$. The specific heat capacity of ice is $2100 \text{ J kg}^{-1} \text{ K}^{-1}$. What is the heat required to change 1kg of ice from -10°C to 50°C ?
- a. $5.04 \times 10^5 \text{ J}$ b. $5.67 \times 10^5 \text{ J}$ c. $6.51 \times 10^5 \text{ J}$ d. $7.56 \times 10^5 \text{ J}$
10. When two blocks of ice are pressed together, they join to form one block. What may be the reason behind it?
- a. due to heat production
b. due to an increase in specific heat capacity
c. due to a decrease in melting point of ice
d. due to an increase in the melting point of ice
11. Two substances have specific heat $s_1 \text{ J kg}^{-1} \text{ K}^{-1}$ and $s_2 \text{ J kg}^{-1} \text{ }^{\circ}\text{F}$. Which of the following relation is valid?
- a. $s_1 = s_2$ b. $s_1 < s_2$ c. $s_1 \leq s_2$ d. $s_1 > s_2$
12. Water falls from a height of 84m. Half of the KE of falling water converts into heat energy ($g=10 \text{ ms}^{-2}$). What is the rise in temperature of falling water on striking the surface?
- a. 0.10°C b. 0.20°C c. 0.30°C d. 0.40°C
13. Two liquids are at temperatures 20°C and 40°C . When the same mass from both of them is mixed, the temperature of the mixture is 32°C . What is the ratio of their specific heat capacities?
- a. $\frac{1}{3}$ b. $\frac{2}{3}$ c. $\frac{2}{5}$ d. $\frac{3}{2}$
14. 10g of ice at -20°C is dropped into a calorimeter containing 10g of water at 10°C . The specific heat of water is twice that of ice. Which of the following is the content of the calorimeter at equilibrium?
- a. 10g of ice and 10 g of water
b. 20g of water
c. 5g of ice and 15g of water
d. 20g of ice
15. 1kW immersion heater can heat 1l water in a kettle for 5 minutes. If the temperature is raised by 60°C , what is the value of the heat capacity of the kettle?
- a. 400 JK^{-1} b. 600 JK^{-1} c. 800 JK^{-1} d. 1000 JK^{-1}

QN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	c	d	a	b	a	a	a	d	b	c	d	a	b	a	c

16. (a) What do you mean by the statement “The specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$ ”?

(b) Why do we fry raw peanuts in a sand bath?

(c) When an 80 kg man suffers from the flu, he has a fever of 39.0°C instead of the normal body temperature of 37.0°C . Assuming that the human body is mostly water, how much heat is required to raise his temperature by this amount? (Sp. Heat of water $= 4200 \text{ J kg}^{-1} \text{ K}^{-1}$)
 $[6.7 \times 10^5 \text{ J}]$

17. (a) State and explain Newton’s law of cooling.

(b) A body initially at 100°C cools to 60°C in 5 minutes, and to 40°C in 10 minutes. What will be the temperature of the body at the end of 15 minutes? $[30^\circ\text{C}]$

18. The figure below represents the plot of temperature versus energy added when 1.00g of ice initially at -30°C is converted to steam at 120°C .

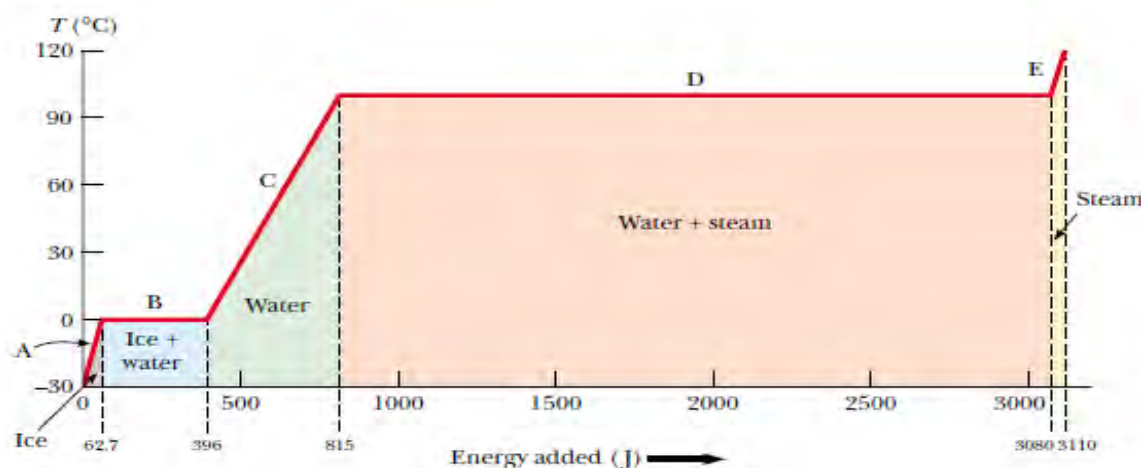
(a) What does the portion B and D represent?

(b) Are the slopes of portions A, C and E equal or different, and why?

(c) Why is the length of portion D more than that of portion B?

(d) What are the temperatures corresponding to the portion B and D called?

(e) From the given information, calculate the values of the specific latent heat of fusion of ice and the specific latent heat of vaporization of water.



19. (a) To raise the temperature of an object, do you need to add heat to it? If you add heat to an object, do you need to raise its temperature? Explain.

(b) A nail driven into a board increases the temperature. If we assume that 60% of the kinetic energy delivered by a 1.80 kg hammer with a speed of 7.80 ms^{-1} is transformed into heat that flows into the nail and does not flow out, what is the temperature increase of an 8.00 g aluminum nail after it is struck ten times? (Sp heat of aluminum $= 910 \text{ J kg}^{-1} \text{ K}^{-1}$)
 $[45.1^\circ\text{C}]$

20. (a) A beaker of water at room temperature is placed in an enclosure, and the air pressure in the enclosure is slowly reduced. When the air pressure is reduced sufficiently, the water begins to boil, but the temperature of the water does not rise. In fact, the temperature drops slightly, why?
- (b) What is the resulting temperature of the mixture of 1g of steam at 100°C and 5 g of ice at 0°C? [Temperature of 6g of water is at 40 °C]
21. (a) Explain the significance of latent heat in the process of changing phases of matter, and how it influences the temperature during phase transitions.
- (b) Design an experiment to calculate the energy required to convert 1 kg of ice from the solid state to the liquid state. Outline the steps of your experiment, including the materials and equipment needed, the variables to be controlled and the measurements to be taken.
- (c) Finally, propose potential interpretations of the results, and their implications for material science.
- (d) Discuss the relationship between pressure and the melting point of substances.
22. 1 kW immersion heater can heat one litre of water in a kettle for 5 minutes. If the temperature rises by 60°C, calculate the heat capacity of the kettle in J K⁻¹. (Sp. Heat of water = 4200 J kg⁻¹K⁻¹)
[0.8 × 10³ JK⁻¹]
23. A lead bullet strikes a target with a velocity of 500 ms⁻¹. Calculate the rise in temperature of the bullet assuming that 40% of the heat produced is used in heating the bullet. [Specific heat of lead = 126 J kg⁻¹K⁻¹]
[396.8°C]
24. You are designing an electronic circuit element made of 23 mg of silicon. The electric current through it can add energy at the rate of 7.4 × 10⁻³ Js⁻¹. If your design doesn't allow any heat transfer out of the element, at what rate does its temperature increase? (Specific heat of silicon is 705 J kg⁻¹K⁻¹.)
[0.46 K s⁻¹]
25. A geologist working in the field drinks her morning coffee out of an aluminum cup. The cup has a mass of 0.120 kg and initially is at 20.0°C. When she pours 0.300 kg of coffee initially at 70.0°C what is the final temperature after the coffee attains thermal equilibrium? (Assume that coffee has the same specific heat as water, and that there is no heat exchange with surroundings).
[66.0°C]

Chapter 12

Rate of Heat Flow

Josef Stefan was born to Slovenian parents in St. Peter, Austria, and he graduated from the university of Vienna in 1857. At the University of Vienna, he rose from lecturer in mathematical physicist in 1858 to professor Ordinarius of physics in 1863 and to the director of the Physical Institute in 1866. Five years after he derived his law of black body radiation empirically, it was derived theoretically by Ludwig Boltzmann of Austria, and hence became known as the Stefan-Boltzmann law.

Stefan-Boltzmann law states that the radiant energy of a black body (a theoretical object that absorbs all radiation that falls on it) is proportional to the fourth power of its absolute temperature. This law was one of the first important steps toward the understanding of the black body which gave the base for the quantum idea of radiation.

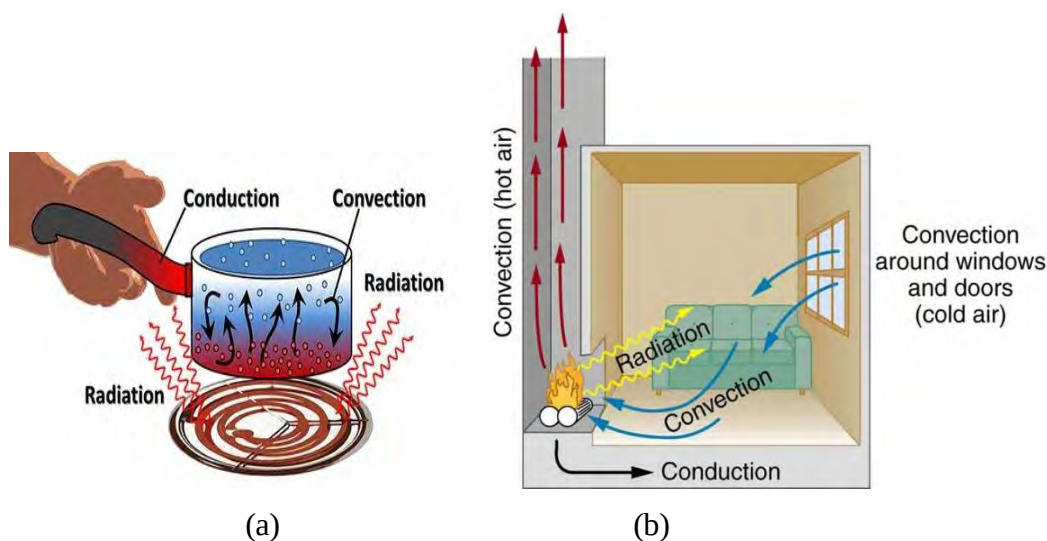


Josef Stefan (1835-1893)

Do you know?

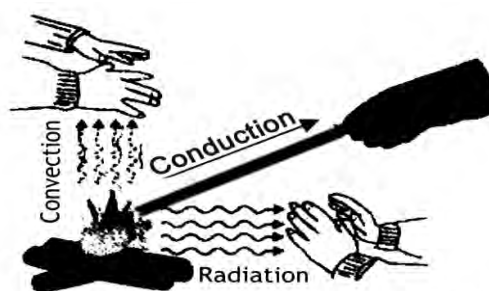
1. When a coil of copper is kept at a certain distance above the flame of a candle, the candle keeps burning, but when the coil is placed over the flame, it is extinguished.
2. Tanks of gasoline are generally painted with aluminum paints.
3. The earth constantly receives heat radiations from the sun and gets warmed up.
4. Water can be boiled in a paper cup.

12.1 Introduction



[Fig:12.1 Methods of heat transfer]

We have many examples in our periphery where heat is continuously being transferred from one body to another. Heat always flows till the temperature difference is maintained, and cannot be confined even with the support of the best conductor or the best insulator. That means naturally heat always flows from a body of a higher temperature to a body of a lower temperature. The expense of heat energy from a hotter body will result in the warming of a colder body.



[Fig. 12.2, Different modes of transfer of

There are three different methods of transfer of heat. For example; when one end of a metallic rod is heated, the other end also gets heated. When a liquid in a vessel is heated from the bottom, the upper surface of the liquid also gets heated. We receive the heat from a very hot object, and from the sun directly.

Transfer of heat due to molecular vibrations in a solid as in the case of a metallic rod is called conduction. Transfer of heat due to actual molecular translation in fluid is called convection. Boiling of water is an example of convection. Transfer of heat without the support of any material medium is called radiation e.g., receiving heat from the sun.

Project work:1

Visit your locality. Observe and list out the spots where the activities of heat transfer are carried out. Discuss among your friends and categorize the method of transfer of heat; conduction, convection, and radiation. Interact with your teacher for the conclusion.

SN	Spot where heat-related devices are used.	Method of transfer of heat	Mechanism of transfer of heat	Remarks
1.	Heating of fry pan in kitchen	Conduction	Molecular vibration	
2.				
3.				
4.				
5.				

12.2 Conduction

When one end of a solid is heated, atoms in this hotter region vibrate with greater amplitude and have more kinetic energy than the neighboring atoms in the colder region. The atoms in the hotter region collide with the neighboring atoms in the colder region. During the process, hotter molecules give up some energy to colder molecules. The same phenomena occur between each set of neighboring atoms and the process continues up to the next end. This method of heat transfer is called conduction. Conduction also takes place between two different bodies if they are in thermal contact.

Therefore, conduction is defined as the process of transfer of heat from one point to another point of a body, carried out by means of collisions between rapidly vibrating atoms in the hotter region and slowly vibrating atoms in the colder region. There is no

actual transfer of particles during conduction.

Material medium is required for the conduction. Molecular collisions enhanced by molecular vibrations are the means of conduction of heat.

The ability of anybody to conduct heat is measured in terms of **thermal conductivity**. Generally, metals have a high value of thermal conductivity. Therefore, metals are called good conductors of heat. The materials which do not conduct heat are called bad conductors. For example; paper, air, plastic, wood, etc.

Examples and Applications of Thermal Conduction in Our Daily Life

1. Two thin jackets are warmer than a single jacket of the same thickness because the air particles trapped between two jackets minimize the conduction.
2. The Eskimo houses are double-walled so that the air trapped between two walls behaves as a bad conductor, and prevents the flow of heat from inside to outside.
3. In cold countries, windows have two panes of glass with a thin layer of air in between.
4. Thick walls are constructed in cold storage rooms.
5. To remove a stopper fitted tightly to the bottle, the neck is immersed in warm water and gently heated. The glass neck being a bad conductor expands but does not conduct heat to the stopper. So that the stopper can be removed easily.
6. Sauce pans, hot water buckets, kettles, etc. are made of metal but their handles are of wooden or ebonite or Bakelite so that heat from utensils cannot conduct to hand.
7. Woolen clothes have fine pores filled with air. Air and wool both are bad conductors of heat. So that the flow of heat from the body to outside the atmosphere is prevented. Therefore, woolen clothes provide good insulation and keep our body warm in winter.
8. In a thermos bottle, a vacuum is created between two surfaces to avoid the transfer of heat due to conduction.
9. We feel iron chairs colder than wooden chairs in winter.
10. Ice is packed in saw-dust inside double-walled ice box wrapped in gunny bags to prevent from melting.
11. Quilts are filled with fluffy cotton. Cotton can trap air in its pores. Both cotton and air are bad conductors and provide good insulation. As a result, the flow of heat is prevented, and keeps our body warm. A newly built quilt can trap more air as it contains more pores than an old one. Therefore, a newly built quilt is warmer than an old one.

Sample Questions:

1. When water is placed in an ice-cube tray in a freezer, why does the water not freeze all at once when the temperature has reached the freezing point? In fact, the water freezes first in the layer adjacent to the sides of the tray, why?
2. Why is snow a better heat-insulator than ice?
3. A plank of wood lying in the sun absorbs more heat than a shiny metal does. But the wood feels less hot than the metal when you pick it up. Explain.
4. You are requested to sit on a chair at a wedding party in a cold winter evening. Would you like to choose a metal chair or a wooden chair?
5. Why does a piece of paper wrapped tightly on a wooden rod burn more quickly when held over a flame than a similar piece wrapped on a metal rod?

6. At what condition will a piece of metal and a piece of wood feel equally hot to us while touching?
7. Why are mud-houses colder in summer and warmer in winter?

12.3 Temperature Gradient

A temperature gradient is a physical quantity that describes the direction and the rate with which temperature changes the most rapidly around a particular location. It is defined as *the rate of change in temperature with displacement in a given direction*. Symbolically, the temperature gradient is expressed as $\frac{d\theta}{dx}$. The SI unit of temperature gradient is Km^{-1} . Here $d\theta$ is the small change in temperature over a small distance dx . The temperature gradient is a vector quantity.

Temperature Gradient for a Conducting Rod

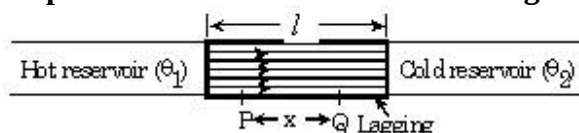
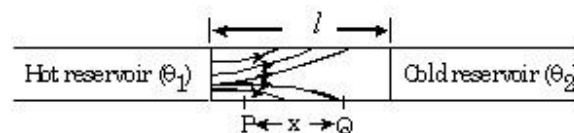
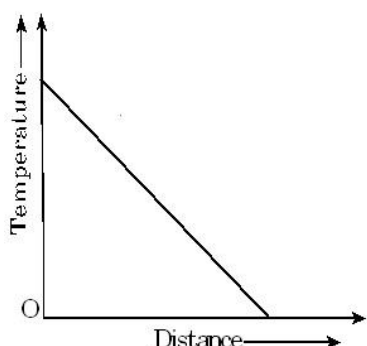


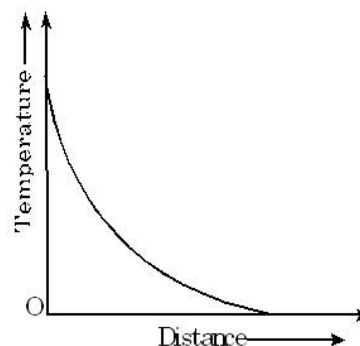
Fig. 12.3 (a), Flow of heat through a lagged conducting rod]



[Fig. 12.3. (b) Flow of heat through an unlagged conducting rod]



(c)



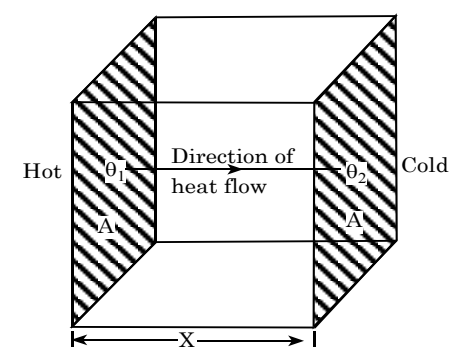
(d)

Suppose a conducting rod of length l and well lagged from its curved surface as in figure 12.3 (a). Let one end of the rod be in thermal contact with a hot reservoir maintained at temperature θ_1 , and another end is in thermal contact with a cold reservoir maintained at temperature θ_2 . At steady state conditions, a graph of temperature along the length versus corresponding distance is the straight line as in figure 12.3 (c). In this case the temperature gradient is uniform $\frac{\Delta\theta}{\Delta x} = \frac{\theta_1 - \theta_2}{l} = \text{constant}$. But for an unlagged conducting rod as in figure 12.3(b), the temperature gradient is not uniform, and the temperature falls exponentially as shown in fig 12.3 (d).

12.4 Thermal Conductivity

Thermal conductivity refers to the ability of a given material to conduct or transfer heat. Thermal conduction generally occurs in solids.

Suppose a parallel-sided conducting slab of length x as shown in figure 12.4 has a uniform cross-section area A such that its first face is at higher temperature θ_1 and next face is at lower temperature θ_2 . Heat flows from the hotter face to the colder face. If loss of heat from the side of the slab is made zero, then, at equilibrium, the rate of heat flow across the opposite faces $\left(\frac{Q}{t}\right)$ is,



[Fig. 12.4, Thermal conduction]

- i. directly proportional to the temperature gradient.

$$\text{i.e., } \frac{Q}{t} \propto \frac{\theta_1 - \theta_2}{x} \quad \dots (1)$$

- ii. directly proportional to area of cross-section of each face,

$$\frac{Q}{t} \propto A \quad \dots (2)$$

Combining equations (1) and (2) we get,

$$\text{or, } \frac{Q}{t} \propto \left(\frac{\theta_1 - \theta_2}{x}\right) A$$

$$\text{or, } \frac{Q}{t} = k \left(\frac{\theta_1 - \theta_2}{x}\right) A \quad [\text{Where } k \text{ is proportionality constant.}]$$

$$\text{or, } k = \left(\frac{Q}{t}\right) \frac{1}{A} \left(\frac{x}{\theta_1 - \theta_2}\right) \quad \dots (3)$$

$$\text{or, } k = \frac{\left(\frac{Q}{t}\right)}{A \left(\frac{\theta_1 - \theta_2}{x}\right)}$$

k is also known as coefficient of thermal conduction or conductivity of the material of the slab, and its value depends on the nature of the material.

If we take a condition that $A = 1 \text{ m}^2$, $\frac{\theta_1 - \theta_2}{x} = 1 \text{ K m}^{-1}$, then, $k = \frac{Q}{t}$

Hence, the coefficient of thermal conduction or thermal conductivity (k) is defined as the rate of flow of heat per unit cross-section area along a bar maintained at a unit temperature gradient at a steady state.

$$\text{SI unit of } k = \frac{\text{J} \times \text{m}}{\text{m}^2 \times \text{K} \times \text{S}} = \text{J S}^{-1} \text{m}^{-1} \text{K}^{-1} = \text{W m}^{-1} \text{K}^{-1}$$

$$\text{Dimensions of } k = \text{Dimensions from } \text{W m}^{-1} \text{K}^{-1} = \frac{[\text{ML}^2 \text{T}^{-2} \times \text{L}]}{[\text{L}^2 \text{K T}]} = [\text{MLT}^{-3} \text{K}^{-1}]$$

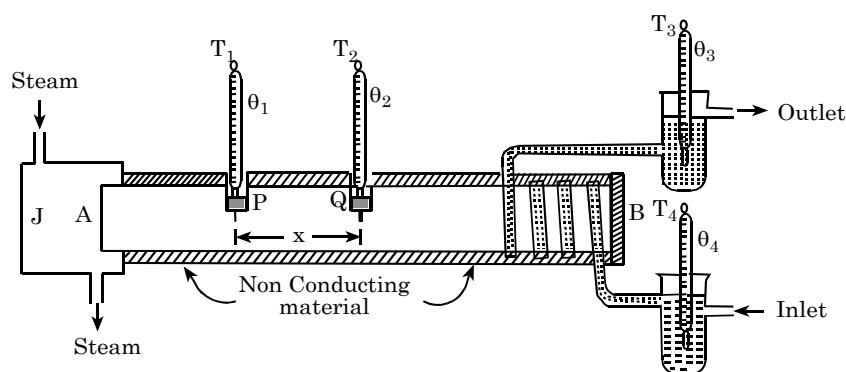
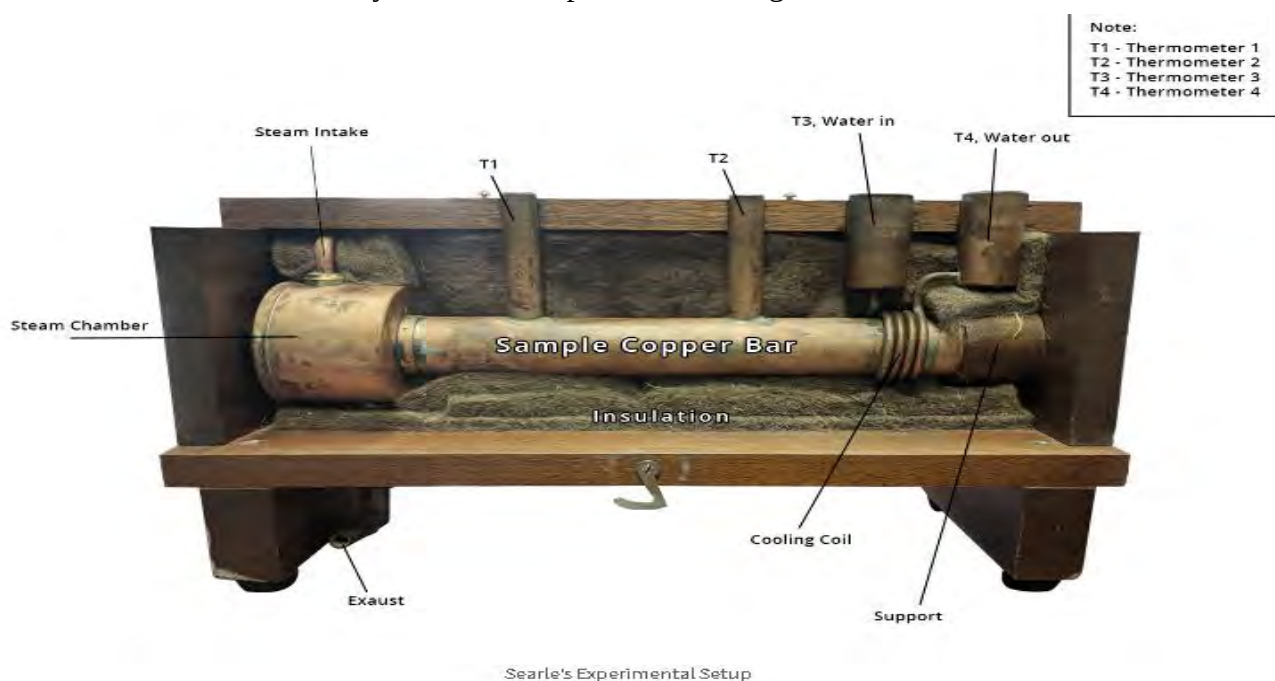
Sample Questions:

1. Birds often puff up their feathers in winter, why?
2. During winter, animals curl into a ball, why?
3. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to 300°C . Which of these will cool fastest and which one slowest, when left in air at room temperature? Give reasons.

12.5 Determination of Thermal Conductivity of Good Conductor; Searle's Method

The coefficient of thermal conductivity of a body is defined as the amount of heat flowing per second at a steady state across the opposite faces of a unit cube maintained at unit temperature difference.

The thermal conductivity of a conducting solid in the form of a rod can be determined by Searle's method in the laboratory from the setup as shown in figure 12.5.



[Fig. 12.5, Searle's Apparatus]

Suppose, there is a metal rod AB whose thermal conductivity (k) is to be determined. Except for cross-section at A and B, the whole rod is kept inside an insulated box to prevent any loss

of heat from its sides. The rod taken has a uniform cross-section (say A) and it has two cavities P and Q at a distance x apart as in figure 12.5. Thermometers T_1 and T_2 respectively are inserted in cavities P and Q . The cavities P and Q are filled with mercury so that accurate measurement of the temperature can be achieved. Also, steam is circulated continuously at end A whereas a cooling pipe of copper is wound near end B , and cold water is circulated continuously through it. The flowing water absorbs heat from the rod, and becomes warm. The warm water is collected into a vessel. Two thermometers, T_4 and T_3 are used to measure the temperature of cold water used, and warm water collected in the vessel respectively. The mass of water collected for a given time is measured. Let t be the time for which heat flows from point P to Q to reach to a steady state.

After the circulation of steam, the temperature readings on thermometer T_1 , T_2 and T_3 increase continuously. When a steady state of heating is achieved, the rate of heat flow through any cross-section of the rod is the same, and all the thermometers show a constant temperature readings.

If θ_1 , θ_2 , θ_3 and θ_4 are the respective steady-state temperature readings of thermometers T_1 , T_2 , T_3 and T_4 then,

$$\text{Rate of flow of heat from P to Q, } \left(\frac{Q}{t}\right)_1 = \frac{kA(\theta_1 - \theta_2)}{x} \dots (1) \quad [\theta_1 > \theta_2]$$

Again, if m and S be the mass and specific heat capacity of water respectively then,

$$\text{Rate of heat absorbed by water, } \left(\frac{Q}{t}\right)_2 = \frac{ms(\theta_3 - \theta_4)}{t} \dots (2) \quad [\theta_3 > \theta_4]$$

Working principle;

At a steady state, the rate of heat flow from point P to Q must be equal to the rate of absorption of heat by warm water collected.

$$\text{i.e. } \left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2$$

$$\text{or, } \frac{kA(\theta_1 - \theta_2)}{x} = \frac{ms(\theta_3 - \theta_4)}{t}$$

$$\therefore k = \frac{smx(\theta_3 - \theta_4)}{At(\theta_1 - \theta_2)}$$

In this expression, all the parameters on RHS are measured. Hence the thermal conductivity k of the rod can be determined by Searle's method.

Workout Examples:

Q. Estimate the rate at which ice would melt in a wooden box 2.5cm thick of inside measurement $100\text{cm} \times 60\text{cm} \times 40\text{cm}$ assuming that the external temperature is 35°C and thermal conductivity of wood is $0.168\text{ Wm}^{-1}\text{K}^{-1}$.

✎ Solution: Here,

✎ Thickness of wooden box (x) = $2.5\text{ cm} = 2.5 \times 10^{-2}\text{ m}$

Internal temperature (θ_2) = 0°C

Total surface area of the walls of the wooden box is,

$$A = 2(lb + bh + lh) = 2(100 \times 60 + 60 \times 40 + 40 \times 100)\text{ cm}^2 = 24800\text{ cm}^2 = 2.48\text{ m}^2$$

External temperature (θ_1) = 35°C

Thermal conductivity of wood (k) = $0.168\text{ Wm}^{-1}\text{K}^{-1}$

Here, the rate at which the wooden box takes heat from surroundings = rate at which ice takes heat from box

$$\text{or, } \frac{kA(\theta_1 - \theta_2)}{x} = \frac{d}{dt} (m L)$$

$$\text{or, } L \frac{dm}{dt} = \frac{kA(\theta_1 - \theta_2)}{x}$$

$$\text{or, } \frac{dm}{dt} = \frac{kA(\theta_1 - \theta_2)}{xL} = \frac{0.168 \times 2.48 (35 - 0)}{336000 \times 2.5 \times 10^{-2}} = 1.74 \times 10^{-3} \text{ kgs}^{-1}$$

Hence ice would melt at the rate of $1.74 \times 10^{-3} \text{ kgs}^{-1}$.

Q. An ice box is made of wood 1.75 cm thick lined inside with cork 2 cm thick. If the temperature of inner surface of the cork is steady at 0°C and that of the outer surface of the wood is steady at 12°C , what is the temperature of the interface? The thermal conductivity of wood is five times that of cork.

✎ Solution: Here,

✎ Thickness of wood (x_w) = 1.75 cm = $1.75 \times 10^{-2} \text{ m}$

Thickness of cork (x_c) = 2 cm = $2 \times 10^{-2} \text{ m}$

Thermal conductivity of cork (k_c) = k

Thermal conductivity of wood (k_w) = $5k$

Temperature inner part of cork (θ_1) = 0°C

Temperature of outer part of wood (θ_2) = 12°C

Temperature of interface (θ) = ?

We know that for steady flow of heat,

Rate of flow of heat from wood = Rate of flow of heat from cork

$$\text{or, } \frac{k_w A (q_2 - q_1)}{x_w} = \frac{k_c A (\theta - \theta_1)}{x_c}$$

$$\text{or, } \frac{5k \times A (12 - \theta)}{1.75 \times 10^{-3}} = \frac{k \times A (\theta - 0)}{2 \times 10^{-3}}$$

$$\text{or, } 10(12 - \theta) = 1.75\theta$$

$$\text{or, } 120 - 10\theta = 1.75\theta$$

$$\text{or, } 11.75\theta = 120$$

$$\text{or, } \theta = \frac{120}{11.75} = 10.2^\circ\text{C}$$

Hence the temperature of the interface between wood and cork is 10.2°C .

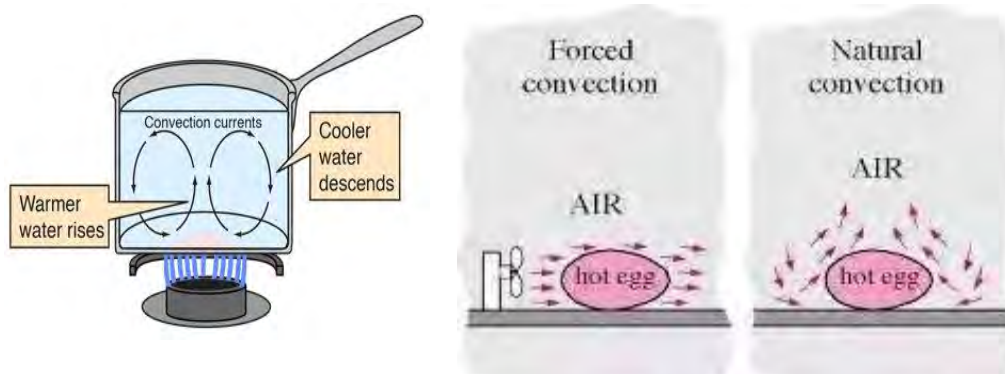
Sample Question:

1. Assuming that, the thermal insulation provided by a woollen glove is equivalent to a layer of quiescent air 3 mm thick; determine the rate of heat loss from a man's hand of surface area 200 cm^2 on a winter day when the atmospheric air temperature is -3°C . The skin temperature is to be taken as 35°C and the thermal conductivity of air as $24 \times 10^{-3} \text{ Wm}^{-1}\text{K}^{-1}$. [6.08 Js^{-1}]

12.6 Convection

Convection is the process of transfer of heat in fluids by means of the actual motion of heated particles from high temperature region to low temperature region. Heating of water, land

breezes, sea breezes, winds, etc. are some examples of convection. In convection, heated particles carry heat and move from the hotter region to the colder region, but the cold particles move in opposite directions. Convection is possible only in liquid and gas but not in solid and vacuum. There are two types of convection; natural convection and artificial convection.



[Fig;12.6 convection; forced convection and natural convection]

Natural or Free Convection;

Natural convection is also called free convection. The primary circulation of the Earth's atmosphere causing daily weather results, land breezes, sea breezes, and trade winds are examples of natural convection. Natural convection is the most commonly brought about by local change in density which is ultimately associated with pressure difference or temperature difference between two regions of fluid. Buoyant force is responsible for movement of particles during natural convection, and it is impossible in gravity-free region.

Artificial or Forced Convection;

If convection is carried out by maintaining pressure difference mechanically by a pump or a blower or by any means of external aid then the convection is called artificial convection or forced convection. Pumping up the blood by the heart and its circulation through the different parts is an example of forced convection. This process keeps our body warm. Also, the use of fans to cool an electronic equipment using water heaters or geysers for instant heating of water are examples of forced convection.

Factors Affecting Convection

Transfer of heat from a surface during convection depends on many factors however experimentally it is found that, the rate of transfer of heat ($\frac{Q}{t}$) is,

- i. directly proportional to the temperature difference between two parts of fluid
i.e. $(\frac{Q}{t}) \propto (\Delta\theta) \dots\dots(1)$
- ii. directly proportional to the surface area of the fluid exposed
i.e. $(\frac{Q}{t}) \propto A \dots\dots(2)$

Combining equation (1) and (2),

$$\frac{Q}{t} \propto A \Delta \theta$$

or,

$$\frac{Q}{t} = h A \Delta \theta$$

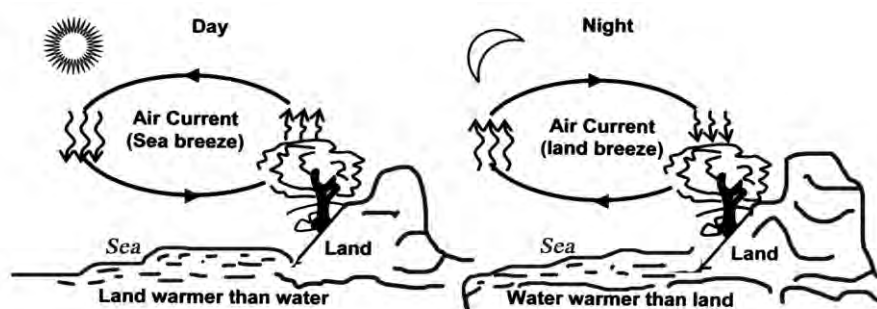
Where h is a constant called the convection coefficient and it depends on nature of fluid.

Examples of Convection:

a) Trade winds

Winds are due to convection current set up in the atmosphere by uneven heating in different locations. The Earth's surface gets heated more at the equator than at the poles. Therefore, warm air rises from the equator. To fill up the vacant space, the cold air from the poles moves towards the equator. These motions of air are known as trade winds because these winds were used by ancient traders for sailing their vessels. Due to the west-to-east motion of the Earth, trade wind seems coming from the northeast in the northern hemisphere but it seems coming from the southwest in the southern hemisphere.

b) Sea breezes and land breezes:



[Fig. 12.7, Land breeze and sea breeze]

EXAMPLE 1

Water has a large specific heat capacity. Because of this, during day time, seawater gets heated up slowly, and becomes less warm than the land surface near the sea. Therefore, the air molecules in contact with land become comparatively hotter and lighter. The lighter air molecules move upward. The vacant space above the land surface is filled by the comparatively cold and heavier air molecules coming from the sea surface. As a result, a large mass of air moves towards the sea-shore from sea. This is called sea breeze. During nighttime, a similar process takes place but in the opposite direction, and air mass moves towards the sea surface from the sea-shore. The process is called land breeze.

c) Ocean Currents:

In the ocean which extends from the pole even up to the equator, water gets heated more at the equator than at the poles. The water level rises at the equator due to a decrease in density. This water flows towards the poles. A current of water is set up which is known as hot current or gulfstream. At the mean time, the cold water moves to the opposite direction beneath the surface which is called undercurrent.

d) Ventilation:

Ventilators are provided near the ceiling of the rooms so that moisturized warm air and carbon dioxide in the upper layer go out through them. As a result, fresh air comes inside

continuously from the windows and the doors.

e) Chimney:

A chimney is provided above the fire in a room or kitchen so that continuous circulation of oxygen for combustion of fuel is possible.

f) Monsoons:

Monsoons are based on the principle of convection.

Sample Questions:

1. Why is it hotter above the fire than by its side?
2. If air is a bad conductor of heat, why do people not feel warm without clothes?
3. Ventilations are provided near the ceilings in a room. Explain this statement.

12.7 Radiation

The heating of the Earth by the Sun and the transmission of heat around the fire are examples of transfer of heat by radiation. The heating of a room with an open-hearth fireplace is another example. The heat radiated by the flames, coals, and hot bricks are examples of transfer of heat by radiation. The radiation takes place even in a vacuum. Radiation does not heat the medium through which heat passes. Hence the radiation is the process of transmission of heat from one point to another without need of any material medium. During radiation, heat is transferred in the form electromagnetic wave and travel with speed of $3 \times 10^8 \text{ ms}^{-1}$ in vacuum. Heat energy radiated by an object is called radiant energy or thermal radiation. Radiation stands for heat energy as well as for the process of transfer of heat.

Thermal Radiation

All objects can radiate energy continuously in the form of electromagnetic radiations at all temperature above zero kelvin. This form of energy by virtue of temperature is called thermal radiation. Thermal radiation exhibits the phenomena like reflection, refraction, interference, polarization and diffraction. Wavelength and energy of thermal radiation depend upon temperature and nature of the radiating body. Thermal radiation belongs to infrared region of electromagnetic spectrum.

Examples of Transfer of Heat by Radiation

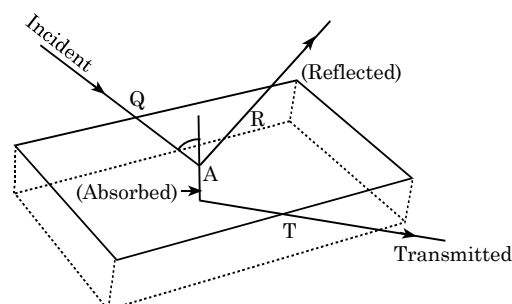
- a. Reflectors of electric heaters are polished so that they reflect maximum heat to the room.
- b. White clothes are preferred in summer but black clothes are preferred in winter.
Radiations get reflected back from white clothes and this support us from overheating during summer. But radiations are absorbed by black clothes so that it keeps us warm during winter.
- c. Hot water pipes and radiators used in rooms.
These pipes are painted black inside the room so that they radiate maximum heat to the room, but they are painted white outside the room to prevent the loss of heat to the surroundings.
- d. The junction of the thermocouple is painted black so that it can absorb maximum heat.

12.8 Reflection, Transmission and Absorption of Heat Radiations

Out of the total heat radiation incident on a surface of a body, some part gets reflected on the surface, some part gets absorbed by the body, and the remaining part is transmitted through the body as shown in figure 12.8.

If Q is the total amount of heat radiation incident on the surface and R , A and T are the reflected, absorbed, and transmitted part of Q respectively then, from the principle of conservation of energy,

$$R + A + T = Q \quad \dots (1)$$



[Fig. 12.8, Reflection, transmission and absorption]

1. Coefficient of Reflection (or Reflectance):

Reflectance is the ratio of heat energy reflected from the surface to the total heat energy incident on the surface during the same time. It is denoted by r and is given by, $r = \frac{R}{Q}$.

2. Coefficient of Absorption (or Absorptance):

Absorptance is the ratio of heat energy absorbed by a body to the total heat energy incident on it during the same time. It is denoted by a and is given by, $a = \frac{A}{Q}$.

3. Coefficient of Transmission (or Transmittance):

Transmittance is the ratio of heat energy transmitted through a body to the total heat energy incident on it during the same time. It is denoted by t and is given by, $t = \frac{T}{Q}$.

EXAMPLE 2

Now dividing equation (1) by Q ,

EXAMPLE 3

$$\frac{R + A + T}{Q} = \frac{Q}{Q}$$

or,

$$\frac{R}{Q} + \frac{A}{Q} + \frac{T}{Q} = 1$$

or,

$$r + a + t = 1$$

EXAMPLE 4

Therefore, the sum of reflectance, absorptance, and transmittance is always equal to 1. The value of r , a and t depends on the nature of the surface and wavelength of incident radiation, but not on the nature of material of the body.

EXAMPLE 5

Analysis of a body on the basis of reflectance, absorptance, and transmittance:

absorptance, and transmittance:

- For a body if $t = 0$ then, $r + a = 1$. Therefore for ' a ' to be greater r must be smaller and vice versa. This means good reflectors are bad absorbers and vice-versa.
- For a body if $t = 0$ and $r = 0$ then, $a = 1$. That is, if a body absorbs all the radiant energy falling on it, neither it reflects nor it transmits it. As a result, a body appears black. In this situation, the given body is known as a black body. Not in all cases the black body appears black. For example, the sun behaves as a black body but does not appear black.

Sample Questions:

- We are lucky that the Earth is not in thermal equilibrium with the sun (which has a surface temperature of about 5800 K). But why are they not in thermal equilibrium?
- How do glass- houses help in keeping plants warm?

3. Why is it hard to see, a house painted white, on sunny days?
4. What would be the temperature of the Earth if the atmosphere were not present?
5. Cooking utensils are blackened at the bottom and polished on the upper surface, why?
6. Although aluminum is a good conductor of heat; how can, aluminum foil with a shining surface, be used to keep food hot for a longer time?
7. . During hot sunny days, a house can be kept cooler if the windows are closed and the curtain pulled to cover the windows. Explain.

12.9 Black Body and Black Body Radiation

A perfectly black body is an idealized physical body that absorbs all incidents of heat radiation, regardless of wavelengths or angle of incidence. Also, a perfectly black body emits radiations of all possible wavelengths when heated. For a perfect black body, the value of absorptance is unity, and that of emissivity is also unity. A good absorber is a good emitter. The sun emits radiations of all wavelengths. So, it can be considered as a black body. A black body is a good emitter so that it can emit radiation of all possible wavelengths. A perfectly black body cannot absorb all the radiant energy but neither it reflects nor it transmits. Hence a perfect body appears black at room temperature and is known as a black body. However, at higher temperatures, it can actually glow at visible wavelengths. Therefore, the term 'black body' is also referred to an object that glows. For example; the sun.

Black body radiation is given off not only by extremely hot objects but also by anything which is above the absolute zero temperature. From stars to ice cubes, they may give black body radiation.

A perfectly black body cannot be realized in actual practice. The nearest approach to a perfect black body is a surface coated with lamp black or platinum black. Such a surface absorbs 96% to 98% of the incident radiation. Ferry's black body is an example of perfect black body realized in practice.

Emissive Power

Emissive power is defined as the rate of heat radiation radiated by a unit area of a body at a particular temperature. It is denoted by E . Mathematically,

$$\text{Emissive power, } E = \frac{\text{Energy radiated (Q)}}{\text{Area (A)} \times \text{time (t)}} = \frac{Q}{A \times t} = \frac{Q}{t \times A} = \frac{P}{A}$$

$$\text{Unit of } E = \frac{\text{Unit of heat}}{\text{Unit of area} \times \text{unit of time}} = \frac{J}{s \times m^2} = Wm^{-2}$$

Emissivity

Emissivity of a body is defined as the ratio of emissive power of given body to the emissive power of a perfectly black body of same size at the same temperature. It is denoted by e and mathematically,

$$\text{Emissivity, } e = \frac{\text{Emissive power of body}}{\text{Emissive power of a perfectly black body}} = \frac{E_b}{E_{bb}}$$

Emissivity is unit less as well as dimensions less quantity. The value of emissivity depends upon the nature of the emitting surface. Emissivity ranges from 0 to 1.

For a perfectly reflecting surface, $e = 0$

For a black body, $e < 1$

For a perfectly black body, $e = 1$

12.10 Stefan's Law of Black Body Radiation

Stefan's law was formulated in 1879 by Austrian physicist Josef Stefan after his experimental studies. The same law was derived theoretically in 1884 by Austrian physicist Ludwig Boltzmann from thermodynamic considerations. Hence, the law was named as the Stefan-Boltzmann Law. This law was one of the first important steps towards the understanding of the black body radiation from which the quantum idea of radiation was raised. Stefan's declared experimentally that energy radiated by a body depends only on its temperature.

Statement: *"The emissive power of a black body is directly proportional to the fourth power of its surface temperature measured in Kelvin scale."*

Suppose, E is the emissive power of a black body at absolute temperature T . Let the surroundings of the body be at zero Kelvin.

According to Stefan's law,

$$E \propto T^4$$

$$\text{or, } E = \sigma T^4 \quad \dots (1)$$

σ is proportionality constant known as Stefan's constant.

In SI unit system, $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$.

For a body of emissivity e ,

$$E = e\sigma T^4 \quad \dots (2)$$

$$\text{Also, } E = \frac{Q}{A t} = e\sigma T^4$$

$$\text{or, } \frac{Q}{tA} = e\sigma T^4$$

$$\text{or, } \frac{\text{Power (P)}}{A} = e\sigma T^4$$

$$\text{or, } P = A e\sigma T^4 \quad \dots (3)$$

If the surrounding temperature is above zero kelvin at temperature T_0 then, the surrounding also emits heat radiation, which is absorbed by a black body. Hence, the above relation was modified by Boltzmann for the net emissive power. The modified relation is known as Stefan - Boltzmann's law. It is given by;

$$E' = \sigma (T^4 - T_0^4) \quad \dots (4)$$

For a body of emissivity e net emissive power is,

$$E' = e \sigma (T^4 - T_0^4) \quad \dots (5)$$

$$\text{Also, } E' = \frac{Q'}{tA} = e \sigma (T^4 - T_0^4)$$

$$\text{or, } \frac{Q'}{tA} = e \sigma (T^4 - T_0^4)$$

$$\text{or, } \frac{\text{Power (P')}}{A} = e \sigma (T^4 - T_0^4)$$

$$\text{or, } P' = A e \sigma (T^4 - T_0^4) \quad \dots (6)$$

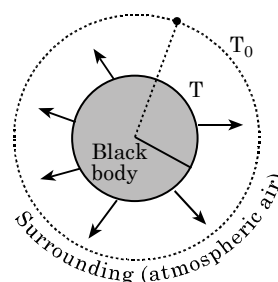


Fig 12.9 Stefan's law of black body radiation

12.12 Solar Constant and Determination of Apparent Temperature of Sun

Solar constant is the solar radiation received per unit time by a unit area of a perfectly black body placed at a distance of 1 astronomical unit from the sun. The area of a black body must be normal to the radiation. It is denoted by S and its approximate value is 1361 Wm^{-2} . The concept of a solar constant and Stefan's constant are used to approximate the surface temperature of the sun.

Sun is taken as a perfect spherical black body of radius R . Let the surface temperature of the sun be T . Suppose r be the mean distance of the Earth from the sun as shown in figure 12.10

Emissive power of the sun,

$$E = \sigma T^4 \quad [\text{According to Stefan's law}]$$

$$\text{or } \frac{Q}{t \times A} = \sigma T^4$$

$$\text{or, } \left(\frac{Q}{t}\right) = A \sigma T^4$$

$$\left(\frac{Q}{t}\right) = 4\pi R^2 \times \sigma \times T^4 \quad \dots (1)$$

[σ is the Stefan's constant]

The sun radiates energy in all directions. If we suppose a sphere of radius r then, it can receive all the radiations from the sun. Therefore, the total energy received per unit time is given by,

$$\left(\frac{Q}{t}\right)' = S \times 4\pi r^2 \quad \dots (2) \quad [S \text{ be the solar constant}]$$

If we assume there is no absorbing medium between the the Earth and the sun then,

$$\left(\frac{Q}{t}\right) = \left(\frac{Q}{t}\right)'$$

$$\text{or, } \sigma T^4 \times 4\pi R^2 = S \times 4\pi r^2$$

$$\text{or, } T^4 = \frac{S}{\sigma} \left(\frac{r}{R}\right)^2$$

$$\text{or, } T = \left[\frac{S}{\sigma} \left(\frac{r}{R}\right)^2 \right]^{1/4}$$

Using this relation, the surface temperature of the sun can be estimated. Its approximate value is 5800 K .

Workout Examples:

Q. Calculate the apparent temperature of the sun from the following information:

Sun's radius $7.04 \times 10^5 \text{ km}$. Distance from the Earth: $14.72 \times 10^7 \text{ km}$. (Solar constant 1400 Wm^{-2} , Stefan's constant: $5.7 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$)

Solution: Here,

$$\text{Solar constant (S)} = 1400 \text{ Wm}^{-2}$$

$$\text{Stefan's constant } (\sigma) = 5.7 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$$

$$\text{Radius of the sun (R)} = 7.04 \times 10^5 \text{ km} = 7.04 \times 10^5 \times 10^3 \text{ m} = 7.04 \times 10^8 \text{ m}$$

$$\text{Distance from the Earth (r)} = 14.72 \times 10^{10} \text{ m}$$

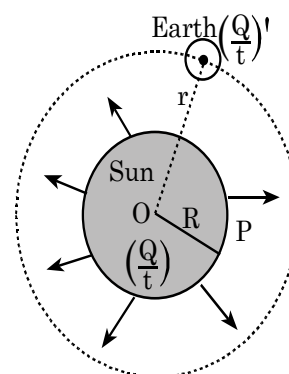


Fig. 12.10 Heat radiation received by Earth from the sun

$$\text{Then temperature of the sun (T)} = \left[\frac{S}{\sigma} \left(\frac{r}{R} \right)^2 \right]^{1/4}$$

$$, \quad T = \left[\frac{1400}{5.7 \times 10^{-8}} \left(\frac{14.72 \times 10^{10}}{7.04 \times 10^8} \right)^2 \right]^{1/4} = [1.07 \times 10^{15}]^{1/4} = 5724 \text{ K}$$

Hence the apparent temperature of the sun is 5724 K.

Q. A spherical black body of radius 5 cm has its temperature of 127°C, and its emissivity is 0.6. Calculate its radiant power. (Stefan's constant, $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$)

✎ Solution: Here,

$$\text{✎ Radius (R)} = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$\text{Temperature (T)} = 127^\circ\text{C} = (127 + 273) \text{ K} = 400 \text{ K}$$

$$\text{Radiant power (P)} = ?$$

$$\text{Emissivity (e)} = 0.6$$

$$\text{Stefan's constant (}\sigma\text{)} = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

We know, from Stefan's law, the emissive power per unit area is,

$$\frac{P}{A} = e \sigma T^4$$

$$\text{or, } P = A e \sigma T^4 = 4\pi R^2 e \sigma T^4 = 4 \times 3.14 \times (5 \times 10^{-2})^2 \times 0.6 \times 5.67 \times 10^{-8} \times (400)^4 = 27.36 \text{ W}$$

Hence radiant power is 27.36 W

Q. A man the surface area of whose skin is 2 m² is sitting in a room where the air temperature is 20°C. If his skin temperature is 35°C, find the rate at which his body loses heat. The emissivity of his skin is 0.97. ($\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4}$)

✎ Solution: Here,

$$\text{✎ Surface area of skin (A)} = 2 \text{ m}^2$$

$$\text{Room temperature (T}_0\text{)} = 20^\circ\text{C} = (273 + 20) \text{ K} = 293 \text{ K}$$

$$\text{Temperature of skin (T)} = 35^\circ\text{C} = (273 + 35) \text{ K} = 310 \text{ K}$$

$$\text{Emissivity of skin (e)} = 0.97$$

$$\text{Stefan's constant } \sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

$$\text{Rate of heat loss (E)} = ?$$

We have relation,

$$E = \sigma A e (T^4 - T_0^4) = 5.67 \times 10^{-8} \times 2 \times 0.97 [(310)^4 - (293)^4] = 92.23 \text{ W}$$

Hence, the body loses heat at the rate 92.23 W.

Exercise

- Which of the following methods of heat transfer is based on gravity?
 - convection
 - conduction
 - radiation
 - conduction and convection
- Which of the following has no effect on the heat radiated by a surface?
 - temperature of surroundings
 - surface area
 - nature of surface
 - Stefan's constant
- On which of the following is the total radiation emitted by a perfectly black body proportional to?
 - absolute temperature
 - square of absolute temperature
 - cube of absolute temperature
 - fourth power of absolute temperature
- On which of the following, does the thermal conductivity depend on?
 - cross-sectional area
 - temperature difference
 - nature of matter
 - It is a pseudo effect.
- Select from the following, the correct option about good absorbers?
 - good reflectors
 - bad reflectors
 - bad emitters
 - good emitters
- By what factor, does the rate at which heat radiated increase, when the temperature of a black body is doubled?
 - 2
 - 4
 - 8
 - 16
- At which of the following temperatures, does a black body not radiate heat?
 - 0°C
 - 0K
 - 273°C
 - 273K
- What does the measurement of the rate of heat flow through a metal bar of the area of cross-section 1m^2 maintained at the temperature gradient of 1°Cm^{-1} in a steady state represent?
 - thermal conductivity
 - Stefan's constant
 - specific heat capacity
 - solar constant
- Why is it hotter above the fire at the same distance than on its side?
 - Heat radiates upward.
 - Air radiates heat upward.
 - Convection takes more heat upward.
 - All the processes conduct upward.
- If a Solid and a hollow sphere of the same material and the size are heated to the same temperature and allowed to cool in the same atmosphere which of the following will be correct?
 - Hollow cools faster.
 - Solid cools faster.
 - Both cool at the same rate.
 - It's insufficient information.

11. Heat is flowing through two cylinders of the same material. The ratios of diameters and that of lengths of the cylinders are 1:2 and 2:1 respectively. If the temperature difference between the two ends is the same, what will be the ratio of the heat conducted through them per unit time?
- a. 1:8 b. 1:4 c. 8:1 d. 2:7
12. A rod AB has the length of 150cm, the temperature of end A is maintained at 100°C and that of end B is maintained at 25°C. What is the temperature of point P at a distance of 50cm from end B?
- a. 50°C b. 60°C c. 75°C d. 80°C
13. A wall has two layers A and B, each made of a different material. The thickness of both layers is the same. The thermal conductivity of layer A is one-third of layer B. If the temperature difference across layer B is 15°C, then what is the temperature difference across layer A?
- a. 5°C b. 20°C c. 45°C d. 60°C
14. There are four articles with different colours, blue, red, black and white. When they are heated together and allowed to cool, which article will cool the earliest?
- a. Blue b. red c. black d. white

QN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	a	d	c	c	d	D	b	a	c	a	a	a	c	c	c

15. (a) At a winter night, you feel warmer when clouds cover the sky than when the sky is clear, why?
- (b) The bulbs of two identical thermometers are coated, one with lamp black and the other with silver, compare their readings when exposed (i) to a hot sunny day (ii) to a clear night.
- (c) You can hold your fingers beside the candle without harm but not above the flame, why?
16. (a) At a wedding party in a cold winter evening you are requested to sit on a chair. Would you like to choose a metal chair or a wooden chair? Explain.
- (b) It is well known that the potato bakes faster if a large nail is stuck through it, why?
- (c) Is an aluminum nail better than a steel for baking a potato? Give reason.
- (d) A newly built quilt is warmer than an old one, why?
- (e) A pot with a steel bottom 8.5 mm thick rests on a hot stove. The area of the bottom of the pot is 0.15 m². The water inside the pot is at 100°C, and 390 g of water is evaporated in every 3 minutes. Find the temperature of the lower surfaces of the pot which is in contact with the stove. [105.5°C]
18. (a) How does tea in a thermos flask remain hot for a long time?
- (b) Why do the Eskimos make double-walled ice houses?
- (c) Estimate the rate of heat loss through a glass window of area 2.0m² and thickness 3.0 mm when the temperature of the room is 20°C and that of the air outside is 5°C. [Thermal conductivity of glass K = 1.2 Wm⁻¹ K⁻¹] [1.2×10⁴W]
- (d) A squirrel wraps its bushy tail around its body during its winter sleep, why?
19. (a) The earth constantly receives heat radiation from the sun and gets warmed up. Why does the earth not get as hot as the sun?
- (b) Does a body at 25°C radiate in room, where the room temperature is 35°C? If yes, why does its temperature not fall further?

- (c) Standing in the sun is more pleasant on a cold winter day than standing in a shade. Is the temperature of the air in the sun considerably higher than that of the air in the shade?
- (d) The element of a 1 kW electric fire has a surface area of 0.006 m^2 . Estimate its working temperature. [The Stefan's Constant $\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$] [1307.6K]
20. (a) We are lucky that the Earth is not in thermal equilibrium with the sun (which has a surface temperature of 5800 K). But why are they not in thermal equilibrium?
- (b) How do the glass-houses help us in keeping plants warm?
- (c) A sphere of radius 2.00 cm with a black surface is cooled, and then suspended in a large evacuated enclosure with the black walls maintained at 27°C . If the rate of change of the thermal energy of the sphere is 1.85 J s^{-1} when its temperature is -73°C , calculate a value for Stefan's constant. [$5.60 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$]
- (d) Tanks of gasoline are generally painted with aluminum paints, why?
21. (a) Stainless steel cooking pans are preferred with extra copper bottom, why?
- (b) What special characters must a cooking utensil have?
- (c) A metal rod of length 20 cm and cross-section area 3.14 cm^2 is covered with a non-conducting substance. One of its ends is maintained at 100°C , while the other end is put in ice at 0°C . It is found that 25 g of ice melts in 5 minutes. Calculate the thermal conductivity of the metal. [Latent heat of fusion of ice = $3.36 \times 10^5 \text{ Jkg}^{-1}$] [$178.34 \text{ Wm}^{-1}\text{K}^{-1}$]
22. A black body of a surface area 10^{-4} m^2 is heated to a high temperature, and is placed in an enclosure maintained at 300 K. The black body radiates heat to the enclosure at the rate (net) of 0.73 W. What is the temperature of black body? (Given: $\sigma = 6 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$) [600 K]
23. 1 m^2 of the Earth's surface receives heat energy from the sun at the rate of 1400 Js^{-1} . The distance of the earth, from the centre of the sun, is $1.5 \times 10^{11} \text{ m}$, and the sun's radius is $7 \times 10^8 \text{ m}$. Considering the sun as a perfect black body; find the temperature of the surface of the sun. [$5.8 \times 10^3 \text{ K}$]

Chapter 13

Ideal Gas

Robert Boyle was an Anglo-Irish natural philosopher, chemist, physicist, and inventor. He was one of the *most prolific figures in the scientific revolution* and is often considered the first modern chemist, making him one of the founding fathers of modern chemistry and a pioneer of contemporary experimental scientific techniques.

Boyle gave a very popular law which defines the inversely proportional connection between absolute pressure, and the volume of a gas in a closed system under constant temperature. Boyle was a polymath with diverse interests ranging from animal husbandry to underwater respiration, and from the study of ancient languages to finding ways of extending the human lifespan. Furthermore, Boyle had both the intellect and the financial resources to pursue such a wide research agenda.



Robert Boyle (1627-1691)

Do you know?

A kilogram of hydrogen has more atoms than a kilogram of iron.

13.1 Introduction

All the states of matter; solid, liquid and gas, do not exhibit the same behavior on the application of heat. The applied heat cannot bring a significant change in pressure in the case of solid and liquid. But in the case of gas, applied heat brings out the change in temperature, volume as well as pressure. The properties, which the matter exhibits on the application of heat, are called thermal properties. Velocity, kinetic energy, mass, etc. are microscopic properties whereas pressure, temperature, volume, etc. are the macroscopic properties. Charles' law, Boyle's Law, Dalton's law, etc. are the interrelationships among the values of thermal properties. By combining these laws in a special condition, the ideal gas equation can also be derived. The ideal gas equation describes the thermodynamic state of a matter at its equilibrium.

The kinetic molecular theory of a gas assumes that the gas molecules exchange energy and momentum during collisions with the wall of the container, and hence give pressure to the wall. This theory explains about macroscopic properties of gas.

13.2 Volume Coefficient and Pressure Coefficient

Unlikely in the case of solid and liquid, gas shows simultaneous change in pressure and volume due to change in temperature. The study of simultaneous change in pressure and volume with temperature is a difficult one. Therefore, gas is heated either at a constant pressure or at a constant volume. Hence a gas has two thermal coefficients; pressure coefficient and volume coefficient.

Volume Coefficient (γ_p)

The volume coefficient of a gas is defined as the change in volume of a certain mass of gas per unit volume at 0°C per unit change in temperature by keeping pressure constant. It is denoted by γ_p .

Mathematically, $\gamma_p = \frac{V_\theta - V_0}{V_0\theta}$ [V_0 is volume of gas at 0°C and V_θ is volume of gas at $\theta^\circ\text{C}$]

Unit of volume coefficient is K^{-1} or, $^\circ\text{C}^{-1}$ and its value is $= \frac{1}{273.15} \text{K}^{-1}$ for all gases. Therefore, at a constant pressure, the volume of a certain mass of a gas increases or decreases by $\frac{1}{273.15}$ of its volume at 0°C for each unit rise or fall in temperature.

Pressure Coefficient (γ_v)

The pressure coefficient of a gas (γ_v) is defined as the change in pressure of certain mass of gas per unit pressure at 0°C per unit change in temperature by keeping volume constant. It is denoted by γ_v .

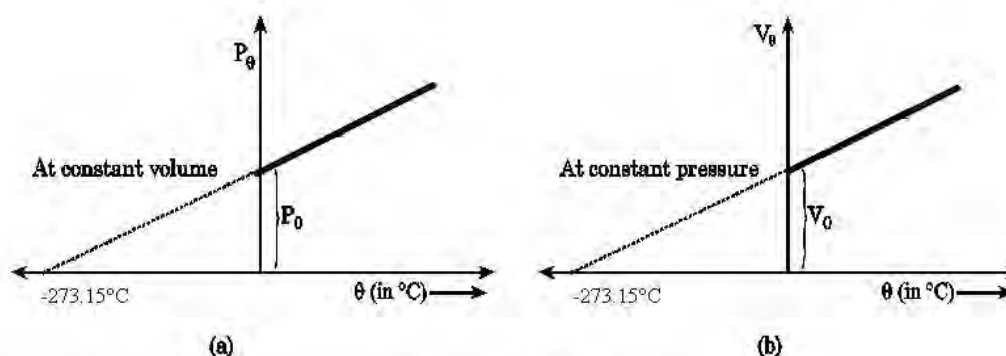
Mathematically, $\gamma_v = \frac{P_\theta - P_0}{P_0\theta}$ [P_0 is pressure at 0°C and P_θ is pressure of gas at $\theta^\circ\text{C}$.]

Unit of pressure coefficient is $^\circ\text{C}^{-1}$ and its value is $= \frac{1}{273.15} \text{K}^{-1}$ for all gases. Therefore, at a constant volume, the pressure of a certain mass of a gas increases or decreases by $\frac{1}{273.15}$ of its pressure at 0°C for each unit rise or fall in temperature.

13.3 Absolute Zero Temperature

Temperature expressed in the Kelvin scale is known as absolute temperature. The zero value of the absolute scale is called absolute zero temperature. At this temperature, the molecular motion stops, and the kinetic energy becomes zero. Also, the absolute temperature is directly proportional to the kinetic energy of its molecules and kinetic energy will never be negative. It means, the absolute temperature cannot be negative. Therefore, absolute zero temperature is the lowest possible temperature in the thermodynamic temperature scale. Hence, in other words, absolute zero temperature is defined as the lowest possible temperature that can exist in the universe. Till the date, the absolute zero temperature has not been practically measured, but the nearest approach is up to 10^{-8}K .

Suppose a certain mass of gas having initial volume V_0 and pressure P_0 at 0°C is heated up gradually at a constant volume, and again at a constant pressure separately. The corresponding volume variation and pressure variation are found as shown in the pressure-temperature graph, and the volume-temperature graph as in figure 13.2.



[Fig. 13.2. Absolute zero temperature from P-T and V-T graph]

From the above graphs, both pressures P_θ and volume V_θ of a gas are the function of a quantity $(273.15 + \theta)$. The quantity $(273.15 + \theta)$ is called absolute temperature. The zero value of this scale is called absolute zero temperature. Hence at absolute zero temperature,

$$\theta + 273.15 = T$$

$$\text{or, } \theta + 273.15 = 0$$

$$\therefore \theta = -273.15^\circ\text{C}$$

Therefore, the absolute zero temperature is -273.15°C .

The extrapolation of *P-T* and *V-T* graph shows that the straight lines cut the temperature axis at a point, which is 273.15 divisions below the origin of the degree Celsius scale, in both cases. The temperature value of that point is taken as absolute zero temperature. Hence absolute zero temperature is that temperature at which the pressure of the given mass of a gas becomes zero at a constant volume or the volume of the given mass of a gas becomes zero at a constant pressure.

Significance of absolute zero temperature

At absolute zero temperature, the molecular motion stops, and there is no collision between gas molecules. So, all the molecules of a gas are at complete rest at the absolute zero temperature. Also, at the absolute zero temperature, matters are highly condensed.

From the above *P-T* and *V-T* graphs, if we go beyond -273.15°C in the temperature axis, then, the volume of a gas at a constant pressure, and the pressure of a gas at a constant volume becomes negative which is practically impossible. Therefore, negative temperature is impossible in absolute scale or Kelvin scale.

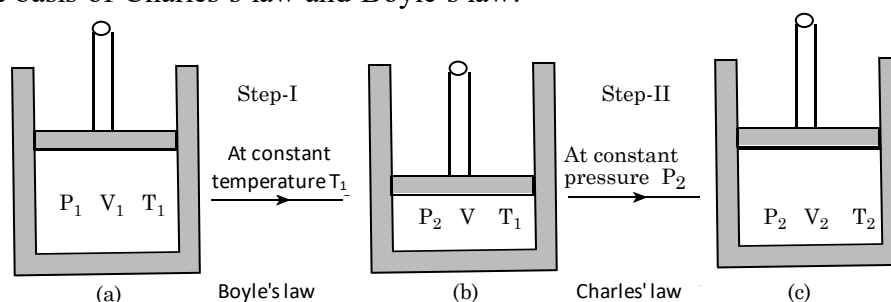
13.4 Ideal Gas

Ideal gases are those having zero intermolecular force of attraction. Ideal gases strictly follow Boyle's law and Charles' law. Ideal gas is an ideal concept. Practically, we cannot get an ideal gas. That means each real gas deviates from the ideal gas behavior. However, at a high

temperature and a low pressure, properties of a real gas are almost similar to those of an ideal gas. For an ideal gas, the intermolecular force of attraction must be zero, but it appears significantly in the case of a real gas.

Ideal Gas Equation

When a certain mass of a gas changes from one equilibrium state to another equilibrium state, its pressure, volume and temperature set up a relation between two equilibrium states on the basis of Boyle's law and Charles' law. Hence, the **ideal gas equation** or **equation of state** is the interrelationship between pressure, volume and temperature of the given mass of an ideal gas on the basis of Charles's law and Boyle's law.



[Fig. 19.5, Different equilibrium states of given mass of a gas]

[Fig. 13.3, Ideal gas at different equilibrium states]

Consider one mole of an ideal gas in a cylinder with a weightless, frictionless, airtight, and movable piston. Let P_1, V_1 and T_1 respectively be the initial pressure, volume, and temperature of a gas as shown in figure 13.3 (a).

In the first step, the gas is compressed to volume V keeping the temperature constant at T_1 . This causes the pressure of a gas increase to P_2 as shown in figure 13.3 (b). Thus, using Boyle's law,

$$P_1 V_1 = P_2 V$$

or, $V = \frac{P_1 V_1}{P_2} \quad \dots (1)$

In the second step, the gas is heated to temperature T_2 keeping the pressure constant at P_2 . This causes the volume of a gas become V_2 as in figure 13.3 (c). Thus, using Charles' law,

$$\frac{V}{T_1} = \frac{V_2}{T_2}$$

or, $V = \frac{V_2 T_1}{T_2} \quad \dots (2)$

Equating the value of V from equation (1) and (2),

$$\frac{V_2 T_1}{T_2} = \frac{P_1 V_1}{P_2}$$

or, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \dots (3) \quad \text{[This is a combined gas equation.]}$

or, $\frac{PV}{T} = \text{constant} \quad \dots (4)$

For one mole, the value of the constant is the same for all gases and is denoted by R. Therefore,

$$\frac{PV}{T} = R$$

or, $PV = RT$... (5)

Equation (5) represents the **ideal gas equation for one mole of an ideal gas**.

Here R is known as the molar gas constant or universal gas constant, and its value depends on the nature and mass of the given gas. The value of R is $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$. Since R is defined for one mole of gas, it is called a molar gas constant. Also, the value of R is the same for all gases which is called the universal gas constant.

For n moles of gas, $PV = n RT$, [V is the volume of n mole of an ideal gas.]

For 1 g of gas, $PV = r T$, $[r = \frac{R}{M}]$ is gas constant for 1 gm of gas and it is also called an ordinary gas constant, and V is the volume of 1 gram of the gas.]

For m gram of gas, $PV = m r T$, $[\frac{R}{M} = r]$ is an ordinary gas constant and V is the volume of m gram of the gas.

Numerical Value of Molar Gas Constant (R)

Taking 1 mol of a gas at NTP into consideration,

Volume of gas (V) = $22.4 \text{ l} = 22.4 \times 10^{-3} \text{ m}^3$

Pressure of gas (P) = $760 \text{ mm of Hg} = 1.01 \times 10^5 \text{ Nm}^{-2}$

Temperature (T) = $0^\circ\text{C} = 273 \text{ K}$

From equation of state for n mole of gas,

$$PV = n RT$$

or, $R = \frac{PV}{nT} = \frac{1.01 \times 10^5 \times 22.4 \times 10^{-3}}{1 \times 273} = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

$$\text{Unit of R} = \frac{\text{Unit of P} \times \text{Unit of V}}{\text{Unit of n} \times \text{unit of T}} = \frac{\text{Nm}^{-2} \times \text{m}^3}{\text{mol} \times \text{kelvin}} = \frac{\text{Nm}}{\text{mol} \times \text{K}} = \text{J mol}^{-1} \text{ K}^{-1}$$

Dimension of R = $[M L^2 T^{-2} K^{-1}]$

Physical meaning of R

From relation $R = \frac{PV}{nT}$, PV represents work done to change the temperature. Therefore, R represents the work done to change the temperature of 1 mole of gas through 1 Kelvin. This is a proportionality constant, which relates the energy scale to the temperature scale.

Calculation of Ordinary Gas Constant (r)

If M is the molar mass of a gas expressed in a gram then, the relation for ordinary gas constant r is given by, $r = \frac{R}{M}$. Therefore,

For Hydrogen gas, M = 2 and $r = \frac{R}{M} = \frac{8.31}{2} = 4.15 \text{ J g}^{-1} \text{ K}^{-1}$

For Oxygen gas, M = 32 and $r = \frac{R}{M} = \frac{8.31}{32} = 0.26 \text{ J g}^{-1} \text{ K}^{-1}$

13.5 One Mole

One mole is a unit for the measurement of a quantity of a substance in terms of volume or mass. A mole corresponds to the mass of a substance that contains 6.02×10^{23} particles of the substance. In other words, one mole is the mass of as much as the number of atoms present in 12 gm of carbon-12 isotope because 12 gm of carbon-12 contains 6.02×10^{23} number of carbon-12 atoms. The molecular weight of a substance expressed in gram is called one mole. Mole is not a number but it is quantity. Mole is a basic unit of SI system and symbol 'mol' is assigned to this. Mass of one mole of any substance is known as molar mass. For hydrogen gas, molecular weight is 2 and molar mass is 2 gm. Therefore,

Molar mass of hydrogen = 1 mole of hydrogen = 2gm of hydrogen = Mass of 6.02×10^{23} hydrogen molecules

Molar mass of oxygen = 1 mole of oxygen = 32 gm of oxygen = Mass of 6.02×10^{23} oxygen molecules

Molar mass of water = 1 mole of water = 18 gm of water = Mass of 6.02×10^{23} water molecules

If m is the total mass of a gas in a container, and M is its molar mass then, number of mole $n = \frac{m}{M}$.

13.6 Avogadro's Law

Statement: "Under identical condition of temperature and pressure, equal volume of all gases contains the equal number of molecules."

If we take 1 mole of a gas at NTP then, any gas has equal volume $22.4\text{ l} = 22.4 \times 10^{-3}\text{ m}^3$ and hence contains equal number of molecules 6.02×10^{23} . For example, at NTP, 1 mole of hydrogen gas (2 gram of hydrogen) occupies the volume of 22.4 l , and contains 6.02×10^{23} molecules of hydrogen. Similarly, 1 mole of oxygen gas (32 gram of oxygen) occupies the volume of 22.4 l , and contains 6.02×10^{23} molecules of oxygen.

Avogadro's number

Avogadro's number is the number of molecules present in one mole of a gas at NTP. It is the same for all gases, and is equal to 6.02×10^{23} molecules. It is denoted by N_A . For example;

1 mole of H_2 gas contains $6.02 \times 10^{23}\text{ H}_2$ molecules

1 mole of O_2 gas contains $6.02 \times 10^{23}\text{ O}_2$ molecules, etc.

Importance of Avogadro's Number N_A

1. To calculate the number of molecules present in a given volume of gas.
2. To calculate the number of atoms present in a given amount of substance.
3. To calculate the number of molecules present in a given amount of substance.
4. To calculate the weight of one atom of an element.
5. To calculate the weight of one molecule of a substance.

Workout Examples:

Q. Two bulbs of equal volume are joined by a narrow tube, and are filled with a gas at STP. When one bulb is kept in melting ice, and the other in boiling water, calculate the new pressure of a gas.

✎ **Solution: Here,**

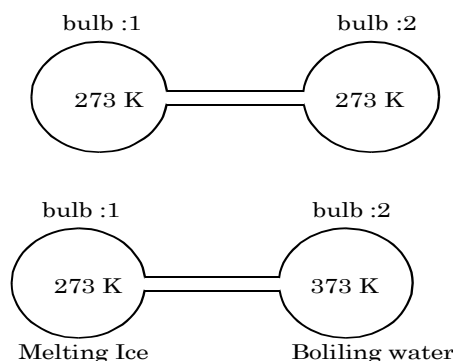
✎ Let, V be the volume of each bulb. Initially, both bulbs are filled with a gas at STP. So,

Standard pressure (P) = 760 mm of Hg

Standard temperature (T) = 273 K

$$\text{Mass of gas in each bulb} = \frac{PV}{rT} = \frac{760V}{r \times 273}$$

$$[PV = nrT]$$



$$\text{Total mass of gas in both bulbs} = 2 \left(\frac{760V}{r \times 273} \right) \quad \dots (1)$$

When a bulb is kept in melting ice (273 K), and the other bulb in boiling water (373 K) then let P' be the new pressure.

$$\text{Now, the mass of gas in the bulb at 273 K} = \frac{P'V}{r \times 273}$$

$$\text{And, the mass of gas in the bulb at 373 K} = \frac{P'V}{373r}$$

$$\text{Total mass of gas in both bulbs} = \frac{P'V}{273r} + \frac{P'V}{373r} \quad \dots (2)$$

But, total mass should remain conserved. So, from equations (1) and (2) we get,

$$2 \times \frac{760V}{r \times 273} = \frac{P'V}{273r} + \frac{P'V}{373r}$$
$$\text{or, } \frac{2 \times 760}{273} = P' \left(\frac{1}{273} + \frac{1}{373} \right)$$

$$\text{or, } P' = 877.65 \text{ mm of Hg}$$

Hence, the new pressure of the gas is 877.65 mm of Hg.

Q.A cylinder of a gas has a mass of 10 kg. and the pressure of 8 atmospheres at 27°C. When some gas is used in a cold room at –3°C, the gas remaining in the cylinder at this temperature has a pressure of 6.4 atmospheres. Calculate the mass of the gas used.

✎ Solution: Here,

✎ Initial mass (m_1) = 10 kg,

Initial pressure (P_1) = 8 atm.

Final pressure (P_2) = 6.4 atm.

Initial temperature (T_1) = 27°C = 300 K

Final temperature (T_2) = –3°C = –3 + 273 = 270 K

Final mass (m_2) = ?

Mass of gas used ($m_1 - m_2$) = ?

Now, using the given condition,

$$P_1V_1 = m_1rT_1 \quad \dots (1) \text{ and } P_2V_2 = m_2rT_2 \quad \dots (2)$$

Dividing equation (1) by (2),

$$\frac{P_1 V_1}{P_2 V_2} = \frac{m_1 r T_1}{m_2 r T_2}$$

$$\text{or, } \frac{8 \times V}{6.4 \times V} = \frac{10}{m_2} \times \frac{300}{270} \quad [\text{Here } V_1 = V_2, \text{ volumes of cylinder and gas are equal}]$$

$$\text{or, } m_2 = \frac{10 \times 300 \times 6.4}{8 \times 270} = 8.9 \text{ kg}$$

Hence, the mass of the gas used, $= m_1 - m_2 = (10 - 8.9) \text{ kg} = 1.1 \text{ kg}$.

Sample Questions:

1. Why does the hot air rising up to a mountain peak cool down?
2. Given a sample of 1cm^3 of hydrogen, and 1cm^3 of oxygen, and if both are at NTP, which sample has a larger number of molecules?
3. The size of a molecule does not enter into the gas laws, why?
4. When a gas is heated in a vessel at constant volume its pressure increases, why?
5. Do you expect the gas in a cooking gas cylinder to obey the ideal gas equation?

13.7 Molecules, Molecular Properties of Matter and Intermolecular Force

Everything is composed of a large number of small identical particles called molecules. The properties of such molecules determine the physical properties of a matter.

Basically, the properties of a matter are of two categories; physical properties (electrical, magnetic, thermal, optical, acoustic, radiological mechanical, etc.), and chemical properties (stability, reactivity, toxicity flammability, etc). Physical properties describe the interaction between matter and energy and are measured in terms of density, temperature; viscosity, surface tension, etc., whereas chemical properties describe the chemical response and interaction. Physical properties explain the rearrangement of molecules whereas chemical properties explain intra-molecular rearrangement. Therefore, generally, less energy is required to bring changes in physical properties than in chemical properties. The intermolecular force of attraction affects and determines the physical behavior of a matter. To understand the physical properties of a matter it is fundamental to understand intermolecular forces.

13.8 Kinetic Molecular Model of Gases

According to the kinetic molecular model of a gas, a gas consists of a large number of molecules. Except at absolute zero temperature, molecules are in continuous random motion with all possible velocities. Because of random motion, the molecules collide with each other as well as with the wall of the container. Both the collisions are elastic in nature. During collision molecules transfer momentum to the wall of the container, therefore, according to Newton's law of motion, the rate of change of momentum results in the force which acts on the wall. Also, the force acting per unit area of the wall gives pressure exerted by the gas in the container.

Since, the diameter of a single gas molecule is very much shorter than the average separation between the molecules, the volume of a single molecule is taken as zero. That is a molecule is supposed to be free to move inside the container. Using this model, different laws of gas (Charles's, Boyle's, etc.) are derived. This model gives ideas about the macroscopic properties like, pressure, volume and temperature, as well as microscopic properties like kinetic energy, velocity, mass, etc. While calculating different parameters of a gas, the kinetic molecular model of gas uses some assumptions which are listed below.

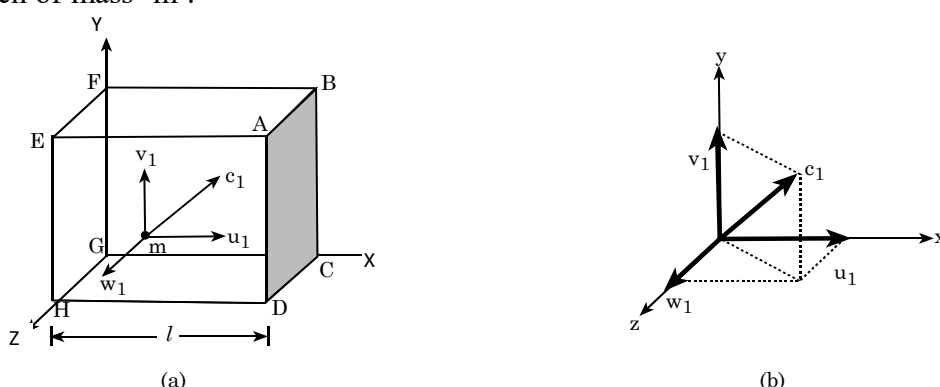
Postulates

1. All gases are composed of a large number of perfectly small elastic spheres called molecules.
2. The attraction between gas molecules is negligible.
3. The volume of a molecule is negligible as compared to the total volume occupied by the gas.
4. The duration of a collision is negligible compared with the time between the collisions.

13.9 Pressure Exerted by a Gas

The pressure exerted by the gas is the force imparted by its molecules per unit area of the wall of the container during collisions.

Suppose, a cubical vessel of length ' l ' as in figure 13.4 (a) contains a gas which has N molecules each of mass ' m '.



[Fig; 13.4, Calculation of pressure exerted by a gas]

The velocity of each gas molecule can be written in terms of their orthogonal components, i.e.,
 $C_1^2 = u_1^2 + v_1^2 + w_1^2$, $C_2^2 = u_2^2 + v_2^2 + w_2^2$, $C_i^2 = u_i^2 + v_i^2 + w_i^2$, $C_N^2 = u_N^2 + v_N^2 + w_N^2$

[Where $i = 1, 2, \dots, N$, is the numbering of gas molecules and u_i , v_i , and w_i are the components of velocity c_i along X, Y and Z axes respectively.]

Let's suppose that a gas molecule moves along X-axis with velocity u_1 so that the momentum of the molecule while striking the wall is mu_1 . After striking the wall, the molecule returns with velocity $-u_1$ because the collision is elastic. Therefore, the momentum of bounce-back of the molecule is $-mu_1$.

Here, the change in linear momentum of the gas molecule during collision = $-mu_1 - mu_1 = -2mu_1$

But, the linear momentum imparted to wall = $2mu_1$

Also, the time taken by the gas molecules between any two successive collisions, $t = \frac{2l}{u_1}$

There, the rate of change of momentum along X-axis for the molecule, i.e.

$$\text{Force } (F_{x_1}) = \frac{2mu_1}{\frac{2l}{u_1}} = \frac{2mu_1}{\frac{2l}{u_1}} = \frac{mu_1^2}{l} \text{ and}$$

Pressure along X – axis,

$$P_{x_1} = \frac{F_{x_1}}{A} = \frac{\frac{mu_1^2}{l}}{l^2} = \frac{mu_1^2}{l^3} = \frac{mu_1^2}{V} \quad [\because \text{Area, } A = l^2 \text{ and Volume, } V = l^3]$$

Therefore, pressure exerted by all molecules of gas along X-axis,

$$P_x = P_{x_1} + P_{x_2} + \dots + P_{x_N}$$

$$\text{Or, } P_x = \frac{mu_1^2}{V} + \frac{mu_2^2}{V} + \dots + \frac{mu_N^2}{V}$$

$$\text{Or, } P_x = \frac{m}{V} (u_1^2 + u_2^2 + \dots + u_N^2)$$

Similarly, $P_y = \frac{m}{V} (v_1^2 + v_2^2 + \dots + v_N^2)$ [pressure exerted by all molecules of gas along Y-axis]

$$P_z = \frac{m}{V} (w_1^2 + w_2^2 + \dots + w_N^2) \quad [\text{pressure exerted by all molecules of gas along Z-axis}]$$

Since, the gas has a uniform density inside the vessel, $P_x = P_y = P_z = P$ and the total pressure exerted by the gas in the vessel can be expressed as,

$$P = \frac{P_x + P_y + P_z}{3}$$

$$\text{or, } P = \frac{m}{3V} [u_1^2 + u_2^2 + \dots + u_N^2 + (v_1^2 + v_2^2 + \dots + v_N^2) + (w_1^2 + w_2^2 + \dots + w_N^2)]$$

$$\text{or, } P = \frac{m}{3V} [u_1^2 + v_1^2 + w_1^2 + (u_2^2 + v_2^2 + w_2^2) + \dots + (u_N^2 + v_N^2 + w_N^2)]$$

$$\text{or, } P = \frac{m}{3V} (C_1^2 + C_2^2 + \dots + C_N^2)$$

$$\text{or, } P = \frac{m}{3V} (N C_{rms}^2) \quad [\because C_{rms} = \sqrt{\frac{C_1^2 + C_2^2 + \dots + C_N^2}{N}} \text{ is root mean square speed of gas molecules}]$$

$$\text{or, } P = \frac{mN}{3V} C_{rms}^2$$

$$\text{or, } P = \frac{M}{3V} C_{rms}^2 \quad [\because M = mN \text{ is mass of all molecules of gas in the vessel}]$$

$$\text{or, } P = \frac{1}{3} \rho C_{rms}^2$$

This is the expression for pressure exerted by a gas on the wall of a container.

Workout Examples:

Q. At a pressure of 700 mm of Hg, the root mean square speed of the molecules of a gas is 400 ms^{-1} . What is the density?

✎ Solution: Here,

$$\text{✎ Pressure (P) = 700 mm of Hg} = 700 \times 10^{-3} \times 13600 \times 10 = 95200 \text{ Nm}^{-2}$$

$$\text{rms speed (C}_{rms}\text{)} = 400 \text{ ms}^{-1}$$

$$\text{Density of gas (}\rho\text{)} = ?$$

We know the relation,

$$P = \frac{1}{3} \rho C_{\text{rms}}^2$$

$$\rho = \frac{3P}{C_{\text{rms}}^2} = \frac{3 \times 95200}{400^2} = 1.785 \text{ kg m}^{-3}$$

Hence the density of a gas is 1.785 kg m^{-3} .

Q. Calculate the pressure in mm of mercury exerted by hydrogen gas if the number of molecules per cm^3 is 6.80×10^{15} , and the root mean square speed of the molecules is $1.90 \times 10^3 \text{ m s}^{-1}$ in a cathode ray tube. (Avogadro's constant = $6.02 \times 10^{23} \text{ mol}^{-1}$, Relative molecular mass of hydrogen = 2.02)

Solution: Here,

$$\text{Number of molecules (N)} = 6.8 \times 10^{15} \text{ cm}^{-3} = 6.8 \times 10^{15} \times 10^6 \text{ m}^{-3} = 6.8 \times 10^{15+6} \text{ m}^{-3} = 6.8 \times 10^{21} \text{ m}^{-3}$$

$$\text{rms speed (C}_{\text{rms}}) = 1.90 \times 10^3 \text{ m s}^{-1}$$

$$\text{Mass of gas (M)} = 2.02 \times 10^{-3} \text{ kg}$$

Mass of each gas molecules,

$$m = \frac{M}{N_A} = \frac{2.02 \times 10^{-3}}{6.02 \times 10^{23}} = 3.36 \times 10^{-27} \text{ kg}$$

Now, Pressure exerted,

$$P = \frac{1}{3} m N C_{\text{rms}}^2 = \frac{1}{3} \times 3.36 \times 10^{-27} \times 6.8 \times 10^{21} \times (1.90 \times 10^3)^2 = 27.46 \text{ Nm}^{-2}$$

Here, $1.01 \times 10^5 \text{ Nm}^{-2}$ is equivalent to 760 mm of Hg

$$1 \text{ Nm}^{-2} \text{ equivalent to } \frac{760}{1.01 \times 10^5} \text{ mm of Hg}$$

$$27.49 \text{ Nm}^{-2} \text{ equivalent to } \frac{760}{1.0 \times 10^5} \times 27.46 \text{ mm of Hg} = 0.2 \text{ mm of Hg.}$$

Therefore, the required pressure is 0.2 mm of Hg.

13.10 Root Mean Square Speed [rms speed]

The square root of the arithmetic mean of the square of the speed of all the molecules of a gas is called rms speed of the gas. If $C_1, C_2, \dots, C_N$ are the speeds of respective gas molecules labeled 1, 2, $\dots$, N then, their rms speed c is given by,

$$C_{\text{rms}} = \sqrt{\frac{C_1^2 + C_2^2 + \dots + C_N^2}{N}} \quad \dots (1)$$

Also, for a gas of density ρ and speed C_{rms} ,

$$\text{Pressure, } P = \frac{1}{3} \rho C_{\text{rms}}^2$$

$$\text{or, } C_{\text{rms}} = \sqrt{\frac{3P}{\rho}} \quad \dots (2)$$

$$\text{Again, } P = \frac{1}{3} \rho C_{\text{rms}}^2$$

$$\text{or, } P = \frac{1}{3} \frac{M}{V} C_{\text{rms}}^2 \quad \left[\because \rho = \frac{M}{V} \right]$$

$$\text{or, } PV = \frac{1}{3} M C_{\text{rms}}^2$$

$$\text{or, } RT = \frac{1}{3} M C_{\text{rms}}^2 \quad [\because PV = RT \text{ for 1 mole of gas}]$$

$$\text{or, } C_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \dots (3)$$

$$\text{or, } C_{\text{rms}} = \sqrt{3R} \times \frac{\sqrt{T}}{\sqrt{M}} \quad \dots (4)$$

Here, for the same gas, mass M is constant at different temperatures, and also $\sqrt{3R} = \text{constant}$. Therefore, from equation (4), we can write,

$$C_{\text{rms}} \propto \sqrt{T} \quad \dots (5)$$

If $(C_{\text{rms}})_1$ and $(C_{\text{rms}})_2$ are rms speeds of the gas at temperature T_1 and T_2 respectively, then,

$$\text{or, } \frac{(C_{\text{rms}})_1}{(C_{\text{rms}})_2} = \sqrt{\frac{T_1}{T_2}} \quad \dots (6)$$

Again, for different gases at the same temperature T , we can write,

$$C_{\text{rms}} \propto \frac{1}{\sqrt{M}} \quad \dots (7) \quad [\text{from equation (4)}]$$

If $(C_{\text{rms}})_1$ and $(C_{\text{rms}})_2$ are rms speeds of gases having respective molar mass M_1 and M_2 , then,

$$\text{or, } \frac{(C_{\text{rms}})_1}{(C_{\text{rms}})_2} = \sqrt{\frac{M_2}{M_1}} \quad \dots (8)$$

Conclusions;

1. The rms speed of molecules of a gas is directly proportional to the square root of the absolute temperature of the gas.
2. The rms speed of molecules of different gases at the same temperature is inversely proportional to the square root of the molecular mass, i.e., the higher the molecular mass, the lesser the rms speed at the same temperature.

Workout Example:

Q. Calculate the rms speed of H_2 molecule at NTP. At what temperature will the rms speed be double at 0°C ?

Solution: Here,

At NTP, Pressure (P) = $1.01 \times 10^5 \text{ N/m}^2$

Temperature (T_0) = 273 K

Relative molecular mass of hydrogen molecule = 2

rms speed at NTP = $(C_{\text{rms}})_0$ (Let)

Therefore,

mass of 1 mole of hydrogen gas, (M) = $2 \text{ gm} = 2 \times 10^{-3} \text{ kg}$

Volume of one mole of hydrogen gas at NTP

$$(V) = 22.4 \text{ l} = 22.4 \times 10^{-3} \text{ m}^3$$

Final temperature (T) = ?

Final rms speed $(C_{\text{rms}}) = 2 (C_{\text{rms}})_0$

Using relation,

$$P = \frac{1}{3} \rho (C_{\text{rms}})_0^2 = \frac{1}{3} \frac{M}{V} (C_{\text{rms}})_0^2$$

$$(C_{\text{rms}})_0^2 = \frac{3PV}{M} = \frac{3 \times 1.01 \times 10^5 \times 22.4}{2}$$

$$\therefore (C_{\text{rms}})_0 = 1842.17 \text{ ms}^{-1}$$

Again, let T be the temperature at which the speed will be double at 0° C. Then

$$\frac{C_{rms}}{(C_{rms})_0} = \sqrt{\frac{T}{T_0}}$$

$$\text{or, } C_{rms} = \sqrt{\frac{T}{T_0}} (C_{rms})_0$$

$$\text{or, } T = \left(\frac{C_{rms}}{(C_{rms})_0} \right)^2 \times T_0$$

$$= \left(\frac{2(C_{rms})_0}{(C_{rms})_0} \right)^2 \times 273 = 4 \times 273 = 1092 \text{ K} = (1092 - 273)^\circ\text{C} = 819^\circ\text{C}.$$

Hence, the required temperature is 1092K or 819°C.

Sample Questions:

1. A closed vessel contains an equal number of molecules of hydrogen and oxygen. A fine hole is made in the vessel from which the gas will leak rapidly.
2. The air that leaks out into the vacuum system contains mainly hydrogen and helium. Explain this on the basis of the kinetic theory of gas.
3. By what factor will the rms speed of a particular gas molecule increase if the temperature is increased from 27°C to 327°C?
[$\sqrt{2}$ times]

13.11 Kinetic Energy of the Gas

For a gas of density ρ and rms speed C_{rms} , pressure is given by, $P = \frac{1}{3} \rho C_{rms}^2 \dots (1)$

If we suppose V as molar volume, M as molar mass, R as molar gas constant, and T as the absolute temperature of a gas, then,

Equation of state is, $P V = R T \dots (2)$

Again, from equation (1), $P = \frac{1}{3} \frac{M}{V} C_{rms}^2$ [$\because \rho = \frac{M}{V}$]

$$\text{or, } PV = \frac{1}{3} M C_{rms}^2$$

$$\text{or, } RT = \frac{1}{3} M C_{rms}^2 \quad [\because P V = R T]$$

$$\text{or, } RT = \frac{2}{3} \left(\frac{1}{2} M C_{rms}^2 \right) = \frac{2}{3} (\text{KE for 1 mole of gas})$$

$$\text{or, } (\text{KE})_{\text{molar}} = \frac{3}{2} RT \dots (3)$$

$$\therefore (\text{KE})_{\text{molar}} \propto T$$

$$\text{For } n \text{ mole of gas, K.E.} = n (\text{KE})_{\text{molar}} = n \left(\frac{3}{2} RT \right) = \frac{3}{2} n RT \dots (4)$$

If N_A is the Avogadro's number and m is the mass of each molecule of the gas, then,

$$M = N_A m$$

$$\text{or, } \frac{1}{2} M C_{rms}^2 = \frac{1}{2} N_A m C_{rms}^2$$

$$\text{or, } \frac{1}{2} M C_{rms}^2 = N_A \frac{1}{2} m C_{rms}^2$$

$$\text{or, } \frac{1}{2} M C_{\text{rms}}^2 = N_A \quad (\text{KE of a molecule})$$

$$\text{or, } (\text{KE})_{\text{molecular}} = \frac{1}{N_A} \left(\frac{1}{2} M C_{\text{rms}}^2 \right) = \frac{1}{N_A} (\text{KE})_{\text{molar}} = \frac{1}{N_A} \left(\frac{3}{2} RT \right) = \frac{3}{2} \left(\frac{R}{N_A} \right) T$$

$$\text{or, } (\text{KE})_{\text{molecular}} = \frac{3}{2} k_B T \quad \dots (5)$$

Where, $k_B = \frac{R}{N_A}$ is Boltzmann's constant and its value is $k_B = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

Hence, the average kinetic energy of gas molecules is directly proportional to its absolute temperature.

Workout Examples:

Q. What is the average translational KE of a molecule of an ideal gas at the temperature of 27°C? What is the total random translational of the kinetic energy of the molecules in 1 mole of this gas? What is the root-mean-square speed of oxygen molecules at this temperature?

✎ Solution: Here,

✎ Average KE of one mole of an ideal gas is

$$KE_1 = \frac{1}{2} M C_{\text{rms}}^2 = \frac{3}{2} RT = \frac{3}{2} \times 8.31 \times 300 = 3739.5 \text{ J} \quad [T = 27^\circ = 300\text{K}]$$

Average KE of a molecule of an ideal gas is,

$$KE_2 = \frac{1}{2} m C_{\text{rms}}^2 = \frac{3}{2} \frac{R}{N} T = \frac{3}{2} \times \frac{8.31}{6.023 \times 10^{23}} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

Again,

$$C_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{32 \times 10^{-3}}} = 483.44 \text{ ms}^{-1}.$$

Hence, the required value of molar kinetic energy, molecular kinetic energy and rms speed are 3739.5 J, $6.21 \times 10^{-21} \text{ J}$ and 483.44 ms^{-1} respectively.

Q. At what temperature will the average speed of an oxygen molecule be sufficient so as to escape from the Earth? Escape velocity from the Earth is 11.2 kms^{-1} [Mass of an oxygen molecule is $5.34 \times 10^{-26} \text{ kg}$ and $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$].

✎ Solution: Here,

✎ Velocity required to escape (v) = $11.2 \text{ kms}^{-1} = 11200 \text{ ms}^{-1}$

Boltzmann's constant (k_B) = $1.38 \times 10^{-23} \text{ J K}^{-1}$

Required temperature (T) = ?

Mass of one oxygen molecule (m) = $5.34 \times 10^{-26} \text{ kg}$

We know, $\frac{1}{2} mv^2 = \frac{3}{2} k_B T$

$$\text{or, } T = \frac{mv^2}{3k_B} = \frac{5.34 \times 10^{-26} \times 11200 \times 11200}{3 \times 1.38 \times 10^{-23}}$$

$$= 161799.4203 = 1.6 \times 10^5 \text{ K}$$

Hence the oxygen molecule will escape from the earth at temperature of $1.6 \times 10^5 \text{ K}$.

Q. Calculate the number of molecules in 1 cm^3 of a perfect gas at 27°C and a pressure of 65 cm of mercury. Given density of the mercury = 13.6 gcm^{-3} . (Boltzmann constant = $1.38 \times 10^{-23} \text{ J K}^{-1}$.)

Solution: Here,

$$\text{Temperature (T)} = 27^\circ\text{C} = (27 + 273) \text{ K} = 300 \text{ K}$$

$$\text{Boltzmann constant (k)} = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{Pressure of gas (P)} = 65 \text{ cm of Hg} = 0.65 \times 9.8 \times 13600 \text{ Nm}^{-2} = 86632 \text{ Nm}^{-2}$$

$$\text{Number of molecules per cm}^3 \left(\frac{N}{V} \right) = ?$$

We know,

$$\begin{aligned} P &= \frac{1}{3} \frac{mN}{V} C_{\text{rms}}^2 \\ &= \frac{1}{3} \left(\frac{N}{V} \right) m C_{\text{rms}}^2 = \frac{1}{3} \left(\frac{N}{V} \right) \times 2 \frac{1}{2} m C_{\text{rms}}^2 \\ &= \frac{2}{3} \left(\frac{N}{V} \right) \times \frac{3}{2} k T = \left(\frac{N}{V} \right) k T \end{aligned}$$

$$\text{Therefore, } \frac{N}{V} = \frac{P}{kT} = \frac{86632}{1.38 \times 10^{-23} \times 300} = 209 \times 10^{23} = 2.09 \times 10^{25}.$$

Hence, the number of molecules is 2.09×10^{25} .

Sample Questions:

1. Would you expect that all gas molecules in a given volume to have same speed?
2. Molecules of different gases have equal average kinetic energies provided their temperature is the same. Do these molecules have equal velocities?
3. The Kinetic temperature of gas in upper atmosphere is of the order of 1000 K, but it is quite cooled up there. Give a solution to this paradox.
4. The temperature of an ideal mono-atomic gas is increased from 20°C to 40°C . Does the average translational of the kinetic energy of each gas atom double?
5. Helium sample has a mixture of two isotopes; one having molar mass 3 gmol^{-1} and the other having molar mass 4 gmol^{-1} . On average, which isotope moves faster?

13.12 Relation between Pressure and Kinetic Energy of a Gas

We know pressure exerted by a gas of density ρ and rms speed c is given by $P = \frac{1}{3} \rho C_{\text{rms}}^2$.

Also, the kinetic energy of a gas having total mass m and volume V is given by,

$$\text{KE} = \frac{1}{2} m C_{\text{rms}}^2$$

$$\text{or, } \text{KE} = \frac{1}{2} V \rho C_{\text{rms}}^2 \quad [\because m = V\rho]$$

$$\text{or, } \frac{\text{KE}}{V} = \frac{1}{2} \rho C_{\text{rms}}^2$$

$$\text{or, } \frac{\text{KE}}{V} = \frac{3}{2} \left(\frac{1}{3} \rho C_{\text{rms}}^2 \right) = \frac{3}{2} P$$

$$\text{or, } E' = \frac{3}{2} P \quad \left[\text{where, } \frac{\text{KE}}{V} = E' \right]$$

Hence, the average kinetic energy per unit volume of a gas (E') is $\frac{3}{2}$ times the pressure exerted by the gas on the wall of the container.

13.13 Kinetic Interpretation Temperature

Average translational molecular kinetic energy of a gas, $\frac{1}{2} m C_{rms}^2 = \frac{3}{2} k_B T$...

(1)

From this, we conclude that the average molecular kinetic energy of a gas is directly proportional to its absolute temperature.

Mathematically, $\frac{1}{2} m C_{rms}^2 \propto T$

This indicates that temperature is the function of the kinetic energy of molecules.

From equation (1) at absolute zero temperature,

$$\frac{1}{2} m C_{rms}^2 = \frac{3}{2} k_B \times 0 = 0, \quad [\because T = 0 \text{ kelvin and } m \neq 0]$$

$$\therefore C_{rms} = 0$$

This suggests that the root mean square speed of the gas molecules becomes zero at absolute zero temperature. Hence, the gas molecules are completely at rest at absolute zero temperature.

Again, from relation (1),

$$m \geq 0 \text{ and } C_{rms}^2 \geq 0 \text{ and } T \geq 0.$$

Hence Kelvin temperature never be negative.

Exercise

- To which of the following is the rms speed of gas molecules proportional?
a. T b. T^2 c. $\sqrt{T}$ d. $\sqrt{M}$
- To which of the following, the pressure exerted by a gas on the wall of the container, is proportional?
a. C_{rms} b. $(C_{rms})^{-1}$ c. $(C_{rms})^{\frac{1}{2}}$ d. $(C_{rms})^2$
- What is the ratio of root mean square velocity of oxygen to hydrogen at NTP?
a. 1:16 b. 4:1 c. 1:4 d. 16:1
- Which of the following is the kinetic energy per unit volume of a gas?
a. P b. $\frac{3}{2}P$ c. $\frac{2}{3}P$ d. $\frac{4}{3}P$
- The kinetic energy of a molecule of an ideal gas at 0°C is E . If the temperature is raised to 273°C , what will be the kinetic energy?
a. E b. $\sqrt{2} E$ c. $2E$ d. $\frac{E}{\sqrt{2}}$
- What is the ratio of the kinetic energy of oxygen to hydrogen molecule at NTP?
a. 1:16 b. 4:1 c. 1:4 d. 1:1
- Which of the following quantities remains the same for all gases at a given temperature?
a. rms speed b. density c. potential energy d. kinetic energy
- At what temperature, will the root mean square velocity of a gas molecule be double to that of 0°C ?
a. 546°C b. 1096°C c. 819°C d. 546K

9. Boltzmann's constant (k_B) is $1.36 \times 10^{-23} \text{ JK}^{-1}$. What is the KE of a molecule of hydrogen gas at 527°C ?
 a. $1.088 \times 10^{-20} \text{ J}$ b. $1.632 \times 10^{-20} \text{ J}$ c. $1.075 \times 10^{-20} \text{ J}$ d. $0.725 \times 10^{-20} \text{ J}$
10. What is the temperature at which the nitrogen molecule will have the same rms speed as the oxygen molecule has at 127°C ?
 a. 77°C b. 127°C c. 273°C d. 350°C
11. At 273K temperature and $1.01 \times 10^5 \text{ Nm}^{-2}$ pressure, 2.70×10^{25} molecules are present per cubic meter. What will be the number of molecules per cubic meter at 223K temperature and $1.33 \times 10^{-4} \text{ Nm}^{-2}$ pressure?
 a. 4.30×10^{12} b. 4.35×10^{16} c. 4.35×10^{20} d. 4.35×10^{22}
12. A cylinder contains 150cc of ideal gas at 27°C and 650mm pressure. What is the volume of the gas at 0°C if the pressure is constant?
 a. 130cc b. 136.5cc c. 140cc d. 150cc
13. Which of the following is a universal constant?
 a. $\frac{C_p}{C_v}$ b. $C_p + C_v$ c. $C_p - C_v$ d. $\frac{C_v}{C_p}$
14. A gas at pressure $1 \times 10^5 \text{ Nm}^{-2}$ and temperature 27°C has density $\rho = 0.09 \text{ kgm}^{-3}$. What will be the density of the gas if the pressure and the temperature change to $4.0 \times 10^5 \text{ Nm}^{-2}$ and 127°C respectively?
 a. 0.94 b. 0.27 c. 0.36 d. 0.15
15. A box contains N molecules of a gas. The number of molecules is increased to $2N$. What will be the pressure of the gas?
 a. become half b. become 4 times
 c. remain unchanged d. become twice

QN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	c	d	c	b	c	d	d	c	b	a	b	b	c	b	d

16. (a) A gas enclosed in a vessel exerts pressure on its walls, why?
 (b) A gas is kept in a rigid cubical container. If a load of 10 kg is put on the top of the container, does the pressure exerted by the gas increase?
 (c) If one wished to keep a mole of an ideal gas at STP, how much big container does he need?

[STP pressure = 760mm of Hg, STP temperature = 273K , $R = 8.3 \text{ Jmol}^{-1} \text{ K}^{-1}$]

[$22.4 \times 10^{-3} \text{ m}^3$]

17. (a) Why is carbon dioxide gas less likely to escape from the earth's atmosphere?
 (b) Practically, there is no atmosphere on the moon, why?
 (c) The correct inflation of a tyre at 20°C is 2 kg cm^{-2} . After driving for several hours, the driver checks the tyres. If the temperature of tyres is 50°C , what should be the pressure readings? [$2.2 \times 10^5 \text{ Nm}^{-2}$]
18. (a) Imagine you have two containers, A and B, each holding an ideal gas at the same temperature and pressure. Container A has twice the volume of container B. Compare

- the number of gas molecules in container A to the number of gas molecules in container B. Justify your answer using the principles of ideal gases.
- (b) A cylindrical tank has a tight-fitting piston that allows the volume of the tank to be changed. The tank originally contains 0.110 m^3 of air at a pressure of 3.4 atm . The piston is slowly pulled out until the volume of the gas increases to 0.390 m^3 . If the temperature remains constant, what will be the final value of the pressure? [0.96 atm .]
19. (a) Would you expect that all gas molecules in a given volume to have the same speed? Explain.
- (b) Consider a sealed container filled with an ideal gas. Discuss the implications of increasing the temperature of the gas according to the kinetic molecular model. How does this affect the average kinetic energy, the speed distribution of gas molecules, and the pressure exerted by the gas? Justify your answer with relevant concepts from the kinetic molecular model.
- (c) Calculate the number of molecules in 1 cm^3 of a perfect gas at 27°C and a pressure of 65 cm of mercury. The given density of mercury = 13.6 gm cm^{-3} . (Boltzmann constant = $1.38 \times 10^{-23} \text{ J K}^{-1}$) [2.09×10^{29}]
20. (a) Explain how the kinetic molecular model of an ideal gas accounts for the relationship between temperature and the average kinetic energy of gas molecules.
- (b) A gas is kept in an enclosure. The pressure of gas is reduced by pumping out some gas. Will the temperature of the gas decrease?
- (c) Assuming the density of helium at STP = 0.1785 kg m^{-3} . Calculate the value for the root mean square speed of thermal agitation of the molecules of helium in a vessel at 30°C . [1372.59 ms^{-1}]
21. The volume of a given mass of a gas at 47°C temperature and pressure 75 cm of mercury is 640 cc . At what temperature, will the pressure of the gas be doubled if the gas is heated keeping the volume constant? ($R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$) [367°C]
22. Find the number of molecules and the number of moles in one cubic meter of air at STP. [$2.68 \times 10^{25} \text{ molecules}$, 44.5 moles]
23. Helium gas with a volume of 2.60 l , under a pressure of 1.30 atm . and at a temperature of 41.0°C , is warmed until both pressure and volume are doubled. (a) What is the final temperature?
- (b) How many grams of helium are there? The molar mass of helium is 4.00 g mol^{-1} . ($R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$) [983°C , $5.23 \times 10^{-4} \text{ kg}$]
24. A balloon, which is partially filled with helium, contains 500 m^3 of helium gas at 27°C , and $1 \text{ atmospheric pressure}$. Its volume goes on increasing and becomes 900 m^3 at a certain height where the temperature is -3°C . What is the pressure of the gas? [0.5 atm .]
25. Determine the density of methane gas at 20°C and $5 \text{ atmospheric pressures}$. (Given: $1 \text{ mol of methane} = 16 \text{ gm}$, $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$, $1 \text{ atm.} = 1.013 \times 10^5 \text{ Nm}^{-2}$) [3.33 kg m^{-3}]
26. A chamber of volume $1 \times 10^{-3} \text{ m}^3$ contains 1×10^{25} of molecules of oxygen. Calculate the pressure of the gas if the molecular mass of oxygen is $2.7 \times 10^{-28} \text{ kg}$, and its rms speed is 400 ms^{-1} . [$14.4 \times 10^4 \text{ Nm}^{-2}$]

Chapter 14

Reflection at Curved Mirror

Born in Darmstadt, Germany on May 12, 1803, Justus von Liebig received his doctorate from the University of Erlangen in Bavaria in 1822. Justus von Liebig was a chemist, who is widely credited as one of the founders of agricultural chemistry. He made crucial contributions to the analysis of organic compounds, and, in his early years, also published several works on the use of inorganic fertilizers in several languages. He discovered that nitrogen was an essential plant nutrient, and presented his famous Law of the Minimum which explained the effect of individual nutrients on crops.

A discovery by Liebig in 1835 made mirrors widely available. Liebig found a way to coat glass with a thin layer of metallic silver by depositing the metal directly by means of a chemical reaction.

[Justus von Liebig, 1803-1873]

Do you know?

Convex mirrors are used in vehicles for rear view but not concave mirrors.

14.1 Introduction

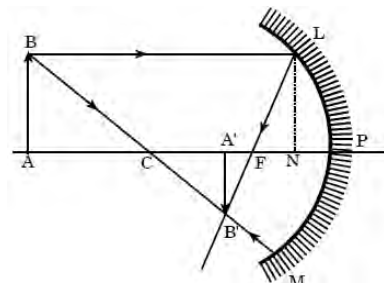
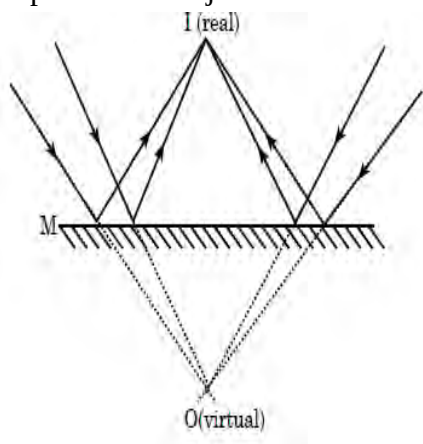
Reflection of light is the fundamental concept in the study of optics and visual perception. When light travelling through one medium approaches to the surface of another medium, it is partly reflected, partly absorbed, and the remaining part is transmitted. "The phenomenon of coming back of light in the same medium when it strikes on a surface is known as reflection of light." The bouncing off of the light occurs in a predictable manner as it follows the laws of reflection. Generally, light reflects from reflecting surfaces (plane, *polished*, smooth shiny surfaces, etc.) like plane mirrors, curved mirrors, etc. The reflection of light from plane mirrors and curved mirrors produces an image of an object. The working of optical instruments is based on the reflection of light. The understanding of the principles of reflection of light is very important in many aspects of the physical world around us from the functioning of our eyes to the behavior of astronomical objects. In this chapter, we will explore the properties and applications of reflected light establishing a foundation for further studies in physics and applied sciences.

14.2 Image

When the reflected rays meet or appear at a point, then the image of an object is formed. There are two types of images. They are real images and virtual images.

Real Image

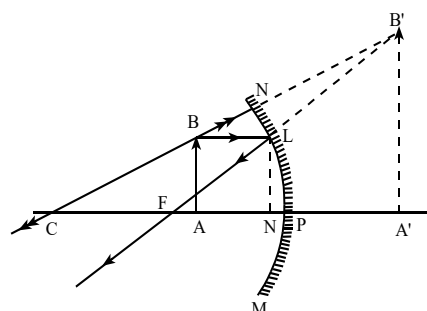
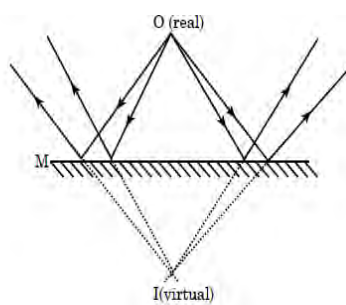
An image formed due to the actual intersection of light rays after reflection or refraction is called real image. A real image can be obtained on a screen. The real image is inverted with respect to the object.



Virtual Image

[Fig. 14.1 (a), Real image]

A virtual image is formed when the light rays after reflection or refraction do not actually intersect but appear to intersect when produced backward. A virtual image cannot be obtained on a screen. A virtual image is erected or upright with respect to the object.



[Fig. 26.16, Ray diagram of concave mirror when virtual image is formed.]

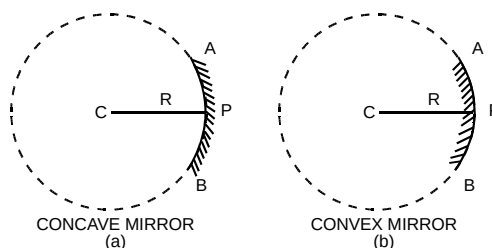
[Fig.14.1(b), Virtual image]

Differences between a real and a virtual image

SN	Real image	S N	Virtual image
1	A real image is formed when light rays converge at a point.	1	A virtual image is formed when light rays appear to converge at a point.
2	A real image can be cast onto a screen.	2	A virtual image cannot be cast onto a screen.
3	A real image is inverted with respect to the object.	3	A virtual image is erected with respect to the object.
4	Images formed on camera film, the retina of the eye are examples of real images.	4	Images formed in a plane mirror and aconvex mirror are examples of virtual images.

14.3 Spherical Mirrors

A spherical mirror is a mirror whose reflecting surface is part of a sphere. A portion of a hollow glass sphere taken out by cutting the sphere along a plane can serve as a spherical mirror if one surface is silvered. Spherical mirrors are of two types; Concave and convex.



[Fig. 14.2, Spherical mirrors]

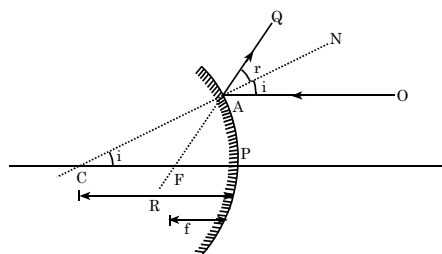
If the outer surface of portion APB of the sphere is silvered as shown in figure 14.2(a) then, it behaves as a concave spherical mirror. On the other hand, if the inner surface of the portion APB of the sphere is silvered as shown in figure 14.2(b) then, it behaves as a convex spherical mirror.

Think?

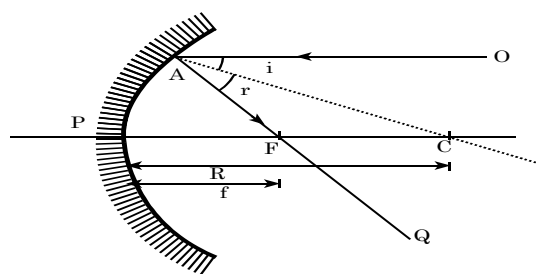
Under what condition does a curve mirror become a plane mirror?

14.4 Relation between F and R

The focal lengths of both concave and convex mirrors of small aperture are related to their respective radii of curvature. The focal length is one half of the radius of curvature. That is $f = \frac{R}{2}$.



[Fig. 14.3(a), Ray diagram of convex mirror]



[Fig. 14.3(b), Ray diagram of concave mirror]

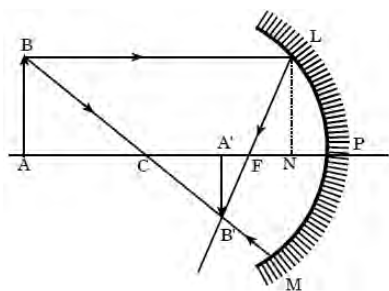
Sample Questions:

1. What type of mirror will you suggest for shaving or make up purpose, and why?
2. Can a curve mirror behave as a plane mirror?
3. If a spherical mirror is immersed in water, does the focal length change?

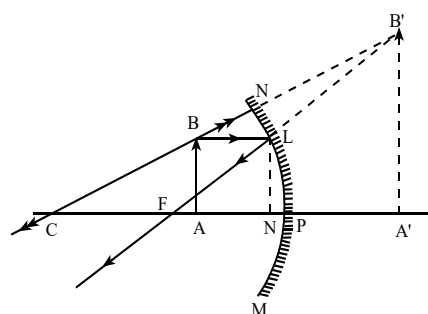
14.4 Mirror Formula

A mirror formula can be defined as the formula which gives the relationship between the distance of object 'u', the distance of image 'v', and the focal length of the mirror 'f'. The mirror formula is applicable for spherical mirrors (concave and convex) as well as plane mirrors.

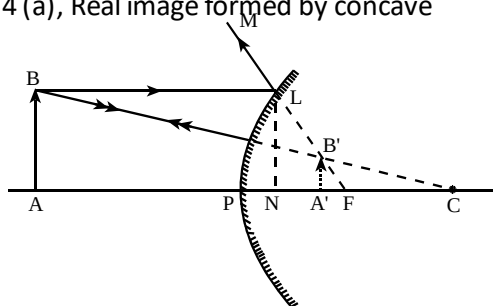
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$



[Fig. 14.4 (a), Real image formed by concave



[Fig.14.4(b), Virtual image formed by
[Fig. 26.16, Ray diagram of concave mirror
when virtual image is formed.



[Fig. 14.4 (c), Virtual image formed by convex

AP= object distance, (u)

A'P= image distance, (v)

FP= focal length, (f)

To derive the mirror formula, the following

assumptions and sign conventions are used:

Assumptions

1. The aperture of the mirror is small.
2. The incident and reflected rays make very small angles with the principal axis of the mirror.
3. The object is a point object situated on the principal axis.

Sign Conventions

1. The entire image distance and object distance are measured from the pole of the mirror.
2. The distances of real objects and real images are taken as positive, and those of virtual objects and virtual images are taken as negative.
3. The radius of curvature and focal length of a concave mirror are taken as positive, and those of convex are taken as negative.

14.5 Magnification

"The ratio of the size of the image (I) formed by a mirror to the size of the object (O) is called magnification of the mirror." Magnification of mirror (m) = $\frac{\text{image size (I)}}{\text{object size (O)}}$

... (1)

The magnification of a mirror is also defined as the ratio of image distance (v) to object distance (u).

The magnification of mirror, (m) = $\frac{\text{image distance (v)}}{\text{object distance (u)}}$

...(2)

a) For the case of a real image, the magnification (m) is taken as positive.

- b) For the case of a virtual image, the magnification (m) is taken as negative.
- c) If magnification is one, it will be of the same size as that of objects.
- d) If magnification is less than one, the image will be smaller in size in comparison to the size of the object. This is the case of diminished image.
- e) If magnification is more than one, the image will be larger in size in comparison to the size of the object. This is the case of an enlarged image.

Magnification in mirrors indicates their ability to produce enlarged or reduced images. The production of enlarged or reduced images is very essential for us in many practical applications, scientific investigations, educational purposes and many more. For examples;

- a) Magnification produced by a concave mirror is used to determine the size and orientation of the image.
- b) Concave mirrors are used in various optical devices such as telescopes, microscopes, and make-up mirrors to get magnified images of small objects as well as distant objects.
- c) Convex mirrors produce virtual and diminished images so that they are used in vehicles for a wider field of rear view.
- d) Convex mirrors are commonly used in traffic mirrors, security mirrors, surveillance, etc.
- e) Magnified image helps to take precise and accurate readings during experiments.

Workout Examples:

Q. Magnification of a plane mirror is always one, why?

A plane mirror can form an image of an object such that the size of the image is equal to the size of the object. Magnification is the ratio of the size of the image to the size of the object.

Hence, magnification of a plane mirror is always one. We know the relation, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ for mirror. But for plane mirror $f = \infty$. Therefore,

$$v = -u \text{ and magnification } m = \frac{v}{u} = 1 \text{ (magnitude).}$$

Q. At what position an object should be placed in front of a concave mirror of radius of curvature 0.4m so that an erected image of magnification 3 is produced?

Solution: Here,

Radius of curvature (R) = 0.4m

$$\text{Focal length (f)} = \frac{R}{2} = \frac{0.4}{2} = 0.2\text{m}$$

$$\text{Magnification (m)} = 3$$

$$\text{Object distance (u)} = ?$$

We know that, for the formation of an erected image, the magnification (m) is taken as negative, i.e.,

$$m = \frac{-v}{u} = 3$$

$$\text{or, } v = -3u$$

Now using mirror formula we get,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{0.2} = \frac{1}{u} - \frac{1}{3u}$$

$$\text{or, } 5 = \frac{3-1}{3u}$$

$$\text{or, } 5 = \frac{2}{3u}$$

$$\text{or, } u = \frac{2}{3 \times 5}$$

$$\text{or, } u = 0.133\text{m} = 13.3 \text{ cm}$$

Hence, the position of an object is 13.3cm in front of a concave mirror.

Q.A convex mirror with a radius of curvature 30cm forms a real image 20cm from its pole. Explain how it is possible and find whether the image is erected or inverted.

Solution: Here,

Radius of curvature of a convex mirror (R) = -30cm

Image distance (v) = 20cm

Object distance (u) = ?

$$\text{Focal length of convex mirror (f)} = \frac{R}{2} = \frac{-30}{2} = -15\text{cm}$$

We know that, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\text{or, } \frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{-15} - \frac{1}{20} = \frac{3-4}{60} = \frac{-1}{60}$$

$$\therefore u = -\frac{60}{1} = -60\text{cm}$$

The negative sign shows that the object is virtual. Since, u and v have opposite signs; the image is erected with respect to the object.

Q. An object 10cm high is placed in front of a convex mirror of focal length 20cm and the object is 30cm from the mirror. Find the height of the image.

Solution: Here,

Height of object (O) = 10cm

Height of image (I) = ?

Focal length of a convex mirror (f) = -20cm

Object distance (u) = 30cm

If v is the image distance, then,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{-20} = \frac{1}{30} + \frac{1}{v}$$

$$\text{or, } \frac{1}{v} = -\frac{1}{20} - \frac{1}{30} = -\frac{5}{60}$$

$$\text{or, } v = -12 \text{ cm}$$

$$\text{Since, magnification (m)} = \frac{\text{Size of image (I)}}{\text{Size of object (O)}} = \frac{\text{Image distance (v)}}{\text{Object distance (u)}}$$

$$\text{or, } \frac{I}{10} = \frac{-12}{30}$$

$$\therefore I = -\frac{120}{30} = -4 \text{ cm}$$

In general, I = 4cm = 4cm

Thus, height of image is 4cm.

Sample Question:

Why is the focal length of a plane mirror infinite? Explain what it means that the focal point is to be at infinity.

14.6 Application of Spherical Mirrors**Concave mirror**

- i. **A concave mirror in reflecting type astronomical telescopes of large aperture:** A concave mirror is used in reflecting type astronomical telescopes of large aperture because a lens of a very large aperture, and free from various defects cannot be obtained in practice.
- ii. **A concave mirror for shaving:** A concave mirror is preferred to a plane mirror for shaving because when a person keeps his face between the pole and the focus of the concave mirror, an erected and highly magnified virtual image is formed. As a result, a proper close shaving will be possible.
- iii. **A concave mirror in the ophthalmoscope:** The concave mirror is used in ophthalmoscope so that by looking through the hole at the centre of the concave mirror, doctors can examine the eyes in a proper way.
- iv. **The concave mirror as reflector:** Concave mirrors are used as reflectors in cinema projectors, magic lanterns, etc. Also, the concave parabolic mirrors are used in searchlights.

Convex mirror in vehicles for rear view:

A convex mirror is used in vehicles for rear view. The objects behind the vehicle are in front of the convex mirror so that the virtual, erected and diminished images are produced between the pole and the focus. This gives the clear view of the most of the rear objects to the driver. Thus, a convex mirror provides a much wider field of view; however, it cannot give the exact idea about the distance and speed of the approaching vehicle.

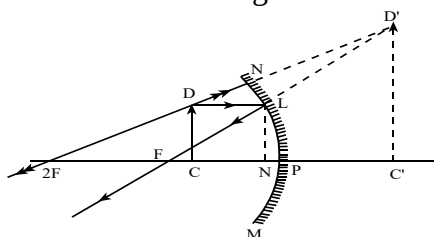
Project Work

Visit the nearest locations and observe mirrors used in different types of devices. Categorize the types of mirrors you observed. Discuss with your friends in class and interact with your teacher about conclusions.

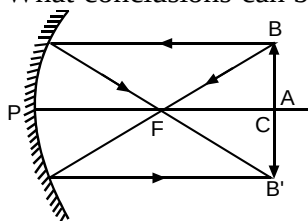
SN	Devices that use mirrors	Use of mirror	Surface of mirror	Type of mirror
1.	Mirrors used in a trial room of a fancy shop	To check the matching of clothes	Plane	Plane
2.	Viewing glass of a car, bus			
3.	Viewing glass of a motor bike, scooter			
4.	Viewing glass of your bath room			
5.	Mirrors used in beauty parlors			
6.	Mirrors used at barbers'			
7.	Mirror used by your mother while doing make up			
8.	Mirror used by your father while shaving			
9.	Dental instruments			
10.				

Exercise

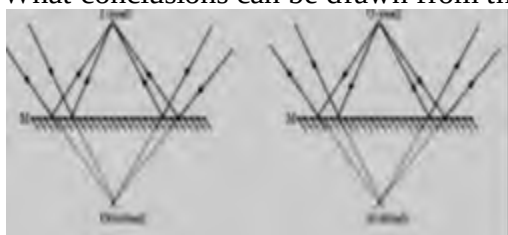
- Which of the following mirrors has the maximum field of view?
 - plane mirror
 - concave mirror
 - convex mirror
 - cylindrical mirror
- What is the nature of image formed in the figure below?



- virtual, erect and magnified
 - real, erect and magnified
 - virtual and inverted
 - virtual, erect and diminished
- What conclusions can be drawn from this ray diagram?



- The size of object is equal to the size of image.
 - The size of object is larger than the size of image.
 - The size of object is smaller than the size of image.
 - The size of object is equal to the size of image when object is placed beyond $2F$.
- What conclusions can be drawn from these figures?



- A plane mirror can form a virtual image only.
 - A plane mirror can form a real image only.
 - A plane mirror can form both types of images in all cases.
 - A plane mirror generally forms a virtual image but for a virtual object it gives a real image.
- Which of the following is the nature of the image produced by a convex mirror for a virtual object?
 - inverted
 - diminished
 - virtual
 - real

6. Which of the following mirrors is used for shaving?
- concave
 - bifocal
 - convex
 - convex cylindrical
7. Which of the following is the nature of image formed by a convex mirror?
- real and virtual
 - virtual and erected
 - real and magnified
 - virtual and magnified
8. A concave mirror of focal length f in air is immersed in water. Which of the following is the focal length of the mirror?
- f
 - $\frac{4f}{3}$
 - $\frac{3f}{4}$
 - $\frac{7f}{3}$
9. Which of the following is true about the focal length of a concave mirror?
- maximum for red color
 - maximum for violet color
 - maximum for yellow color
 - same for all colors
10. An object is placed at 20 cm from a convex mirror of focal length 10 cm. Which of the following is true about the image?
- real and at 20cm from the mirror
 - virtual and 20cm from the mirror
 - virtual and at $\frac{20}{3}$ cm from the mirror
 - real and at $\frac{20}{3}$ cm from the mirror
11. An erected object of size 1m is placed in front of a concave mirror at infinity. Which is true about the size of the image formed?
- of the same size as the object
 - half of the size of the object
 - two times the size of the object
 - a point image of no definite size
12. What will be the distance of the object when a concave mirror produces a real image of magnification m ? (The focal length of the mirror is f .)
- $\frac{(m-1)}{m} \times f$
 - $\frac{(m+1)}{m} \times f$
 - $\frac{m-1}{f}$
 - $(m+1)f$
13. A convergent beam of light converges at a point 12 cm from the pole of a convex mirror. An inverted image of the same size is formed coincident with the virtual object. What is the focal length of the mirror?
- 24 cm
 - 12 cm
 - 6 cm
 - 3 cm
14. A spherical mirror produces a real image of a pointed object placed in front of it. The distances of the object and the image from the mirror are 20cm and 30cm respectively.

What is the focal length and type of spherical mirror?

- a. 120mm, convex mirror
- b. 60mm, convex mirror
- c. 120mm, concave mirror
- d. 60mm, concave mirror

Q N	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Answer	c	a	a	d	d	a	b	a	d	c	d	b	c	c

15. (a) Distinguish between real and virtual images.

(b) Can a plane mirror ever form a real image?

(c) Explain with figure that how a plane mirror can form a real image.

(d) If you are bringing a plane mirror towards your face at right angles to your face with a speed of 10ms^{-1} , at what rate is the image approaching? [20ms^{-1}]

16. (a) What type of mirror will you suggest for shaving or make up purpose and why?

(b) Write mirror formula?

(c) For a spherical convex mirror of focal length 25 cm, an object of height 2cm is placed perpendicular to the principal axis at a distance of 100 cm. Where will the image be formed and what will be the size and nature of the image?

[at 20 cm behind the mirror, 0.4 cm, virtual]

17. (a) Spherical mirror may behave like a plane mirror as a special case. Explain.

(b) A convex mirror is immersed in water; will the focal length change?

(c) A concave mirror of focal length f in air is immersed in water. What will be the new focal length? Explain

18. (a) A convex and a concave mirror are fitted on a wall. How will you distinguish between them without touching?

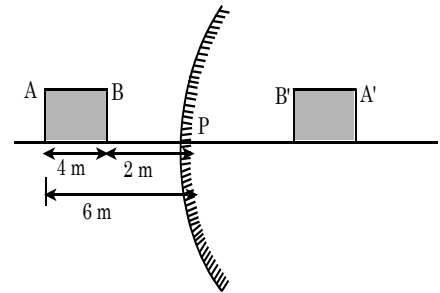
(b) A ray of light falling normally on a mirror retraces its path, why?

19. How far from a lamp must a concave mirror of focal length 3m be placed in order to throw an image on a screen 8 m from the lamp? [4m]

20. An object is placed in front of a convex mirror of 20 cm radius of curvature. Its image is formed at a distance of 8 cm behind the mirror. Find the distance of the object from the mirror. [40 cm]

21. An object is at a distance of 10 cm from mirror and image of the object is at a distance of 30 cm from the mirror on the same side of the object. Is the mirror concave or convex? What is its focal length? [Concave, 7.5 cm]

22. An object 1 cm high is placed at a distance of 1.5 meter from a screen. How far be the concave mirror placed so that it may form a 2 cm high image of the object on the screen? Also find the focal length of the mirror. [at 1.5 m from the object, 1 m]
23. A concave mirror produces magnification of 0.5 when an object is placed at a distance of 60 cm from it. Where should the object be placed so that a virtual image of double the size is formed? [10 cm]
24. A pole of 4m length is laid along the principal axis of a convex mirror of focal length 1m as shown in figure. The end of the pole nearer the mirror is 2m from it. Find the length of the image of the pole. [19cm]



25. An object is situated 40cm from a convex mirror. When a plane mirror is inserted between the object and the convex mirror at a distance of 32 cm from the object, the images in the two mirrors coincide. What is the focal length of the convex mirror? [-60cm]

Chapter 15

Refraction at Plane Surfaces

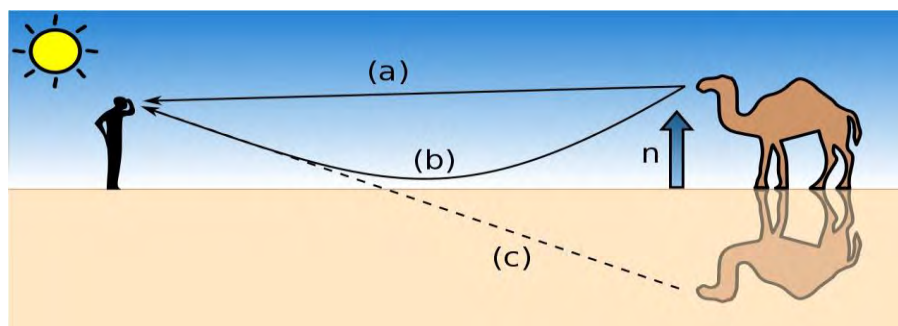
Willebrord Snell was a 17th century Dutch astronomer and mathematician. One of his greatest findings in the world of science is law of refraction. Snell brought about a major revolution when he developed a new method to calculate the mathematical function 'Pi', which led to tremendous development in the field of mathematics. Snell had published several of his works; the two most acknowledged and acclaimed are 'Eratosthenes Batavus' and 'Tiphys Batavus'. He also worked in the field of astronomy.



Willebrord Snell (1580-

Do you Know?

1. A crack in glass appears shining when viewed from a suitable direction.
2. A clear pool of water appears shallower than it actually is?
3. The sun looks a little oval when it is at the horizon?





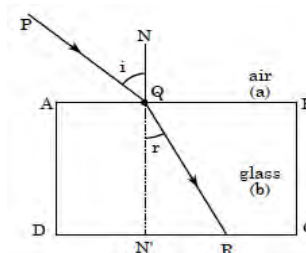
15.1 Introduction

Refraction of light is the fundamental concept in the field of optics which unveils the behavior of light as it travels through different media. Refraction of light, bending of light rays as they pass from one medium to another, plays a vital role in twinkling of stars in the night sky, bending of straw in a glass of water, shimmering of a mirage in a desert. The study of this fundamental phenomenon, refraction, provides a better understanding of the principles involved in the above mentioned phenomena and the mysteries behind them. Also, the study of refraction provides the foundation for the design of optical lenses, prisms, optical pipes, optical fibers and many more. This chapter provides a clear picture of the refraction of light and its influencing behaviors on our world.

15.2 Refraction of light

When a ray of light travels from one medium to another it encounters a change in density. As a result, the velocity of light changes, and light suffers change in initial direction in the second medium. This phenomenon is called refraction of light. **Hence, refraction is the phenomenon of bending of light as it goes obliquely from one medium to another.**

Consider a ray of light PQ incidents on the surface AB of air-glass interface at a point Q so that it bends towards the normal NN' and finally goes along QR as shown in figure 15.1. PQ and QR are incident ray and refracted ray respectively.



[Fig.15.1, Refraction of light from air -glass interface]

Similarly, $\angle PQN$ and $\angle N'QR$ are angles of incidence and angle of refraction respectively. When light encounters change in density of medium, it also suffers a change in velocity. The change in velocity results in the change of direction of light. This change in direction of light as it passes through a medium of varying density is called the refraction of light.

15.3 Laws of Refraction of Light

1. The incident ray, refracted ray, and normal at the point of the incident, all lie in the same plane.
2. When light travels from a rarer medium to a denser medium, it bends toward the normal, and when light travels from a denser medium to a rarer medium; it bends away from the normal.
3. For a given pair media, the ratio of the sine of the angle of incidence ($\sin i$) to the sine of the angle of refraction ($\sin r$) is constant. The constant is called refractive index of the second medium with respect to the first medium i.e. ${}_1\mu_2 = \frac{\sin i}{\sin r}$. This is Snell's law.

15.4 Refractive Index

Refractive index is the fundamental physical property that describes how light propagates through a material. It indicates the speed at which light travels through a particular material relative to the speed of light in a vacuum. For example, the refractive index of water is approximately 1.33 which means that the speed of light in water is about 75% of the speed of light in a vacuum. The materials with higher refractive index are considered more optically dense as they can slow down the propagation of light more than materials with lower refractive index. The refractive index is denoted by μ . It is a dimensionless quantity. The refractive index of the medium b with respect to another medium a, is given by ${}_a\mu_b = \frac{\sin i}{\sin r}$.

Also, the ratio of the velocity of light in a vacuum (c) to the velocity of light in a medium (v) gives the refractive index of a medium (μ). When light travels through a vacuum to any other medium, the refractive index is called the absolute refractive index. Therefore, absolute refractive index,

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{velocity of light in medium}} = \frac{c}{v}$$

$$\text{Again, } {}_a\mu_b = \frac{\mu_b}{\mu_a} = \frac{\sin i}{\sin r} \text{ and } \mu_a \sin i = \mu_b \sin r.$$

Value refractive index changes with the change in wavelength of light, nature of materials, temperature, applied pressure, doping level, etc. The refractive index does not make sense in the case of opaque objects.

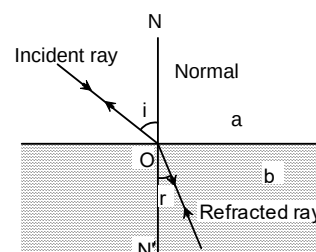
Reversibility of Light

Reversibility of light states that "If the path of light is reversed even after series of refractions and reflections it travels along the same path in opposite direction." The ray diagram of the reversibility of light is shown in figure 15.2.

Suppose, a ray of light enters from medium a to medium b at an angle of incidence i and makes an angle of refraction r in the second medium.

From Snell's law,

$$\text{Refractive index of medium b with respect to medium a, } ({}_a\mu_b) = \frac{\sin i}{\sin r} \dots (1).$$



[Fig15.2, Reversibility of

Again, if a ray of light enters from medium b at an angle of incidence r , then the angle of refraction in medium a will be i . Hence,

$$\text{Refractive index of medium a with respect to medium b, } ({}_b\mu_a) = \frac{\sin r}{\sin i} \dots (2).$$

Multiplying equation (1) and (2) ,

$${}_a\mu_b \times {}_b\mu_a = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1$$

$$\text{Therefore, } {}_a\mu_b = \frac{1}{{}_b\mu_a} \dots (3)$$

Hence the refractive index of medium a with respect to medium b is equal to the reciprocal of the refractive index of medium b with respect to medium a.

Workout Example:

How long will the light take in travelling a distance of 500 m in water? The refractive index of water is 1.33, and the velocity of light in a vacuum is 3×10^8 m/s.

Solution: Here,

Time taken in travelling (t) = ?

Distance (d) = 500m

Refractive index of water (${}_a\mu_w$) = 1.33

Velocity of light in vacuum (c) = 3×10^8 ms⁻¹

$$\text{We have, } {}_a\mu_w = \frac{c}{v}$$

$$\text{or, } v = \frac{c}{{}_a\mu_w} = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8 \text{ ms}^{-1}$$

We have the relation,

$$v = \frac{d}{t} = \frac{500}{2.26 \times 10^8} = 2.2 \times 10^{-6} \text{ s.}$$

Hence, time taken by light in travelling a distance is 500 m is 2.2×10^{-6} s.

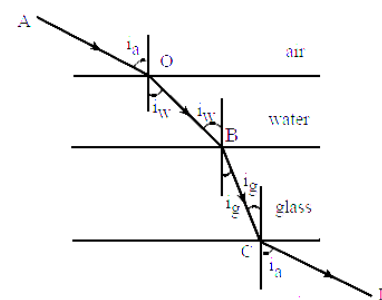
Sample Questions:

1. On what factors does the value of refractive index depend?
2. Differentiate between an optically denser medium, and an optically rarer medium.
3. Is it possible for the absolute refractive index of a medium to be less than 1?

15.5 Relation between Relative Refractive Indices

"Refraction of light is a phenomenon observed as the light passes through different media like air, water, and glass. When a ray of light, represented by AO, passes from air to water, it bends to follow the path OB. Similarly, as it moves from water to glass, the ray follows BC. Finally, emergent ray CD goes parallel to the incident ray AO, with the angle of emergence equal to the angle of incidence as shown in figure 15.3.

Let, i_a and i_w be the angle of incidence and angle of refraction for the air-water interface then, the corresponding refractive index,



[Fig.15.3, Refraction of light through three media: air, water and glass]

$${}_a\mu_w = \frac{\sin i_a}{\sin i_w} \quad \dots (1)$$

$$\text{For water-glass interface, } {}_w\mu_g = \frac{\sin i_w}{\sin i_g} \quad \dots (2)$$

$$\text{For glass-air interface, } {}_g\mu_a = \frac{\sin i_g}{\sin i_a} \quad \dots (3)$$

Multiplying equations (1), (2) and (3), we get

$${}_a\mu_w \times {}_w\mu_g \times {}_g\mu_a = \frac{\sin i_a}{\sin i_w} \times \frac{\sin i_w}{\sin i_g} \times \frac{\sin i_g}{\sin i_a}$$

$$\text{or, } {}_a\mu_w \times {}_w\mu_g \times {}_g\mu_a = 1 \quad \dots (4)$$

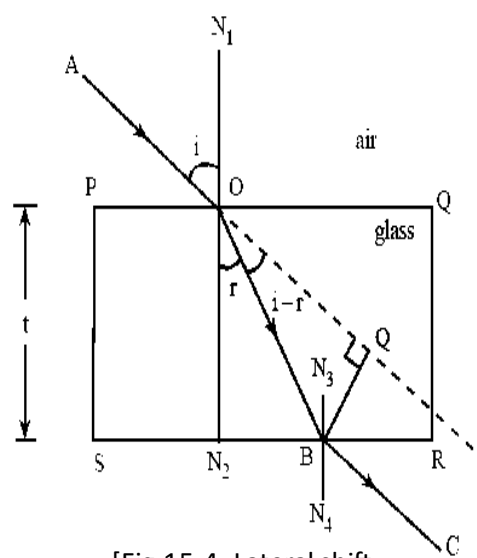
This gives the relation between refractive indices of different media.

Also, from equation (4)

$${}_w\mu_g = \frac{1}{{}_a\mu_w \times {}_g\mu_a}$$

$$\text{or, } {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} \quad \left[\text{Since, } {}_a\mu_g = \frac{1}{{}_g\mu_a} \right]$$

15.6 Lateral Shift during Refraction through a Glass Slab



[Fig.15.4, Lateral shift

When light refracts from a rarer to a denser medium, and then emerges back into the rarer medium, the incident and emergent rays are parallel. "The perpendicular distance between the incident ray and the emergent ray is called the lateral shift."

Consider, a parallel-sided glass slab PQRS of refractive index μ having thickness t . A ray of light AO is incident on the upper face PQ of the slab at angle of incidence i . Due to refraction through the air-glass interface, the ray bends towards the normal and follows the path OB. Finally, the ray emerges out along the path BC as shown in figure 15.4.

$$\text{From } \triangle OBQ, \quad \sin(i - r) = \frac{BQ}{OB}$$

$$\therefore BQ = OB \sin(i - r) \quad \dots (1)$$

$$\text{Again, in } \triangle ON_2B, \cos r = \frac{ON_2}{OB}$$

$$\text{or, } OB = \frac{ON_2}{\cos r}$$

$$\text{or, } OB = \frac{t}{\cos r} \quad \dots (2). \quad [\text{Since, } ON_2 = \text{thickness } t]$$

From equation (1) and (2),

$$BQ = L = \frac{t}{\cos r} \sin(i - r)$$

$$\text{Therefore, lateral shift (BQ)} = L = \frac{t \sin(i - r)}{\cos r} \quad \dots (3)$$

If we take, $i = 90^\circ$ then, equation (3) will be

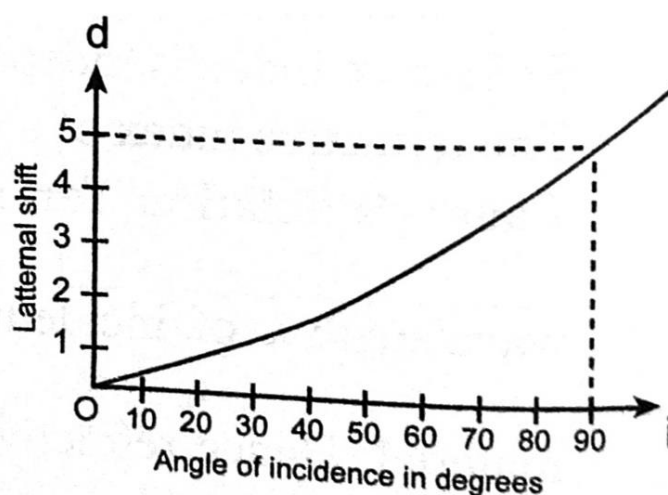
$$L = \frac{t \sin(90^\circ - r)}{\cos r}$$

$$\text{or, } L = \frac{t \cos r}{\cos r}$$

$$\text{or, } L = t$$

$\therefore$ Lateral shift = thickness of a glass slab [It is the case of grazing incidence.]

Hence, when light is incident at 90° on the surface of a glass slab, the lateral shift produced by it is equal to the thickness of the glass slab. The variation of lateral shift with the angle of incidence is shown in the figure below.



[Fig. 15.5; Variation of lateral shift with angle of incidence]

For small angle of incidence i , $\sin i = i$, $\sin r = r$, $\cos r = 1$, $\frac{\sin i}{\sin r} = \frac{i}{r} = \mu$ and

$$\text{Lateral shift, } L = t(i - r) = t i \left(1 - \frac{1}{\mu}\right).$$

Thus lateral shift depends on the nature of the material of the slab (μ), angle of incidence (i) and thickness of the slab (t)

Workout Example:

A light ray is incident at an angle of 45° with the normal to a $\sqrt{2}$ cm thick plate ($\mu = 2.0$). Find the lateral shift in the path of the light as it emerges out from the plate.

Solution: Here,

Angle of incidence (i) = 45°

Thickness (t) = $\sqrt{2}$ cm

Refractive index (μ) = 2.0

Let, r be the angle of refraction corresponding to the angle of incidence.

From Snell's law, we have,

We know that, $\mu = \frac{\sin i}{\sin r}$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45^\circ}{2} = \frac{1}{2\sqrt{2}}$$

$$\therefore r = \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) = 20.7^\circ$$

Lateral shift,

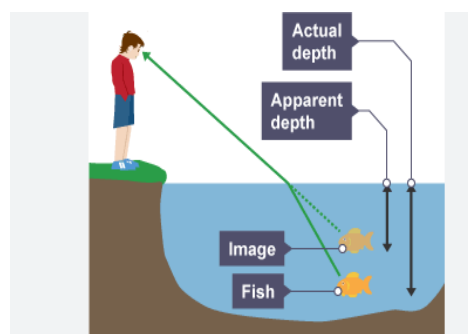
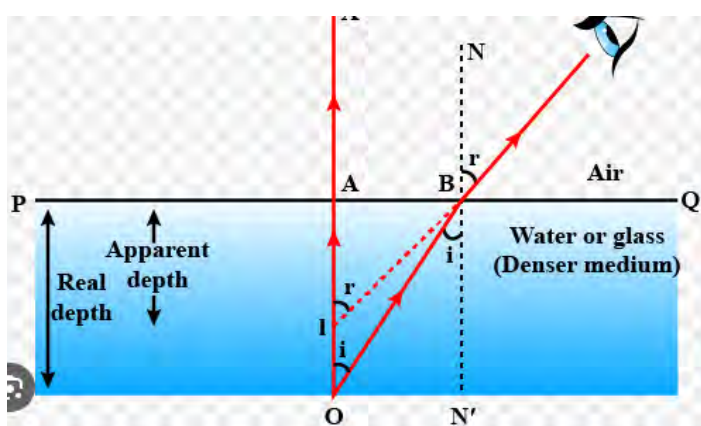
$$L = \frac{\sin(i - r)}{\cos r} \times t = \frac{\sin(45^\circ - 20.7^\circ)}{\cos 20.7^\circ} \times \sqrt{2} = \frac{\sin 24.3^\circ}{\cos 20.7^\circ} \times \sqrt{2} = 0.62 \text{ cm}$$

Therefore, lateral shift = 0.62 cm

Sample Questions:

1. How does the lateral shift change with the change in the angle of incidence?
2. For which angle of incidence, a parallel-sided glass slab produces maximum lateral shift?

15.7 Real Depth and Apparent Depth



[Fig. 15.6, Real and apparent depth]

When an object is viewed from a rarer medium while submerged in a denser medium, it appears shallower than its actual depth. The actual depth is termed as the real depth whereas the perceived depth to the eye is known as the apparent depth.

Hence, the real depth is the actual depth of the object placed in a denser medium and the apparent depth is the virtual depth that is observed as a result of the refraction of light. The apparent depth depends upon the refractive index of the medium.

Consider, a point object O placed at the depth R (=AO) below the surface of a transparent but denser medium like water or glass.

For vertical observation, there is no normal shifting, and no case of apparent depth or shifting of the image.

For oblique observation as in figure 15.6, incident ray OB appears to meet with OA at point I after refraction on the interface. Therefore, I is the virtual image of object O. So AI = a is apparent depth and OI = d is apparent shift (normal shift).

From Snell's law, ${}_w\mu_a = \frac{\sin i}{\sin r} \dots (1)$

In $\triangle AIB$ and $\triangle AOB$, $\sin r = \frac{AB}{BI}$ and $\sin i = \frac{AB}{OB}$

Therefore, $\frac{\sin i}{\sin r} = \frac{\frac{AB}{OB}}{\frac{AB}{BI}} = \frac{AB}{OB} \times \frac{BI}{AB} = \frac{BI}{OB} = \frac{AI}{AO} = \frac{a}{R} \dots (2)$ [where $BI \approx AI$ and $OB \approx AO$]

From equation (1) and (2),

$${}_w\mu_a = \frac{a}{R} \dots (3)$$

$$\text{or, } {}_a\mu_w = \frac{R}{a} = \frac{\text{Real depth}}{\text{apparent depth}} \quad \text{Therefore,}$$

If the object is inside, and the observer is outside the medium, $\mu = \frac{R}{a} = \frac{\text{Real depth}}{\text{apparent depth}}.$

If the object is outside, and the observer is inside the medium, $\mu = \frac{a}{R} = \frac{\text{Apparent depth}}{\text{Real depth}}.$

Apparent shift:

From figure 15.6,

Real depth = Apparent depth + Apparent shift

i.e., $R = a + d$

or, $d = R - a$

or, $d = R - \frac{R}{{}_a\mu_w}$ [From equation (3)]

$$\text{or, } d = R \left(1 - \frac{1}{{}_a\mu_w} \right) \dots (4)$$

$$\text{or, } d = t \left(1 - \frac{1}{{}_a\mu_w} \right)$$

This gives the apparent shift.

If several transparent parallel media of refractive indices $\mu_1, \mu_2, \dots$ and thickness $t_1, t_2, \dots$ placed one above the other, total apparent depth and total apparent shift are given by,

$$a = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \dots,$$

$$d = \left(1 - \frac{1}{\mu_1}\right)t_1 + \left(1 - \frac{1}{\mu_2}\right)t_2 + \dots$$

Consequences of refractions:

1. A clear pool of water always appears shallower than it actually is.
2. A stick partially dipped in water seems to be bent.
3. Person inside water looking at a tree on the shore of water sees the tree much taller than it actually is.

Workout Example:

A microscope is focused on a scratch on the bottom of a beaker. Turpentine is poured into the beaker to a depth of 4cm and it is found necessary to raise the microscope through a vertical distance of 1.25 cm to bring the scratch again into focus. Find the refractive index of the turpentine.

Solution: Here,

Real depth = 4cm

Apparent shift = 1.25 cm

Apparent depth = 4 – 1.25 = 2.75cm

(Since, R.D. = A.D. + A.S. $\Rightarrow$ A.D. = R.D.– A.S.)

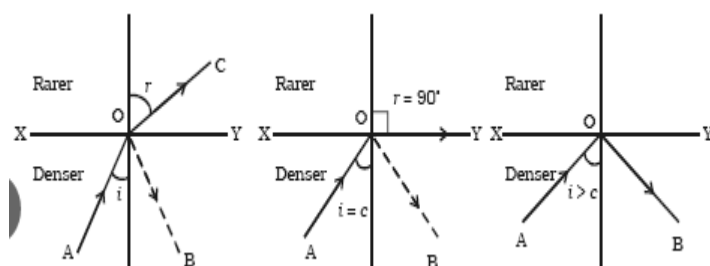
We know, $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{4}{2.75} = 1.45$

Hence, refractive index = 1.45

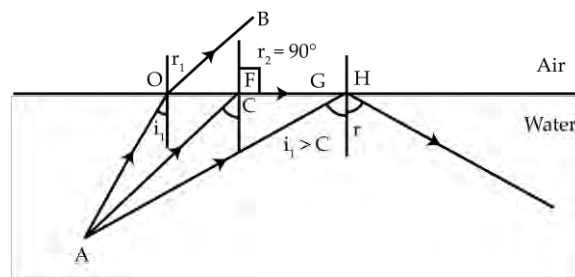
Sample Questions:

1. During midday objects are seen shaking in summer, why?
2. If a plane glass slab is placed on letters of different colours, then violet-coloured letters appear more and more raised up, why?
3. What is the apparent position of an object below a rectangular block of glass 6 cm thick if a layer of water 4cm thick is on the top of the glass? (Refractive index of glass = 1.5 and that of water = 1.33)
[7cm]

15.8 Critical Angle and Total Internal Reflection



While light travels from a denser to a rarer medium, the angle of refraction increases with the angle of incidence. At a particular angle of incidence in the denser medium, the angle of refraction in the rarer medium reaches 90° . This angle of incidence is called critical angle, and it is denoted by C . Here C is the threshold value of the angle of incidence for total internal reflection occurs. Hence critical angle is the angle of incidence in the denser medium beyond which light reflects entirely back into the denser medium. The value of the critical angle varies between different pairs of media but remains constant for a specific pair.



[Fig. 15.7, Total internal

Consider, the ray of light from object A , placed in denser medium water, incidents at a point O on the denser-rarer interface separating two media so that the refracted ray bends away from the normal in rarer medium air as shown in figure 15.7. On gradual increase of angle of incidence, the angle of refraction also goes on increasing, and becomes 90° . The corresponding angle of incidence is the critical angle (C). At this condition,

$${}_w\mu_a = \frac{\sin i}{\sin r_2}$$

$$\text{or, } {}_w\mu_a = \frac{\sin C}{\sin 90^\circ} \quad [i = C \text{ and } r_2 = 90^\circ]$$

$$\text{or, } \frac{1}{{}_a\mu_w} = \frac{\sin C}{\sin 90^\circ} = \frac{\sin C}{1} \quad [\sin 90^\circ = 1]$$

$$\text{or, } \sin C = \frac{1}{{}_a\mu_w}$$

$$\therefore C = \sin^{-1} \left(\frac{1}{{}_a\mu_w} \right)$$

This gives the critical angle for a given pair of media; water and air. Hence, the value of the critical angle for a particular pair of media is constant, but it will be different for different pairs of media.

If the angle of incidence in the denser medium is less than critical angle, the refraction of light takes place in a normal way. But, if the angle of incidence is greater than the critical angle, light undergoes total internal reflection in the same denser medium as shown in figure 15.7. In figure, for ray of light AH , $\angle i$ is greater than $\angle C$ and $\angle r$ is greater than 90° . Therefore, this is the case of total internal reflection. Hence the complete reflection of a light ray reaching an interface with a less dense medium when the angle of incidence exceeds the critical angle is called total internal reflection TIR. TIR occurs *when the light rays travel from a more optically denser medium to a less optically denser medium*. Therefore, following are the conditions for total internal reflection;

1. The ray must pass from a denser medium to a rarer medium or an object must be in a denser medium.
2. The angle of incidence should be greater than the critical angle.

Workout Example:

Calculate the critical angle for (i) an air-glass surface (ii) an air-water surface (iii) a water-glass surface. (${}_a\mu_g = 1.5$, ${}_a\mu_w = 1.33$)

Solution: Here,

- i. For an air – glass surface,

$$\sin C = \frac{1}{{}_a\mu_g} = \frac{1}{1.5} = 0.6667$$

$$C = \sin^{-1}(0.6667) = 41.8^\circ$$

Hence, critical angle of air-glass surface is 41.8°

- ii. For an air-water surface,

$$\sin C = \frac{1}{{}_a\mu_w} = \frac{1}{1.33} = 0.7519$$

$$C = \sin^{-1}(0.7519) = 48.8^\circ$$

Hence, the critical angle of air-water surface is 48.8°

For a water-glass surface,

$$\sin C = \frac{1}{{}_a\mu_w} = \frac{1}{{}_w\mu_a \times {}_a\mu_g} = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{1.33}{1.5} = 0.8867$$

$$\therefore C = \sin^{-1}(0.8867) = 62.5^\circ$$

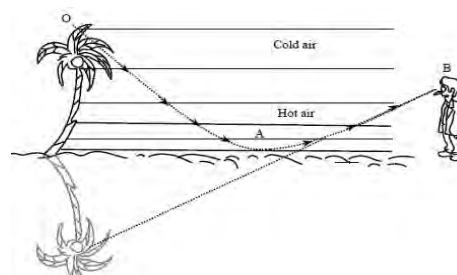
Hence, the required critical angle for a water glass surface is 62.5°

Sample Questions:

1. Why is total internal reflection called so?
2. Light from a luminous point on the lower face of a rectangular glass slab 2.0cm thick strikes the upper face, and the totally reflected rays outline a circle of 3.2cm radius on the lower face. What is the refractive index of the glass?

[1.6]

15.8.1 Mirage



[Fig.15.6, Mirage formation during hot sunny

A mirage occurs on hot sunny days when light from distant objects undergoes total internal reflection in the atmosphere. This creates an optical illusion where the light appears to come from the ground, forming a virtual and inverted image just beneath the object. It's all about how light bends in different temperatures, creating a fascinating illusion.

On a hot day, the air near the Earth's surface heats up, while the layers above cool down. This temperature difference causes variations in air density, which affects the refractive index. As

a result, the refractive index decreases gradually from the upper layers to the lower layers. This setup creates conditions favorable for total internal reflection of light from distant objects.

Consider a ray, OA, from a distant object. As it passes through each layer of air, it bends away from the normal due to refraction. Eventually, when the angle of incidence reaches the critical angle, the ray undergoes total internal reflection, moving upward along AB. This phenomenon creates an illusion where the distant object appears to be reflected as if it were over a body of water.

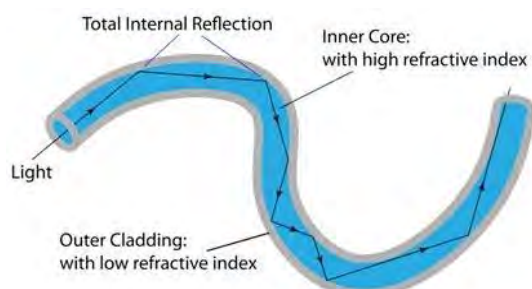
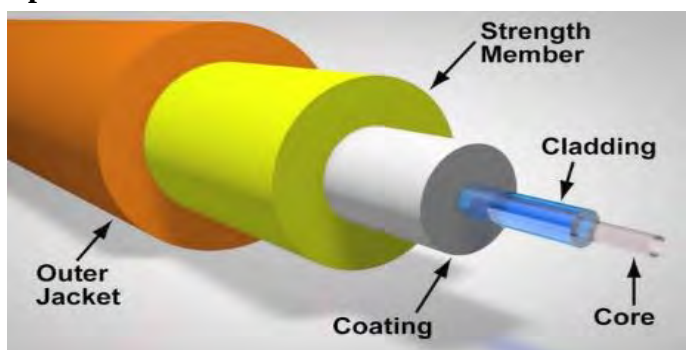
An observer at point B sees the image of the object just below its actual position, accompanied by a portion of the sky surrounding it. This optical illusion mimics the appearance of an object reflected in water, creating what we perceive as a mirage.

Sample Questions:

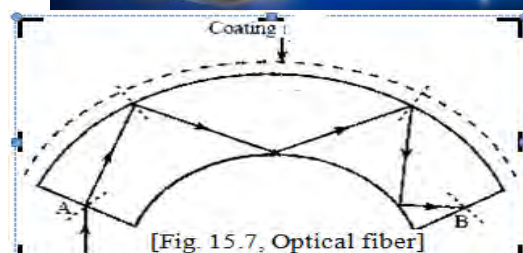
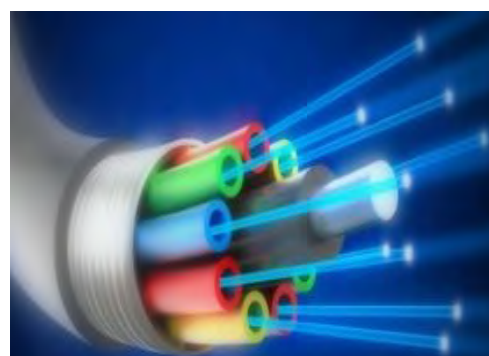
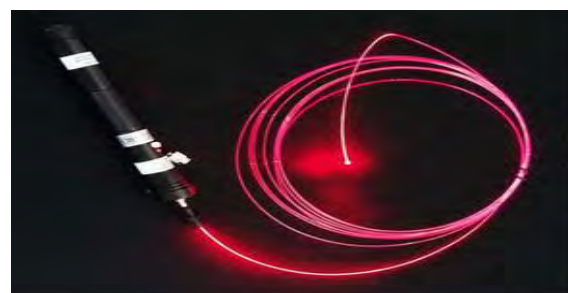
1. Why does a black-topped road often appear to be covered with water when seen from a distance in summer?
2. Describe any natural phenomenon in which total internal reflection plays a part.
3. A ray of light in the air strikes a glass surface. Is there a range of angles for which total internal reflection takes place?
4. Can total internal reflection be achieved if an object originates in rarer medium?

15.7 Applications of Total Internal Reflection

a. Optical Fiber:



An optical fiber is a device based on total internal reflection. With the use of an optical fiber, a light signal can be transmitted from one place to another with a negligible loss of energy. It is a very long and



thin pipe (thickness nearly 10^{-4} m) of a transparent material with a double layer such that the outer layer has a relatively lower refractive index. Large number of such fibers is held together from a light pipe. Optical fibers are used for the signal for communication. A ray of light strikes from one end at a small angle of incidence so that firstly it suffers refraction and then multiple total internal reflections and finally strikes the other end at an angle less than the critical angle. As a result, the ray emerges out in air as shown in figure 15.7. There is no loss of energy during total internal reflection. The optical fiber is used in light pipe. The application of the principle of total internal reflection in optical fibers has revolutionized communication field, medical field, and optical examinations field. Furthermore, the optical fiber has numerous applications as listed below.

i) **Telecommunications:**

Optical fibers are extensively used in telecommunications networks to transmit voice, data, and video signals. High-speed and long-distance communication, telephone systems, and cable television are possible with the use of optical fibers. Optical fiber facilitates provide seamless online activities, streaming, and large file transfer.

ii) **Medical Imaging:**

Optical fibers have a crucial role in medical imaging fields, such as endoscopy and fiber optic microscopy. Endoscopes use flexible light pipe to visualize internal organs.

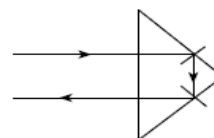
iii) **Data Centers:**

Optical fibers are extensively used in data centers for interconnecting servers. They are also used in storage systems and networking equipments. They make rapid transmission of large volumes of data possible.

b. **Total reflecting prism:**



Total internal reflecting prism is an isosceles right-angled prism made of crown glass. When a ray of light incident at an interface undergoes refraction and makes an angle of incidence greater than critical angle on other interface and it will suffer total internal reflection as shown in figure 15.8. This type of prism is used in binoculars and submarine periscopes.



[Fig.15.8, Inversion by total internal reflection prism]

c. Sparkling of Diamond:



The critical angle for diamond-air interface is very small, nearly 24.4° . Therefore, for most of the cases, when light from an external source enters in a well cut diamond, it suffers multiple total internal reflection inside the diamond. In such situations light entering into diamond from all directions finally emerges out from some particular directions. Therefore, while looking from those directions, the diamond appears sparkling. A diamond cutter utilizes this principle so that multiple total internal reflections occur and sparkling of diamonds becomes possible.

Exercise

- What remains unchanged for light during its refraction from one medium to another medium?
 - speed
 - wave length
 - frequency
 - displacement
- What happens when a monochromatic light travels from air to glass?
 - Frequency and wavelength both decrease.
 - Frequency increases and wavelength decreases.
 - Frequency remains unchanged but wavelength decreases.
 - Frequency and wavelength both remain unchanged.
- Refractive index of glass with respect to air is 1.5. What is the velocity of light in glass?
 - $1.5 \times 10^8 \text{ m/s}$
 - $2 \times 10^8 \text{ m/s}$
 - $3 \times 10^8 \text{ m/s}$
 - $4 \times 10^8 \text{ m/s}$
- A glass slab has thickness of 6 mm, and $\mu = 1.5$. What is the time in nanosecond required for light to pass through it?
 - 0.02
 - 0.03
 - 0.04
 - 0.05
- An air bubble inside a glass slab of refractive index $\mu = 1.5$, appears at 6cm when viewed from one side and 4 cm when viewed from the opposite side. What is the thickness of the slab?
 - 6.67cm
 - 10 cm
 - 15 cm
 - 17cm
- The refractive index of water is $\frac{4}{3}$ and that of glass slab immersed in it is $\frac{5}{3}$. What is the critical angle for a ray of light while going from glass to water?
 - $\sin^{-1}\left(\frac{2}{3}\right)$
 - $\sin^{-1}\left(\frac{3}{4}\right)$
 - $\sin^{-1}\left(\frac{4}{5}\right)$
 - $\sin^{-1}\left(\frac{4}{3}\right)$
- Critical angle for total internal reflection from a medium to a vacuum is 30° . What is the velocity of the light in the medium?
 - $1.5 \times 10^8 \text{ m/s}$
 - $\sqrt{3} \times 10^8 \text{ m/s}$
 - $3 \times 10^8 \text{ m/s}$
 - $6 \times 10^8 \text{ m/s}$
- What principle are the optical fibres based on?
 - interference
 - dispersion
 - refraction
 - total internal reflection

9. For which of the following colours of light, the critical angle is minimum for a given pair of media?

- a. red b. green c. yellow d. violet

10. When a ray of light enters from one medium to another, then its velocity in the second medium becomes double. What must be the maximum value of the angle of incidence so that total internal reflection may not take place?

- a. 30° b. 60° c. 90° d. 180°

Q.N.	1	2	3	4	5	6	7	8	9	10
Answer	c	c	b	b	c	c	a	d	d	a

11. (a) Why does a refraction of light occur when light passes from one medium to another?
 (b) What is lateral shift? How does the lateral shift change with the increase in the angle of incidence?
 (c) Which colour of light travels faster in a medium?

12. (a) Can the refractive index be defined using Snell's law for the case of normal incidence? Explain.

(b) Why does a clear pool of water always appear shallower than it actually is?

(c) If a plane glass slab is placed on letters of different colours, then the violet-coloured letters appear more and more raised up, why?

(d) How long will the light take to travel a distance of 500 m in water? Refractive index of water is 1.33, and velocity of light in a vacuum is 3×10^8 m/s. [2.2×10^{-6} s]

13.(a) On what factors does the value of the refractive index depend?

(b) What is the value of the critical angle for the medium in terms of its refractive index?

(c) Describe any natural phenomenon in which total internal reflection plays a part.

(d) A microscope is focused on a scratch on the bottom of a beaker. Turpentine is poured into the beaker to a depth of 4cm, and it is found necessary to raise the microscope through a vertical distance of 1.25cm to bring the scratch again into focus. Find the refractive index of the turpentine. [1.45]

14. (a) During midday object are seemed to be shacked in summer, why?

(b) Why does the sun look a little oval when it is at the horizon?

(c) An optical fiber ($\mu = 1.72$) is surrounded by a glass coating ($\mu = 1.50$). Find the critical angle for total internal reflection at the fiber glass interface. [60.4°]

15. (a) A ray of light through air strikes a glass surface. Is there a range of angles for which total internal reflection takes place?
 (b) What is a total reflecting prism?
 (c) Why does a crack in glass appear shining when viewed from a suitable direction?
16. (a) A Black-topped road often appears to be covered with water when seen from a distance in summer. Explain
 (b) Why does a piece of diamond sparkle with great brilliance?
 (c) Can total internal reflection be achieved if the object originated in the rarer medium? Explain with a diagram to justify your answer.
 (d) A ray of light from water incidents at an angle of 70° on the water-glass interface. Calculate the angle of refraction in the glass. (${}_a\mu_g = 1.5$, ${}_a\mu_w = 1.33$) [56.4°]
17. The refractive index for water is 1.33, and for glass it is 1.5. A ray of light travelling in water is incident at an angle of 30° upon a water-glass interface. Calculate the angle of refraction into the glass. [26.4°]
18. What is the critical angle for a ray going from glass into water? The refractive indices of glass and water are 1.62 and 1.32 respectively. [54.6°]
19. What is the apparent position of an object below a rectangular block of glass 6 cm thick, if a layer of water 4 cm thick is on top of the glass? (Given, ${}_a\mu_g = 1.5$ and ${}_a\mu_w = 1.33$) [7cm]
20. A glass slab is immersed in water. Find the critical angle at the glass-water interface. (Given ${}_a\mu_g = 1.5$ and ${}_a\mu_w = 1.33$) [62.46°]

Chapter 16

Refraction through Prisms

Augustin-Louis Cauchy was a French mathematician and physicist. He was born on August 21, 1789 in Paris, France, and died on May 23, 1857 in Sceaux. Cauchy was one of the greatest of modern mathematicians. In mathematics, Cauchy pioneered in analysis and the theory of substitution groups. In optics, Cauchy defined an empirical relationship between the refractive index and the wavelength of light for a particular transparent material in 1837. This relation is called Cauchy's transmission equation.

In 1816, Cauchy won the grand prix of the Institute of France for a paper on wave propagation which is now accepted as a classic in hydrodynamics. In 1822 he laid the foundations of the mathematical theory of elasticity.

Cauchy made substantial contributions to the theory of numbers and wrote three important papers on error theory. His work on optics provided a mathematical basis for the workable but somewhat unsatisfactory theory of the properties of the ether, a hypothetical, omnipresent medium once thought to be the conductor of light. His collected works, *Oeuvres complètes d'Augustin Cauchy* (1882–1890), were published in 27 volumes.



Augustin- Louis Cauchy
(21 August 1789 – 23 May 1857)

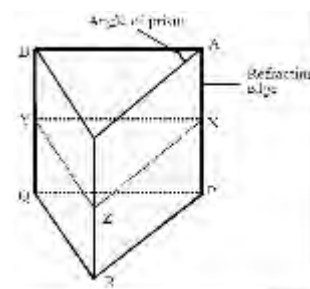
Think?

1. Can a ray of light pass straight through a glass prism?
2. Is there a condition that a ray of light passing through a glass prism suffers minimum deviation?

16.1 Introduction

A prism is a seemingly simple but versatile optical tool. Study of prism provides a gateway to a world of optical phenomena and applications of light. A prism is a simple triangular slab of glass but it has a pivotal role in taking out the mysteries of dispersion of light, spectrum of the rainbow, fundamental properties of light itself and many more. A prism has applications from spectroscopy and photography to telecommunications and beyond. In this chapter we study about the refraction of light through prism in detail.

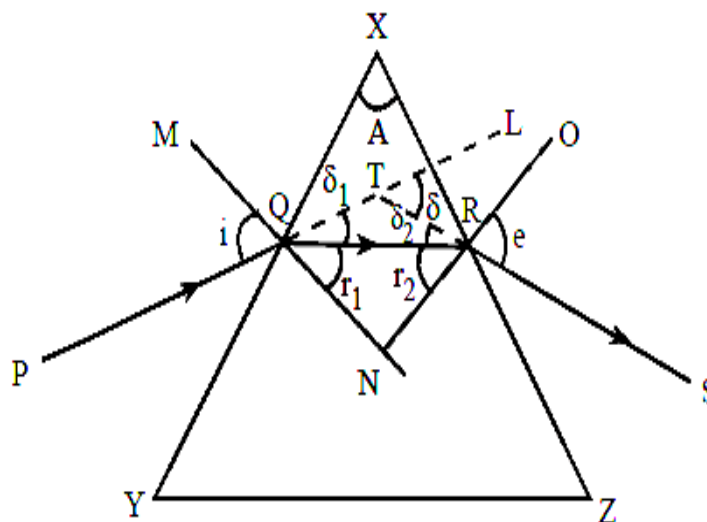
A homogenous transparent refracting medium enclosed by two plane surfaces meeting each other along a straight edge as shown in figure 16.1 is called a prism. The angle between the refracting surfaces is called the refracting angle or angle of prism. The section cut by a plane (XYZ) perpendicular to the refracting surfaces is called the principal section of the prism.



[Fig. 16.1, A glass prism]

16.2 Refraction through a Prism

Consider an equilateral glass prism XYZ having angle of prism A. When a ray of light strikes the face XY at a point Q, it suffers refraction along QR. Refracted ray QR also strikes on the next refracting face XZ and again undergoes refraction. Finally, the ray of light emerges out along RS as shown in figure 16.2.



[Fig.16.2, Refraction through prism]

Let MN and NO be normal at the points Q and R respectively. Suppose angle i and angle e represent angle of incidence in the first face XY and the angle of emergence in the second face XZ. The angle between direction of incident ray and that of emergent ray is called angle of deviation and it is denoted by δ .

From figure 16.2, $\angle LTR = \delta$ is angle of deviation.

From Geometry, $\delta = i + e - A$ and $A = r_1 + r_2$

This is the deviation produced by a glass prism.

Workout Example:

Calculate the angle of emergence e and angle of deviation δ , when light is incident at 90° on the face of a 60° prism of refractive index 1.5.

Solution: Here,

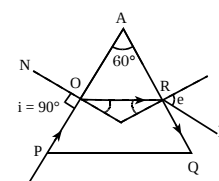
Suppose incident ray PO is refracted at the critical angle C along OR in the glass. From the relations, $\sin c = \frac{1}{\mu} = \frac{1}{1.5} = 0.6667$ and $c = 41.8^\circ$. Since, $A = (c + r_2)$,

Where, $r_2 = 60 - 41.8^\circ = 18.2^\circ$

From, $\frac{\sin e}{\sin r_2} = \mu$, the angle of emergence e is given by, $\frac{\sin e}{\sin 18.2^\circ} = 1.5$

or, $\sin e = 1.5 \times \sin 18.2^\circ = 0.4685$

so, $e = 27.9^\circ$



Angle of deviation,

$$A + \delta = i + e$$

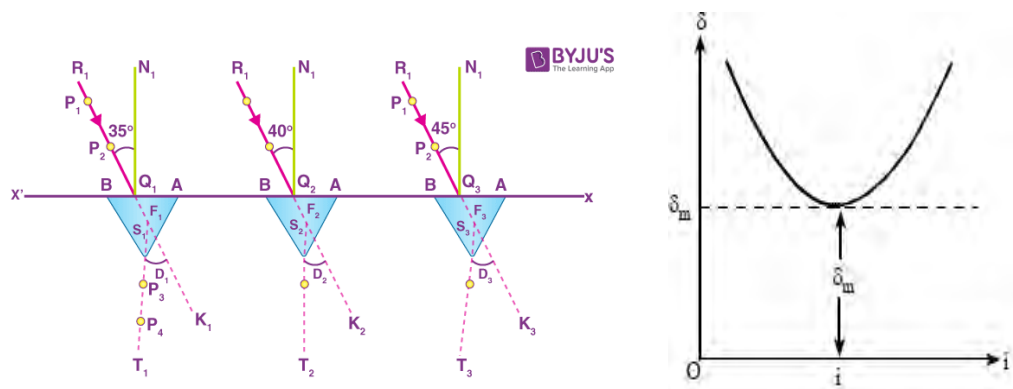
$$\delta = 90 + 27.9 - 60 = 57.9^\circ$$

Hence, angle of emergence = 27.9° and angle of deviation = 57.9° .

Sample Questions:

1. Why don't we consider the lateral shift in prism?
2. A ray of light strikes on the glass prism at an angle of 40° . Find the angle of emergence, if the angle of prism is 60° , and the refractive index is 1.5. [58.5°]
3. An equilateral prism with the refractive index of 1.45 is surrounded by water with the refractive index of 1.33. A narrow beam of light strikes normally on one face of the prism. Find the direction of the emergent ray. [49.3°]

Activity:



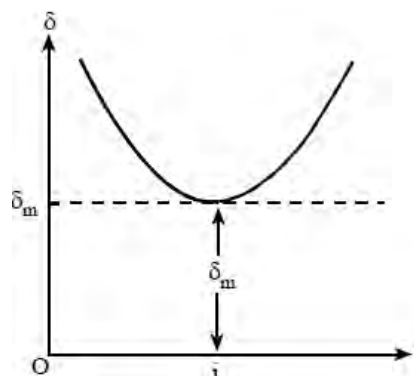
[Fig. 16.3 Deviation produced by a prism]

1. Fix a white sheet on a drawing board using thumb pins.
2. Place the triangular prism resting on its triangular base and using a pencil, draw the outline of the prism.
3. Draw a normal to one face of the prism and make an angle of incidence gradually increasing from 30° and 60° .
4. Fix two pins at a distance of at least 8cm from each other on the line drawn for incident ray and mark these as P_1 and P_2 .
5. Look for the images of the pins P_1 and P_2 through the other face of the prism.
6. Fix two pins at P_3 and P_4 so that they appear erected on the same straight line drawn for an incident ray in the next face of the prism.
7. Remove the pins and the prism.
8. Make a line joining the points P_3 and P_4 .
9. Make necessary extensions of lines, and now mark the angle of incidence $\angle i$, angle of refraction $\angle r$, the angle of emergence $\angle e$, and the angle of deviation $\angle \delta$ from the figure of experimental setup.
10. Repeat the experiment for more angles between 30° and 60° .

11. Draw a graph between the angle of incidence and the angle of deviation. Find the angle of incidence for minimum deviation. Also, measure the angle of incidence and angle of emergence for the condition of minimum deviation. Measure the distance between the refracted ray, and the base of the prism at different points inside the prism.
12. Discuss the findings among your friends in class and interact with the teacher for the conclusion.

16.3 Angle of Minimum Deviation and Refractive Index

During the refraction of light through a prism, the angle of deviation goes on decreasing with an increase in the angle of incidence, becomes minimum for the particular angle of incidence, and then goes on increasing. The graph showing the variation of the angle of deviation with the angle of incidence for the case of prism is shown in figure 16.4. The lowest point of the graph gives the minimum value of the angle of deviation which is called the angle of minimum deviation and it is denoted by δ_m .



If a prism is in a minimum deviation position, the light ray passes symmetrically through the prism. In such situation, the angle of incidence in the first face is equal to the angle of emergence in the next face, and the angle of refraction in first face is equal to angle of refraction in the second face. The refracted ray passes parallel to the base of the prism. Therefore, for minimum deviation conditions,

angle of deviation, $\delta = \delta_m$, angle of incidence, i = angle of emergence, e , angle of refraction in first face, r_1 = angle of refraction in second face $r_2 = r$.

From relation $r_1 + r_2 = A$

$$r + r = A$$

$$\text{or, } r = \frac{A}{2} \quad \dots (1)$$

Again, from relation, $\delta = i + e - (r_1 + r_2) = i + e - A$

$$\delta_m + A = i + i$$

$$\text{or, } i = \frac{A + \delta_m}{2} \quad \dots (2)$$

Using Snell's law,

$$\text{The refractive index, } {}_a\mu_g = \frac{\sin i}{\sin r} \quad \dots (3)$$

From equation (1), (2) and (3) we get,

$${}_a\mu_g = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} \quad \dots (9)$$

This is the expression for the refractive index of a glass prism using a minimum deviation condition.

Workout Examples:

Q. Show that at the minimum deviation condition angle of incidence is equal to the angle of emergence in the case of prism.

$$\text{We know, } \delta = i + e - A = (\sqrt{i})^2 + (\sqrt{e})^2 - A = (\sqrt{i} - \sqrt{e})^2 + 2\sqrt{ie} - A$$

Here, angle of prism A is constant, so the angle of deviation δ becomes minimum only when

$$(\sqrt{i} - \sqrt{e})^2 = 0$$

$$\text{or, } \sqrt{i} - \sqrt{e} = 0$$

$$\text{or, } i - e = 0$$

$$\text{or, } i = e$$

Q. A glass prism of angle A and refractive index 1.5 produces the angle of minimum deviation equal to 40° . Calculate the value of angle of prism A .

Solution: Here,

$$\text{Given, refractive index } (\mu) = 1.5$$

$$\text{Angle of minimum deviation } (\delta_m) = 40^\circ$$

$$\text{Angle of prism } (A) = ?$$

We know that,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}}$$

$$\text{or, } 1.5 = \frac{\sin\left(\frac{A + 40^\circ}{2}\right)}{\sin \frac{A}{2}}$$

$$\text{or, } 1.5 = \frac{\sin\left(\frac{A}{2} + 20^\circ\right)}{\sin \frac{A}{2}} = \frac{\sin \frac{A}{2} \cdot \cos 20^\circ + \cos \frac{A}{2} \cdot \sin 20^\circ}{\sin \frac{A}{2}} = \cos 20^\circ + \cot$$

$$\frac{A}{2} \cdot \sin 20^\circ$$

$$\text{or, } \cot \frac{A}{2} = \frac{1.5 - \cos 20^\circ}{\sin 20^\circ} = \frac{1.5 - 0.94}{0.342}$$

$$\cot \frac{A}{2} = 1.638$$

$$\text{or, } \tan \frac{A}{2} = \frac{1}{1.638} = 0.61$$

$$\text{or, } \frac{A}{2} = \tan^{-1}(0.61) = 31.4^\circ$$

$$A = 62.8^\circ$$

Hence, required angle of prism is 62.8° .

Q. What should be the refracting angle of a prism of refractive index 1.5 in order that the ray incident on the face at an angle of 50° may suffer minimum deviation?

Solution: Here,

Refractive index (μ) = 1.5

Angle of incidence (i) = 50°

Angle of prism (A) = ?

We have, from laws of refraction,

$$\frac{\sin i}{\sin r_1} = \mu$$

$$\sin r_1 = \frac{\sin i}{\mu}$$

$$r_1 = \sin^{-1} \left(\frac{\sin 50^\circ}{1.5} \right) = \sin^{-1} (0.51) = 30.71^\circ$$

Now, since the prism is at the condition of minimum deviation,

$r_1 = r_2 = r$ (say)

Hence, from $A = r_1 + r_2 = 2r$

Thus, $A = 2 \times 30.71 = 61.42^\circ$

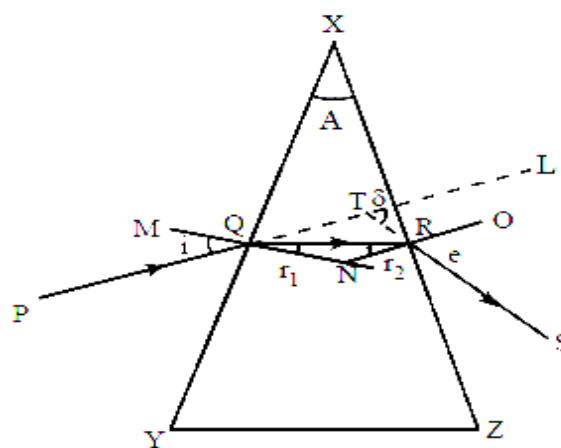
Hence, the refracting angle of the prism is 61.42°

Sample Questions:

- Does the angle of minimum deviation changes if the prism is dipped into water?
- Calculate the refractive index of a prism, if the angle of the prism is 60° and the angle of minimum deviation is 50° . [1.638]

16.4 Deviation Produced by a Small Angled Prism

Consider a small angle glass prism XYZ whose refracting angle is A. Let a ray of light PQ incidents on refracting face XY, refracts along QR and emerges out along RS as shown in figure 16.5. Let i, r_1 , r_2 , δ and e be the angle of incidence, angle of refraction in first face, angle of incidence on second face, angle of deviation and angle of emergence respectively.



[Fig.16.5, Refraction of light by small angle prism]

We know for prism,

$$\delta = i + e - (r_1 + r_2)$$

$$= i + e - A$$

$$\dots (1)$$

$$[\text{Since, } r_1 + r_2 = A]$$

We have at face XY, $\mu = \frac{\sin i}{\sin r_1}$. . . (2)

Since both i and r_1 are very small, $\sin i \approx i$ and $\sin r_1 \approx r$, $\mu = \frac{i}{r_1}$.

Therefore, $i = \mu r_1$. . . (3)

Similarly, at face XZ, $e = \mu r_2$. . . (4)

From equation (1), (3) and (4)

$$\delta = i + e - A = \mu r_1 + \mu r_2 - A = \mu(r_1 + r_2) - A = \mu A - A = A(\mu - 1)$$

or, $\delta = A(\mu - 1)$

This is the expression for the angle of deviation produced by a small angled prism.

Hence, in this case, the angle of deviation is independent of angle of incidence but depends upon refractive index and the angle of prism.

The advantages of totally reflecting prism over plane mirror are:

- i. Comparatively increases the intensity of image.
- ii. Does not cause the image laterally inverted.
- iii. Comparatively reduces the size of optical instrument.

Importance of Deviation through Small Angled Prism

By controlling and manipulating light precisely while passing through small angled prisms, we can achieve desired optical performance.

i. Collimation and Divergence:

A small-angle prism can cause collimation or divergence of light depending on its orientation. This property is used in laser beam expanders, optical metrology systems, etc.

ii. Optical Alignment:

The deviation from a small-angled prism can be precisely controlled and measured. This property enables accurate alignment of optical components like lenses, mirrors, filters, etc. for optimal performance of optical systems.

iii. Dispersion Compensation:

Small-angled prisms improve performance in fiber optics, spectroscopy and telecommunications by reducing chromatic dispersion.

iv. Image Displacement:

Introduction a small angled prism into the optical path allows shifting of the image with minimal degradation. As a result, alignment of optical components, adjustment of path of a beam and redirecting of light in imaging systems become possible.

v. Prism-Based Optical Systems:

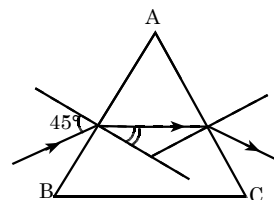
Small angled prisms serve as fundamental components in optical systems like telescopes, binoculars, and periscopes to create complex functions such as image rotation, light path manipulation, etc.

Exercise

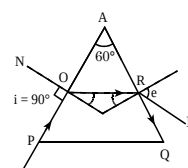
- Which of the following must be equal to the angle of emergence for minimum deviation of light passing through a prism?
a. angle of incidence b. angle of prism c. angle of refraction d. angle of deviation
- Which of the following is false for a prism placed in the position of minimum deviation?
a. $i_1 = i_2$ b. $r_1 = r_2$
c. $i_1 = r_1$ d. light passes symmetrically through the prism
- The critical angle for the material of a prism is C , and the angle of prism is A . In which of the following situations there may not be an emergent ray?
a. $A < 2C$ b. $A = 2C$ c. $A > 2C$ d. $A \geq 2C$
- What should be the refracting angle of a prism of refractive index 1.5 so that the ray of incident on a face at an angle of 50° may suffer minimum deviation?
a. 59.42° b. 60.42° c. 61.42° d. 70.42°
- An equilateral prism has refractive index $\sqrt{2}$. What is the angle of incidence for minimum deviation to occur?
a. 60° b. 45° c. 30° d. 20°
- The angle of prism is A and angle of minimum deviation is $(180^\circ - 2A)$. What must be the refractive index of the material of the prism?
a. $\sin\left(\frac{A}{2}\right)$ b. $\cos\left(\frac{A}{2}\right)$ c. $\tan\left(\frac{A}{2}\right)$ d. $\cot\left(\frac{A}{2}\right)$
- The refracting angle of prism is A and refractive index of the material of prism is $\cot(A/2)$. What may be the angle of minimum deviation?
a. $(180^\circ - 3A)$ b. $(180^\circ - 2A)$ c. $(90^\circ - A)$ d. $(180^\circ + 2A)$

QN	1	2	3	4	5	6	7
Ans	a	c	c	c	b	d	b

- (a) Can you ever have situation in which a light ray goes undeviated through a prism?
Explain.
(b) One face of a prism ($\mu = \sqrt{2}$) is silvered, and the refracting angle of the prism is 30° . At what angle should the ray be incident on the silvered face so that it retraces its path after refraction and then reflection on the silvered face?
[45°]
- (a) What are the factors on which the deviation produced by a prism depends on?
(b) Given figure is an equilateral glass prism ($\mu=1.5$) in which light incidents at an angle of 45° on one face. What are the angles of emergence and the angle of deviation? [$e = 52.4^\circ$; $\delta = 37.4^\circ$]
- (a) Why don we not consider the lateral shift in prism?
(b) What is the relation of angle of prism, and the critical angle for normal incidence and 90° angle of emergence?
(c) A ray of light is inclined to one face of the prism at an angle of 50° . If the angle of the prism is 60° and the ray is deviated through an angle of 42° , find the angle which the emergent ray makes with the second face of the prism.
[28°]



11. (a) State the factors on which the deviation produced by a prism depends.
 (b) A ray of light strikes at an angle of 60° on one face of a prism which has refractive angle of 30° . The angle between the emergent ray and incident is 30° . What is the angle between the emergent ray and face from which ray emerges? $[0^\circ]$
12. (a) A glass prism is immersed in water. What happens to the value of angle of minimum deviation?
 (b) In an isosceles right-angled prism, light falls from the flat face, and undergoes total internal reflection on the second face. What is the value of refractive index of prism? $[\sqrt{2}]$
13. (a) Does the angle of deviation depend on the angle of prism? Explain.
 (b) In an equilateral prism, angle of incidence is equal to angle of emergence. Also angle of incidence is three-fourth of angle of prism. What is the angle of deviation produced? $[30^\circ]$
14. (a) Does the angle of deviation depend on the angle of prism?
 (b) List out the conditions of minimum deviation through a prism?
 (c) What must be the values of angle of emergence 'e' and angle of deviation from the given prism as shown in figure, where refractive index of prism is 1.5? $[e = 27.9^\circ, \delta = 57.9^\circ]$
15. Calculate the value of an angle of minimum deviation for a ray of light passing through a prism of angle 60° , and the refractive index 1.5. What is the corresponding angle of incidence? $[37.2^\circ, 48.6^\circ]$
16. A ray of light refract through a prism placed in a minimum deviation position. If the angle of prism is 60° , and the deviation produced is 34° , find the angle of incidence and refractive index. $[47^\circ, 1.46]$
17. A ray of light falls normally on the face of a glass prism having a refractive index of 1.5. Find the angle of prism if the ray just fails to emerge from the second face. $[41.8^\circ]$
18. What should be the refracting angle of a prism of refractive index 1.5 so that the ray incident on the face at an angle of 50° may suit minimum deviation? $[61.4^\circ]$



Chapter 17

Lens

Joseph Ritter von Fraunhofer was a German physicist and optical lens manufacturer. Fraunhofer was born in Straubling, Bavaria, on March 6, 1787, and he died in Munich on June 7, 1826. He made optical glass, an achromatic telescope, and objective lenses. He also invented the spectroscope, and developed diffraction grating for producing and analyzing light spectra. He was the first to study and classify the dark lines that appear in the spectra of the sun. During his lifetime he was known as the maker of the finest refractor telescopes in Europe.



Joseph Ritter von Fraunhofer (1787-1826)

Do you know?

1. Why do people use glasses?
2. What are the basic components of microscope and telescope?

Project Work: 1

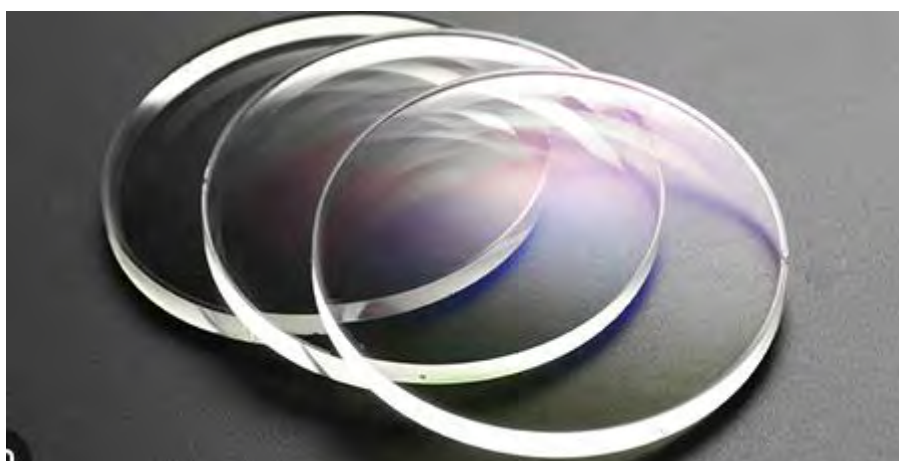
Visit each class in your school. Make a name list of people who use eyeglasses. Ask them about the type of lens they have used in their glasses. Touch the surfaces of the lens used in eyeglasses. Complete the following table. Discuss the results among your friends in the class, and interact with teacher about the conclusion.

S N	Name of the person who uses eyeglasses	Type of lens used in glasses	Surface of the lens	Problem of eyes	Remarks
1.	Ram	Convex	Thicker in the middle and thinner in the edges	Hypermetropia	
2.					
3.					
4.					
5.					

17.1 Introduction

Lens is a homogenous transparent medium enclosed by two spherical refracting surfaces. Lenses are commonly used in our daily life. For example, eyeglasses, contact lenses, cameras, smartphones, microscopes, binoculars, telescopes, projectors, reading glasses, magnifying glasses, headlights, spotlights and flashlights of vehicles, eyepieces in binoculars and

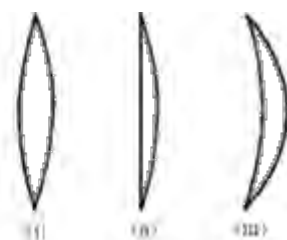
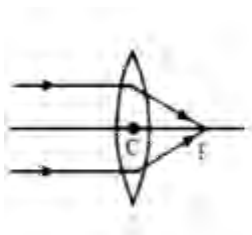
microscopes, etc. Lenses are essential optical components that enable us to see objects and images clearly, capture images, explore the world around us, and enhance our visual experiences in various ways. There are mainly two types of lenses; concave and convex.



1. Convex Lens

A lens thicker in the middle and thinner at the edges as shown in figure 17.1 is called a convex lens. There are of three types convex lenses.

Bi-convex lens: A lens having both surfaces convex as shown in figure 17.2 (i) is a **bi-convex lens**. Radii of curvatures of two surfaces may or may not be equal.



Plano-convex lens: A lens having one surface plane and the other surface convex as shown in figure 17.2 (ii) a **plano-convex lens**.

Concavo - convex lens: It is a lens having one surface concave and the other convex as shown in figure 17.2(iii). For this lens radius of curvature of the convex surface is less than that of the concave surface.

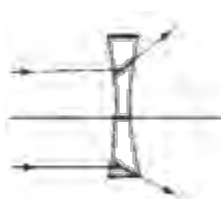
[Fig.17.1, A convex lens]

[Fig. 17.2, Types of convex lenses]

Concavo - convex lens: It is a lens having one surface concave and the other convex as shown in figure 17.2(iii). For this lens radius of curvature of the convex surface is less than that of the concave surface.

2. Concave Lens

A lens which is thinner in the middle and thicker at the edges as shown in figure 17.3 is called a concave lens. Concave lenses are of three types.



[Fig.17.3, A concave lens][Fig.17.4, Types of concave lenses]

Bi-concave lens: A lens having both surfaces concave as shown in figure 17.4 (i) is a **bi-concave lens**. Radii of curvatures of two surfaces may or may not be equal.

Plano-concave lens: It is a lens having one surface plane and the other surface concave as shown in figure 17.4 (ii).

Convexo – concave lens: It is a lens having one surface convex and the other concave as shown in figure 17.4 (iii). The radius of curvature of the concave surface is less than that of the convex surface.

Sample Questions:

1. Why is the concave lens called a diverging lens?
2. Why is the convex lens called a converging lens?

Properties of Spherical Lenses

The properties of lenses determine their optical behavior. Therefore, it is important to understand the properties of spherical lenses for their proper design, and for using them in various applications effectively to ensure proper focus, image clarity, and optical performance. Important properties of spherical lenses are explained below.

Focal Length (f): It is the distance between the optical centre and the principal focus of a lens. The focal length determines the converging as well as the diverging power of a lens. A lens with a shorter focal length has a stronger converging power whereas a lens with a longer focal length has a weaker converging power.

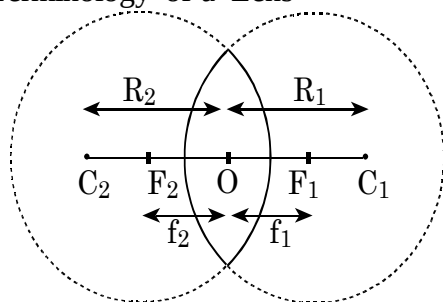
Optical Power (P): The optical power is the measure of the ability of a lens to bend (converge or diverge) the rays of light. It is expressed as reciprocal to the focal length so that the focal length must be in meter. Therefore, the power of lens, $P = \frac{1}{f}$. A lens with a higher optical power bends the light more strongly than a lens with a lower optical power.

Shape of Lens: Spherical lenses have a characteristic shape. One or both the surfaces of a spherical lens are the portions of a sphere. The shape of a lens may be concave or convex.

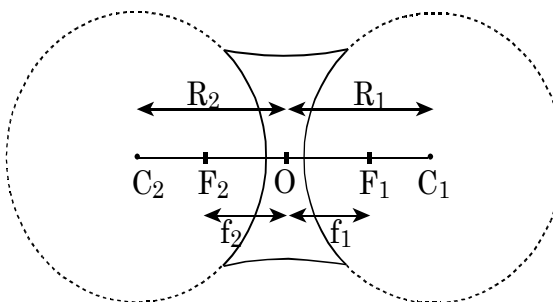
Aperture: The aperture of a lens is the effective diameter of the lens through which light passes. The amount of light that can enter the lens depends on its aperture. It also affects the brightness of the image formed.

Good to know:

Terminology of a Lens



Convex lens



Concave lens

$$OC_2 = R_2$$

$$C_1C_2 = \text{Principal axis}$$

$$OF_2 = f_2 \text{ and } OF_1 = f_1 \text{ (focal length)}$$

$$OC_1 = R_1$$

$$O = \text{Optical centre}$$

$$F_1 \text{ and } F_2 = \text{Principal focus}$$

[Fig. Formation of convex and concave lenses]

Principal axis: The line joining the centers of curvature of the two refracting surfaces of a lens is called the principal axis (C_1C_2).

Principal focus (F): A point on the principal axis is such a point, where all the rays parallel to the principal axis intersect after refraction through a convex lens. In case of a concave lens, the rays of light appear to come from the principal focus. Every lens has two principal foci, one for each refracting surface. The foci of a convex lens are real and the foci of the concave lens are virtual.

Optical centre (O): The optical centre of the lens is its geometrical centre from where the rays of the light pass without deviation. It is denoted by O.

Focal length (f): The distance between the optical centre and the principal focus of a lens is called its focal length. It is denoted by f. The focal length of convex lenses is taken as positive, and that of concave lenses is taken as negative.

Focal plane: The plane passing through the focus and which is perpendicular to the principal axis of a lens is called the focal plane.

Centre of curvature: Lenses are formed by the combination of two curved surfaces. Those curved surfaces are the parts of two spheres. The centers of those spheres are called centers of curvature of the lens. Therefore, a lens has two centers of curvature. In figure 17.5, C_1 and C_2 are centers of curvature.

Radius of curvature: The radius of the sphere of which a part is a lens is called the radius of curvature. A lens has two radii of curvature R_1 and R_2 . The radii of curvature of convex lenses are taken as positive, and that of concave lenses are taken as negative.

17.2 Sign Conventions of Lens

The sign convention is a bunch of rules to set signs for different distances like image distance, object distance, focal length, etc. *in ray diagrams of spherical lenses* for the mathematical analysis of an image formation.

Sign conventions for the lens are given below:

- i. The aperture of the lens is small.
- ii. All the distances are measured from the optical centre of the lens.
- iii. All the distances of real objects and real images are taken as positive whereas all the distances of virtual objects and virtual images are taken as negative.
- iv. Convex lenses can form both real and virtual images but concave lenses can form only virtual images.
- v. Focal length and radius of the curvature of a convex lens are taken as positive whereas those for a concave lens are taken as negative.

17.3 Lens Formula

The lens formula is a well-designed relation between object distance (u), image distance (v) and the focal length (f) of the lens taking the optical centre as the origin. The lens formula is applicable for both concave as well as convex lenses to find image distance, the type of image formed, and the focal length (f).

Lens formula is given by, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Good to know;

Lens formula for a convex lens

for the formation of a real image

Let's place an object on the principal axis of a convex lens so that the rays of light coming from the object strike on the refracting surface of the lens and hence a real image is formed on the other side of the object as shown in figure 1.

In figure, A'B' is the image of object AB. The image is real and inverted.

Let, AO represent object distance (u) and A'O represent image distance (v), OF = represent the focal length (f) and OQ represent the height of the object (h). Then, in similar triangles, $\triangle ABO$ and $\triangle A'B'O$

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \quad \dots (1)$$

Again, in similar triangles $\triangle OQF$ and $\triangle A'B'F$,

$$\begin{aligned} \frac{A'B'}{OQ} &= \frac{A'F}{FO} \\ \text{or, } \frac{A'B'}{AB} &= \frac{A'F}{FO} \quad \dots (2) \text{ [Since } AB = OQ \text{]} \end{aligned}$$

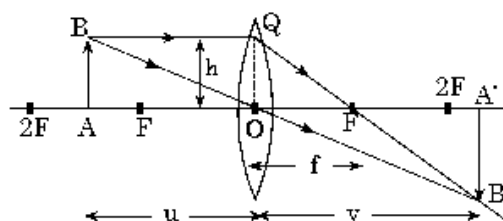
Equating equation (1) and (2), we get

$$\begin{aligned} \frac{A'O}{AO} &= \frac{A'F}{FO} \\ \text{or, } \frac{A'O}{AO} &= \frac{A'O - FO}{FO} \\ \text{or, } \frac{v}{u} &= \frac{v - f}{f} \\ \text{or, } vf &= uv - uf \\ \text{or, } uv &= vf + uf \quad \dots (3) \end{aligned}$$

Dividing equation (3) by fuv on both sides, we get

$$\begin{aligned} \frac{uv}{uvf} &= \frac{vf + uf}{uvf} \\ \text{or, } \frac{uv}{uvf} &= \frac{vf}{uvf} + \frac{uf}{uvf} \\ \therefore \frac{1}{f} &= \frac{1}{u} + \frac{1}{v} \quad \dots (4) \end{aligned}$$

This is the required lens formula for the formation of a real image by convex lens.



[Fig. Ray diagram of convex lens when real image is formed]

Workout Example:

The real image obtained by a lens of power 5D is three times the length of the object. Calculate the object and image distances.

Solution: Here,

Given, power of lens (P) = 5D

Focal length (f) = $\frac{1}{P} = \frac{1}{5} = 0.2\text{m}$

Magnification (m) = 3

Object distance (u) = ?

Image distance (v) = ?

We have, magnification

$$m = \frac{v}{u} = 3$$

or, $v = 3u$

Again, using lens formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or, } \frac{1}{0.2} = \frac{1}{u} + \frac{1}{3u}$$

$$\text{or, } 5 = \frac{4}{3u}$$

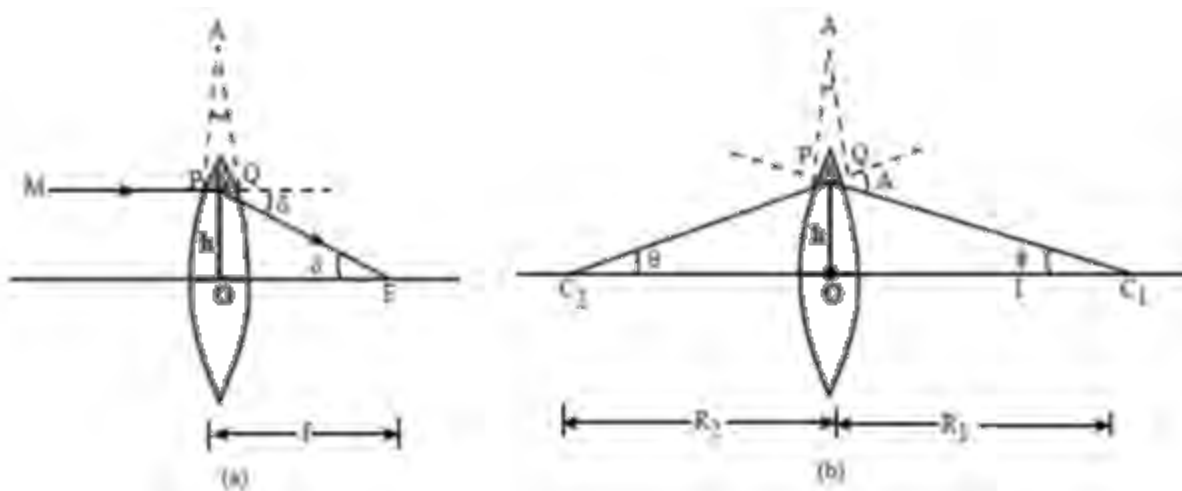
Therefore, $u = 0.267\text{m} = 26.7\text{cm}$ and $v = 3u = 80\text{cm}$.

Sample Questions:

1. At what distance from a convex lens should an object be placed on the axis, so as to form a real image of the same size? Illustrate your answer with a ray diagram.
2. A 5 cm long object is placed at the principal focus of a concave lens of focal length 20 cm. Find the nature, position and the size of the image. [virtual. -10 cm, 2.5 cm]

17.4 Lens Maker's Formula

Lens maker's formula is the relation between focal length of a lens to the refractive index of its material and radii of curvature of its two refracting surfaces. Lens manufacturers use the lens maker's formula to make the lenses of particular power from the glass of a given refractive index.



[Fig.17.5, (a) Deviation produced by convex lens(b) Convex lens as small angle prism]

Let's think a convex lens of focal length (f) and refractive index (μ). Suppose, a ray MP, parallel to the principal axis, strikes on the lens at a small height (h) above optical centre. The ray will pass through the focus (F) after refraction, and gets deviated through an angle δ as shown in figure 17.5(a).

From geometry, $\tan \delta = \frac{h}{f}$

For small angle $\tan \delta = \delta$

Thus, $\delta = \frac{h}{f} \dots (1)$

Since convex lens also behaves as a small angled prism, angle of deviation δ is given by;

$$\delta = A(\mu - 1) \dots (2), \text{ [where } A \text{ is angle of prism]}$$

Equating equation (1) and (2), we get,

$$\frac{h}{f} = A(\mu - 1)$$

$$\text{or, } \frac{1}{f} = \frac{A}{h} (\mu - 1) \dots (3)$$

If C_1 and C_2 are the centers and R_1 and R_2 are the corresponding radii of the curvature of two spherical surfaces then, from figure 17.5 (b) $C_1O = R_1$, $C_2O = R_2$. Also, $KO = h$, $\angle KC_2O = \theta$, $\angle KC_1O = \phi$. By using the geometry of figure,

$$\angle PAQ = \angle QKC_1 = A, \tan \phi = \frac{h}{R_1}, \tan \theta \approx \theta = \frac{h}{R_2}$$

$$\text{and } \angle KC_2O + \angle KC_1O = A$$

$$\text{or, } \theta + \phi = A \dots (4)$$

$$\text{or, } \frac{h}{R_2} + \frac{h}{R_1} = A$$

$$\text{or, } h \left(\frac{1}{R_2} + \frac{1}{R_1} \right) = A$$

$$\text{or, } \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{A}{h} \dots (5)$$

Using equation (5) in equation (3) we get,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \dots (6)$$

This is the lens maker's formula derived by using a convex lens.

Lens maker's formula for concave lens:

$$\frac{1}{f} (\mu - 1) \left(-\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = -1(\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Workout Examples:

Q. The surfaces of sunglasses are curved but power is zero, how?

In the case of sunglasses both surfaces are curved to the same side so that their centers of curvature lie on the same side. Therefore, radii of curvature are equal in magnitude but opposite in sign that is; $R_1 = R$ and $R_2 = -R$. Hence, the focal length of sunglasses is,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = (\mu - 1) \left(\frac{1}{R} - \frac{1}{R} \right) = 0$$

And corresponding power, $P = \frac{1}{f} = 0$.

Q. A lens made of glass is immersed in water. Will its power increase or decrease?

For a lens of refractive index with respect to air μ_g and radii of curvature of two surfaces R_1 and R_2 the focal length (f_1) in air is given by, $\frac{1}{f_1} = (\mu_g - 1) \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$ and corresponding

power (P_1) in air is given by, $P_1 = (\mu_g - 1) \left[\frac{1}{R_1} + \frac{1}{R_2} \right] \quad \dots (1)$

Similarly, the power (P_2) of the lens when it is immersed in water is given by,

$P_2 = (\mu_{wg} - 1) \left[\frac{1}{R_1} + \frac{1}{R_2} \right] \quad \dots (2) \quad [\mu_{wg} \text{ is the refractive index of glass with respect to water}]$

Dividing (2) by (1), we get,

$$\frac{P_2}{P_1} = \frac{(\mu_{wg} - 1)}{(\mu_g - 1)}$$

Since the refractive index of glass with respect to air is greater than that with respect to water, the power of lens decreases when it is immersed in water.

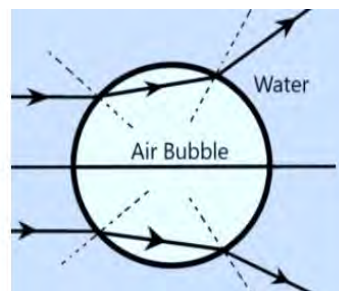
Q. An air bubble is formed inside water. If refraction of light takes place through this bubble, does it act as a converging lens or a diverging lens?

According to the lens maker formula, the focal length of the lens is given by,

$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$. Where, μ is the refractive index of air lens with respect to the water.

Here $\mu < 1$ and hence f becomes negative. This means light rays will diverge in such arrangement. Therefore, air bubble inside water will act as a diverging lens.

But due to total internal reflection air bubbles commonly glow inside water.



An air bubble inside water behaves like a diverging lens.

Q. A convex lens of focal length 24 cm (refractive index = 1.5) is totally immersed in water (refractive index = 1.33). Find its focal length in water.

Solution: Here,

Focal length of a convex lens (f) = 24cm

Refractive index of glass (${}_a\mu_g$) = 1.5

Refractive index of water (${}_a\mu_w$) = 1.33

Focal length of the lens in water (f_1) = ?

When the lens is in air, then using lens maker formula,

$$\frac{1}{f} = ({}_a\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\text{or, } \frac{1}{24} = (1.5 - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\text{or, } \frac{1}{24} = 0.5 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\text{or, } \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12} \quad \dots (1)$$

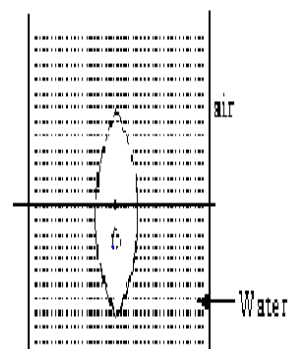
When the lens is in water. Then lens maker formula,

$$\frac{1}{f_1} = ({}_w\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

We know that, ${}_a\mu_w \times {}_w\mu_g \times {}_g\mu_a = 1$ or ${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w}$.

Then,

$$\frac{1}{f_1} = \left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1 \right) \times \frac{1}{12} = \left(\frac{1.5}{1.33} - 1 \right) \times \frac{1}{12} = 0.0106$$



$\therefore f_1 = 93.88\text{cm}$. Hence, focal length of convex lens when it is in water is 93.88 cm.

Q. An equiconvex lens has a power of 4 D. What will be the radius of curvatures of each surface if the lens is made of glass of refractive index 1.5?

Solution: Here,

Power of lens (P) = 4 D

Radius of curvature (R) = ?

Refractive index (μ) = 1.5

We have, lens maker's formula,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

For equiconvex lens, $R_1 = R_2 = R$

$$\text{or, } \frac{1}{f} = (\mu - 1) \left(\frac{1}{R} + \frac{1}{R} \right)$$

$$\text{or, } P = (\mu - 1) \frac{2}{R}$$

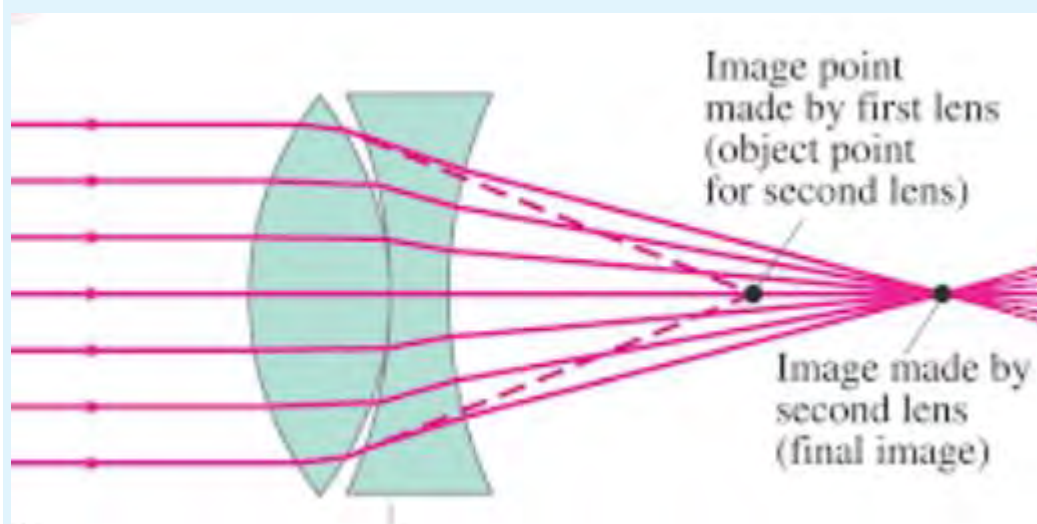
$$\text{or, } R = (\mu - 1) \frac{2}{P} = (1.5 - 1) \times \frac{2}{4} = \frac{0.5 \times 2}{4} = \frac{1}{4} = 0.25\text{m}.$$

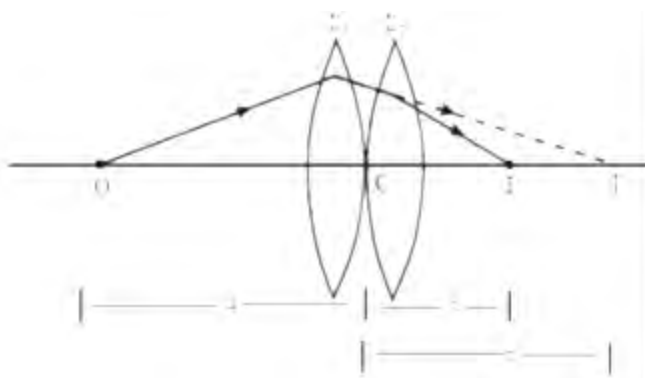
Hence, required radius of curvature is 25 cm.

Sample Questions:

1. A converging lens is dipped in a liquid having refractive index greater than that of the lens. Does its power change?
2. A thin equiconvex glass lens of refractive index 1.5 whose surfaces have radii of curvature of 24cm is placed on a horizontal plane mirror. When the space between the lens and the mirror is filled with a liquid and a pin is hung 40 cm vertically above. The lens is found to coincide with its own image. Calculate the refractive index of the liquid. [1.4]

17.5 Combination of two lenses (both convex or, both concave or concave and convex):





[Fig.17.6, Combination of two convex lenses]

Let us consider two lenses L_1 and L_2 of corresponding focal length f_1 and f_2 are placed coaxially in contact to each other as shown in figure 17.6. In this case both the lenses are convex.

Suppose a point object (O) placed at a distance u from optical centre C of the lens L_1 produces an image (I') at a distance v' on the other side of the lens as in figure 17.6. Also, it behaves as a virtual object for lens L_2 and produces an image (I). Therefore, from the lens formula,

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \dots (1)$$

$$\frac{1}{f_2} = \frac{1}{-v'} + \frac{1}{v} \quad \dots (2)$$

Adding equation (1) and (2) gives,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v} \quad \dots (3)$$

If we suppose the combination of L_1 and L_2 as a single lens of focal length F then I will be the image of object O . Therefore,

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \dots (4)$$

Comparing equations (3) and (4) we get,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots (5).$$

This is an expression for combined focal length of two convex lenses in contact.

17.6 Power of a Lens

Do you know why some people in your locality use spectacles?

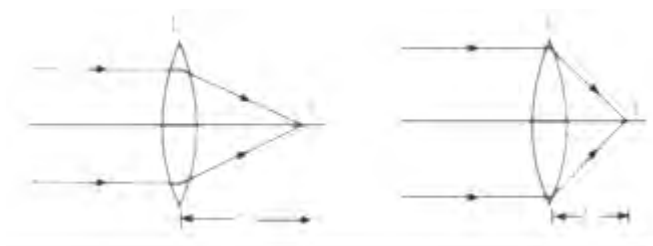
Do you know the power of the lens they used in their spectacles?



Every ray of light falling on the convex lens deviates towards the principal axis and tends to converge, but every ray of light falling on the concave lens deviates away from the principal axis and tends to diverge. Therefore, the main function of the lens is to deviate the light rays incident on it and hence power of a lens is measured in terms of its ability to deviate the rays.

The ability of a lens to converge or diverge a ray of light after refraction through it is called the power of that lens. It is defined as the reciprocal of focal length. A lens of a short focal length produces more deviation than that of a lens of a larger focal length as shown in figure 17.7. Lens L_2 has less focal length than lens L_1 so that L_2 bends the rays more than L_1 , and the power of lens L_2 is greater than that of L_1 .

$$\therefore \text{power (p)} = \frac{1}{f}$$



[Fig.17.7, Ray diagram for measurement the power of lens]

The power of a lens is measured in Dioptre (D) unit such that the focal length is expressed in metre. Therefore, power of a lens $P = 1 \text{ D}$, its focal length $f = 1\text{m}$. For the combination of more than two lenses with equivalent power is an algebraic sum of powers of the individual lense which is given by,

$$P = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$$

$$P = P_1 + P_2 + P_3 + \dots$$

A convex lens deviates the incident rays towards the axis and its power is taken positive while a concave lens deviates the incident rays away from the axis and its power is negative.

If a convex lens and a concave lens of equal power are combined then, equivalent power of combination,

$$p = p_1 + (-p_2) = p_1 - p_2 = 0$$

Therefore, the equivalent power will be zero for the combination of a convex lens and a concave lens both having equal power. It means the combination behaves as a glass plate.

Workout Example:

How will the combination behave if a converging and a diverging lenses having the same focal length are in contact?

The focal length of a combination of two lenses is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$. Here, the lenses are a converging lens and a diverging lens, and they have equal focal lengths. So, $f_1 = -f_2$. This gives,

$$\frac{1}{F} = \frac{1}{-f_2} + \frac{1}{f_2} = -\frac{1}{f_2} + \frac{1}{f_2} = 0$$
$$\text{or, } \frac{1}{F} = \frac{0}{1}$$
$$\therefore \frac{1}{F} = \frac{1}{0} = \infty \text{ and power } P = \frac{1}{F} = \frac{1}{\infty}$$

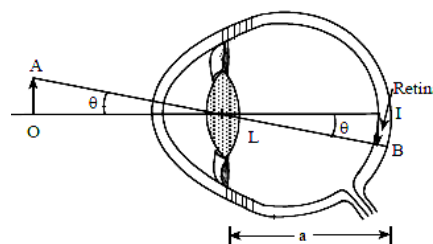
This makes the focal length equal to infinity, making power zero. The combination thus will act as a simple plane glass plate, or a watch glass.

Sample Questions:

1. Can a convex lens and a concave lens be combined so that the combination behaves like a diverging lens? Justify your answer.
2. How do lenses combine to increase the resultant power?

17.7 Visual Angle

Visual angle is the angle subtended by an object or an image at the eye or eyepiece. It is measured in terms of angle formed by two rays of light or two straight lines drawn from the extreme points of the viewed object or image to the nodal point of the eye. The visual angle is influenced by two things; *size of the object or image, and the distance of the object or image from the eye*. The larger the visual angle, the larger is the size of the image. So, the apparent size of the object is determined by the visual angle.



[Fig.17.8, Visual angle]

In figure 17.8, θ is the visual angle of object OA at human eye. IB is the image of OA at the retina.

Here, the height of object OA is h , and that of the image is given by,

$$h' = IB = \theta IL = \theta a \quad \left[\text{Since, } \theta = \frac{l}{r} = \frac{IB}{a} \text{ and } IB = \theta a \right]$$

Where, $IL = a$ is a diameter of the eye.

Since a is constant, $h' \propto \theta$. Thus, the size of image is proportional to the angle subtended by the object at the eye.

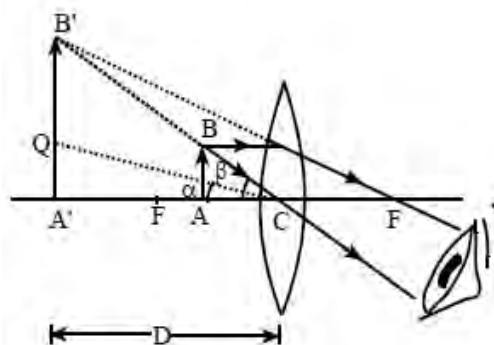
17.8 Magnification Produced by a Lens

Magnification is defined as the ratio between the dimensions of the image and the object. The process of magnification can occur in lenses, telescopes, microscopes, etc. Magnification produced by a lens depends on the position of objects. There are two types of magnification produced by a lens.

1. Angular magnification
2. Linear magnification

1. Angular magnification:

The magnifying power or angular magnification of a lens is defined as the ratio of the visual angle subtended by the image at the eye (β) to the visual angle subtended by the object at the eye (α) when both lie at the least distance of distinct vision.



EXAMPLE 6

is, magnifying power or angular magnification of

a lens, $M = \frac{\beta}{\alpha}$

That [Fig.17.9, Ray diagram of simple microscope]

2. **Linear magnification:** The linear magnification is defined as "the ratio of the size of an image to the size of an object."

That is linear magnification

$$m = \frac{\text{size of image (I)}}{\text{size of an object (O)}} = \frac{I}{O} \dots (1)$$

Consider, an object AB and its image A'B' as formed by a convex lens as shown in figure 17. 10.

In similar triangles ABC and A'B'C

$$\frac{A'B'}{AB} = \frac{A'C}{AC}$$

$$\text{or, } \frac{A'B'}{AB} = \frac{v}{u}$$

$$\text{or, } \frac{\text{size of an image (A'B')}}{\text{size of an object (AB)}} = \frac{v}{u} \dots (2)$$

From equation (1) and (2)

$$\text{Linear magnification, } m = \frac{\text{size of image}}{\text{size of an object}} = \frac{I}{O} = \frac{A'B'}{AB} = \frac{v}{u}$$

$$\therefore m = \frac{v}{u} \dots (3)$$

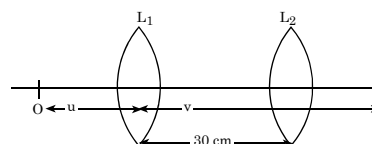
Thus, linear magnification is also defined as the ratio of the **image distance to the object distance**.

Workout Example:

A luminous object and a screen are placed on an optical bench and a converging lens is placed between them to produce a sharp image of the object on the screen. The linear magnification of the image is found to be 2.5. The lens is now moved 30 cm nearer the screen and a sharp image is again formed. Calculate the focal length of the lens.

Solution: Here,

Magnification produced (m) = 2.5.



First case:

If u and v are the object and image distance respectively, then magnification $m = \frac{v}{u}$

$$\text{or, } 2.5 = \frac{v}{u} \text{ or, } v = 2.5u \quad \dots (1)$$

Using lens formula, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{2.5u}$$

$$\therefore \frac{1}{f} = \frac{1.4}{u} \dots (2)$$

Second case:

When the lens is moved towards the screen from position L_1 to position L_2 such that $L_1L_2 = 30\text{cm}$. So that,

new object distance $u' = u + 30\text{ cm}$

new image distance (v') = $v - 30 = 2.5u - 30$. [Since $v = 2.5u$]

Using the lens formula, we get $\frac{1}{f} = \frac{1}{u'} + \frac{1}{v'}$

$$\therefore \frac{1}{f} = \frac{1}{u + 30} + \frac{1}{2.5u - 30} \quad \dots (3)$$

Equating and solving equation (2) and (3), we get, $u = 20\text{ cm}$.

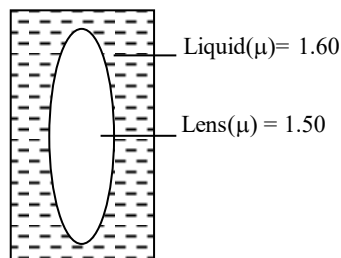
From equation (2)

$$\frac{1}{f} = \frac{1.4}{20} \text{ and } f = 14.28\text{cm} = 0.143\text{m}$$

Hence, the focal length of the lens is 14.3cm.

Exercise

- Two lenses of powers + 12 D and –2 D are placed in contact. What is the focal length of the combination?
a. 8.33cm b. 10.00cm c. 12.50cm d. 16.60cm
- If we put an object at the focus of a convex lens then, what may be the nature of the refracted rays?
a. parallel
b. converging
c. diverging
d. both converging and diverging
- What is the minimum distance between an object and its real image of the same size formed by a thin convex lens of focal length f ?
a. $\frac{1}{2} f$ b. f c. $2f$ d. $4 f$
- A convex lens of focal length 16 cm forms a virtual image of double the size of the object. What is the object distance?
a. 8 cm b. 16 cm c. 24 cm d. 32 cm
- A converging lens is placed inside a cell filled with a liquid of refractive index 1.60 as shown in figure. The focal length of the lens is 20 cm and its material has the refractive index 1.50 with respect to air. What is the focal length of the system?



- 100 cm b. -90 cm c. -80 cm d. 24 cm
- The line magnification of an image is m . What will be the magnification of its area?
a. $\sqrt{m}$ b. m c. m^2 d. m^4
 - Two thin lenses of focal length f_1 and f_2 are in contact and co-axial. What will be the equivalent power of a single lens of the combination?
a. $f_1 + f_2$ b. $\frac{f_1 f_2}{f_1 + f_2}$ c. $\frac{f_1 + f_2}{2}$ d. $\frac{f_1 + f_2}{f_1 f_2}$

8. How does a convex lens behave when its second face is silvered?
- convex lens
 - concave mirror
 - convex mirror
 - as a plate
9. A convex lens is placed in a mirror. What will be the power of the lens if the space between the lens and the mirror is filled with water?
- Remains constant
 - Increases
 - Decreases
 - May increase or decrease depending on the focal length of the convex lens
10. A convex lens is dipped in a liquid whose refractive index is equal to the refractive index of the lens. What will be its focal length?
- remain unchanged
 - become zero
 - become small but not zero
 - become infinite
11. A convex lens of focal length f_1 is placed in contact with a concave of focal length f_2 . In which of the following conditions, the combination will act as a convergent lens?
- $f_1 < f_2$
 - $f_1 = f_2$
 - $f_1 > f_2$
 - $f_1 < 2f_2$

QN	1	2	3	4	5	6	7	8	9	10	11
Ans	b	a	d	a	a	c	d	b	b	b	a

12. (a) In which case a lens has higher power, when it is (i) in air or (ii) in water?
- (b) Why is the power of a lens measured as a reciprocal of focal length?
- (c) What is the effective power in diopetre if lenses of focal lengths +10 cm and -20cm are combined? [5Diopetre]
13. (a) Which type of lens can be a spherical water droplet in air and a spherical air bubble in water?
- (b) Under what condition does a lens become invisible when immersed in a transparent liquid?
- (c) A convex lens of power + 6D is placed in contact with a concave lens of power -4D. What is the nature and focal length of the combination? [Convex and 50cm]
14. (a) A converging lens is dipped in a liquid having a refractive index greater than the lens. Does its power change? Explain
- (b) A convex lens is immersed in water, will its focal length change? [1]
- (c) An object is placed at a distance of 20cm from a convex lens of focal length 10cm. At what distance will the image be formed? [20 cm on the other side of the lens]
15. (a) Can convex and concave lenses be combined so that the combination behaves like a diverging lens? Justify your answer.
- (b) If a lens made of glass is immersed into water, what will happen to its power? Explain.

- (c) A lens of 12 cm focal length produces a virtual image whose linear dimensions are one-third that of the object. What kind of lens is it? Determine the position of the object and the image.
[diverging, 24 cm, 3 cm]
16. (a) If a converging lens and a diverging lens both having the same focal length are in contact, how will the combination of lenses behave?
- (b) At what distance from a convex lens should an object be placed on the axis to form a real image of the same size? Illustrate your answer with a ray diagram.
- (c) What are conjugate centers and radii?
- (d) The radii of curvature of the two surfaces of a convex and a concave lens are 15 cm and 30 cm. Find the focal length of the lens, if the refractive index of its material is 1.6.
[50 cm]
17. A lens has a power of +5 dioptre in air. What will be its power, if completely immersed in water? Given, refractive index of glass = 1.5 and refractive index of water = 1.33.
[1.25 dioptre]
18. A converging lens of focal length 5 cm is in contact with a diverging lens of focal length 10 cm. Find the focal length of the combination. Is it a converging lens or diverging lens after combination?
[10 cm, converging]
19. The radius of curvature of the convex face of a plano-convex lens is 12 cm and the refractive index of its material is 1.5. Find the focal length of the lens. [24 cm]
20. A lens placed at a distance of 20 cm from an object produces a virtual image two-thirds the size of the object. Find the position of image, the kind of lens, and its focal length.
[-13.3 cm, diverging. -40 cm]

Chapter 18

Dispersion

Augustin-Jean Fresnel was a French civil engineer and physicist. Fresnel pioneered in optics and did much to establish the wave theory of light advanced by English physicist Thomas young.

Fresnel presented his work on diffraction in 1819. A mathematician Simeon-Denis Poisson (a prominent advocate of Newton's corpuscular model of light) pointed out that Fresnel's model predicted a different result: when a parallel beam of light falls on a small spherical obstacle, there will be a bright spot at the centre of the circular shadow. The spot was nearly as bright as if the obstacle was not there at all. An experiment performed by the French physicist Arago and validated that spot (subsequently called Poisson's spot).



Augustin-Jean Fresnel
(10 May 1788 – 14 July 1827)

Can you answer?

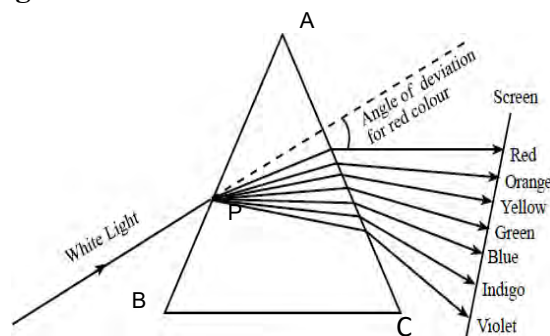
3. How and in which situation does a rainbow appear in the sky?
4. Why white light splits into different colours?



18.1 Introduction

The dispersion of light is an important phenomenon in the field of optics, which provides the spectrum of colors that forms the essence of our visual world. When white light encounters a medium like a prism or a droplet of water, its composite wavelengths undergo a mesmerizing transformation, and give an array of colors ranging from violet to red, each representing a unique wavelength of light. This phenomenon is known as dispersion, and it was famously demonstrated by Isaac Newton in the 17th century. The study of dispersion explores the secrets behind rainbows, colour spectrum from prisms, and a number of natural phenomena.

18.2 Dispersion of light



[Fig.18.1, Dispersion of light through a glass prism]

When a beam of white light falls on the refracting face of a prism, it splits into seven colours as shown in figure 18.1. Those seven colours are violet, indigo, blue, green, yellow, orange, and red. The band of seven colours obtained on the screen is called the spectrum of white light. The phenomenon is called the dispersion of light. Therefore, **dispersion of light is defined as the phenomenon of splitting white light into its constituent colors.** The formation of a rainbow is an example of the dispersion of light.

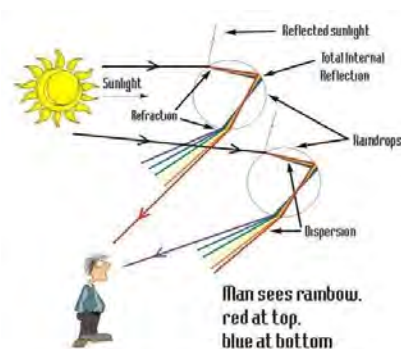
18.3 Cause of Dispersion

The refractive index of a medium is highly colour dependent. It is inversely proportional to the wave length of the light (from Cauchy's empirical relation $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \dots$). Since, the wave length of orange light is smaller than that of red light, the refractive index of red colour μ_r is less than that of orange colour μ_o . Similarly, the refractive index of yellow light is less than that of orange light and so on. Therefore from the relation $\delta = A(\mu - 1)$, $\delta_v > \delta_i > \delta_b > \delta_g > \delta_y > \delta_o > \delta_r$. That is different colour of light deviate along different directions with different values of angle of deviation. Therefore, when white light is incident on a refracting face of a prism, different colours are refracted through different angles. As a result, a white light splits and dispersion takes place.

Results of dispersion of light

a. Rainbow

The natural phenomenon of dispersion of light is the dispersion of sunlight in the sky which gives a rainbow formation. Rainbow is a natural spectrum which is seen in the sky. During rainfall, a rain drop behaves as a dispersive medium like a prism, and the sky acts as screen so that at a particular and suitable angle of incidence, a white ray of light incident on rain drop splits into its seven constituent colours, and a rainbow is seen in the sky.



[Fig. 18.2 Rainbow formation]

b. Blue colour of sky

The sky looks blue due to the dispersion of light followed by scattering. When sunlight passes through the atmosphere it suffers dispersion. Also, it suffers scattering from dust particles and air molecules. According to Rayleigh empirical formula of scattering is $I \propto \frac{1}{\lambda^4}$. Red light scatters least, and blue light scatters most. The blue colour, which scatters most, covers the sky. Due to this the sky seems blue.
Blue sky due to dispersion]



[Fig.18.3

c. Sun appears red during Sunrise and Sun-set

The Sun light suffers dispersion during its long travel through the atmosphere. During sunrise or sunset, when the Sun is close to the horizon, the light must travel a longer distance through the atmosphere than it does when the Sun is overhead. This results in comparatively more scattering of light. Red colour scatters the least, and becomes capable to reach our eyes, but other colours scatter away. Hence, during sunset and sunrise the sun appears red.



[Fig.18.4 Red appearance of the sky due to dispersion of light]

Sample Questions:

1. Why does white light disperse w passing through a prism?
2. A rainbow is never observed on the surface of the moon, why?
3. Why is there no dispersion of monochromatic light?
4. Which light; blue or red will be deviated more by a prism? Give a reason for your answer.

18.4 Spectrum of Light

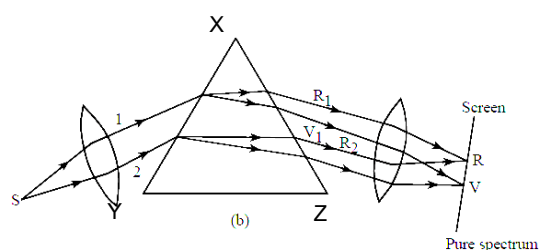
The band of seven colors obtained, when a white light splits into its constituent colors on striking the surface of a glass prism is called a spectrum." In other words, a spectrum is a collection of dispersed light that gives wavelength composition. When white light is incident on a prism, it is splitted into different constituent colours which are deviated through different angles. Such dispersed light, when received on the screen or photographic plate, forms a spectrum. There are two types of spectrum; pure spectrum and impure spectrum.

Pure Spectrum

When the white light is dispersed through a glass prism, and split colours do not overlap to each other (all colors can be seen distinctly) on the screen as shown in figure 18.2, then the corresponding spectrum is pure spectrum of light. A better method of obtaining a pure spectrum is to use a parallel beam of light as in an instrument called a spectrometer. For this an illuminated slit is placed at the principal focus of a converging lens so that a parallel beam of light emerges from it, and falls on a prism. Refraction through the prism splits up the light into

separate parallel beams of different colours, each of which is brought to its own focus in the focal plane of a second lens. The resulted pure spectrum may be formed on a white screen, or can be viewed through a magnifying eyepiece. Therefore, for obtaining pure spectrum following conditions should be satisfied.

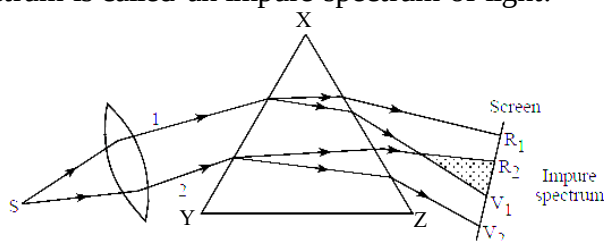
1. Slit should be narrow.
2. Rays coming out from convex lens should have parallel incidence.
3. Prism should be in condition of minimum deviation.
4. Emerging light rays should be focused by achromatic lens.



[Fig.18.5, Pure spectrum]

Impure Spectrum

When the white light is dispersed through a glass prism and the split colours overlap on the screen so that all colours cannot be seen distinctly as in figure 18.6, then the corresponding spectrum is called an impure spectrum of light.

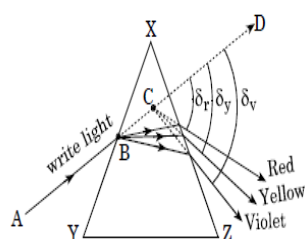


[Fig. 18.6, Impure spectrum]

Sample Questions:

1. How does a pure spectrum differ from an impure one?
2. Is a rainbow a pure spectrum?

18.5 Angular Dispersion



[Fig.18.7, Angular dispersion produced by a prism]

Angular dispersion is defined as the difference in the angles of deviation of two extreme colours (violet and red) of the spectrum of white light. That is

Angular dispersion, $\delta_a = \delta_v - \delta_r \dots (1)$

Consider a beam of white light passing through a glass prism so that different colours are deviated by different angles. Deviation of yellow colour is nearly average of deviation of all other colours. Therefore, yellow colour is taken as mean colour, and mean deviation of light is measured by the deviation of yellow light. The angle of deviation produced by violet light is more than that produced by red light as shown in figure 18.7. That means $\delta_r < \delta_y < \delta_v$ and $\delta_a = A(\mu_v - \mu_r)$.

This expression gives the angular dispersion of light through a glass prism.

18.6 Dispersive Power

A white light passing through a prism suffers both deviation and dispersion. The dispersive power of the material of prism is *its ability to disperse the constituent colours of incident light*, and it is measured in terms of *the ratio of angular dispersion between two extreme colours to the deviation produced by mean colour*. Dispersive power has no unit and dimension. It is denoted by ω and given by

$$\omega = \frac{\delta_v - \delta_r}{\delta} \dots (1)$$

$$\text{or, } \omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)} \dots (2)$$

This is an expression for dispersive power of the material of a prism.

Sample Questions:

1. How is dispersive power related to refractive index of the material?
2. On what factors does angular dispersion and dispersive power depend?

18.7 Aberration of Light in Lenses

The deviation in the shape, size and position of the actual image from the image predicted by lens formula is called aberration. It is defined as a property of optical systems such as lenses that causes light to be spread out over some region of space rather than being focused to a point. An optical system with aberration will produce a blurred image that is not sharp, and cannot be seen with clarity. There are different types of aberration; Chromatic aberration, Spherical aberration, etc.

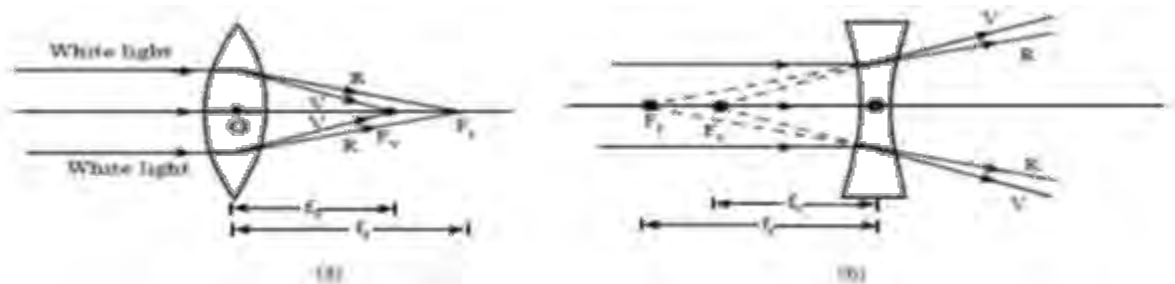
a. Chromatic Aberration

A lens may be considered as the combination of prisms so it exhibits prismatic behavior. If rays of light parallel with the principal axis incident on the surface of the lens then they split up into different constituent colours after refraction. "An inability of a lens to focus all colours of light at a single point on the principal axis is called chromatic aberration. It is also known as axial or longitudinal aberration. The chromatic aberration is measured in terms of the difference in focal lengths of red colour (f_r) and violet colour (f_v).



i.e., chromatic aberration = $f_r - f_v$... (1).

Consider a parallel beam of white light along the principal axis incident on the refracting surface of the lens. Light rays suffer dispersion after refraction through the lens and come to focus or appear to focus at different points on the principle axis as shown in figure 18.8. F_r and F_v be the focus points for red and violet colour.



[Fig.18.8, (i) Chromatic aberration in a convex lens (ii) Chromatic aberration in a concave lens]

Let, f_r , f , f_v and μ_r , μ , μ_v represent the focal lengths and refractive indices of red, mean and violet colours of light respectively then, using lens maker's formula for a lens of the radius of curvature R_1 and R_2 we get,.

For mean light,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

or, $\left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{f(\mu - 1)}$... (1)

For red light,

$$\frac{1}{f_r} = (\mu_r - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \dots (2)$$

From equation (1) and (2),

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{f(\mu - 1)} \dots (3)$$

Similarly, for violet light,

$$\frac{1}{f_v} = \frac{(\mu_v - 1)}{f(\mu - 1)} \dots (4)$$

Subtracting equation (3) from equation (4) we get,

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{(\mu_v - 1)}{f(\mu - 1)} - \frac{(\mu_r - 1)}{f(\mu - 1)}$$

or, $\frac{f_r - f_v}{f_r f_v} = \frac{(\mu_v - 1 - \mu_r + 1)}{f(\mu - 1)}$

or, $f_r - f_v = \frac{(\mu_v - \mu_r)}{f(\mu - 1)} f_r f_v$

or, $f_r - f_v = \frac{(\mu_v - \mu_r)}{f(\mu - 1)} f^2$ [Focal length of seven colours are in geometric progression, $f = \sqrt{f_r f_v}$]

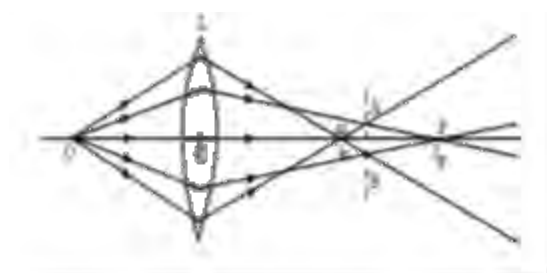
$$\text{or, } f_r - f_v = \frac{(\mu_v - \mu_r)}{(\mu - 1)} f$$

$$\therefore f_r - f_v = \omega f \quad \dots (5) \quad \left[\text{since, } \omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)} \right]$$

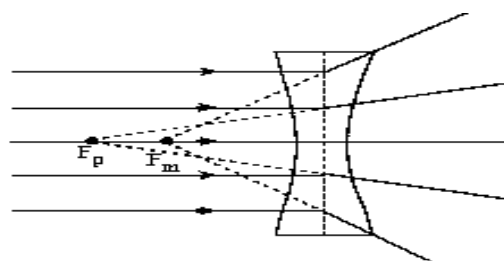
This is the condition of chromatic aberration in the lens. Hence, chromatic aberration is the product of focal length of mean colour of light and dispersive power.

b. Spherical Aberration in a Lens

Spherical aberration is an inability of a lens to focus the paraxial rays and marginal rays at the same point on the principal axis. The spherical aberration of the lens is due to its spherical shape. The rays passing through the lens close to its centre (paraxial rays) are focused farther away than rays passing through a circular zone near its rim (marginal rays). Images formed by the lens having large apertures are therefore not sharp but get sharper at smaller apertures. Spherical aberration is found in case of near objects and it is positive for the convex lens and negative for the concave lens.



[Fig.18.9, spherical aberration in a convex lens]



[Fig.18.10, Spherical aberration in a concave lens]

Consider a pointed object (O) has been placed on the principal axis of convex lens L so that paraxial rays from O are focused at point I_p (longer distance) whereas marginal rays are focused at point I_m (shorter distance) on the other side of the lens. The image is not sharp at any point on the axis. If the screen is placed perpendicular to the axis, then the image appears as a circular patch of diameter AB as shown in figure 18.9. The spherical aberration in the convex lens is shown in figure 18.10.

To remove spherical aberration in a lens, essentially its focal length for each zone must be made equal so that refracted rays meet at the same point. Practically it is impossible. Therefore, the spherical aberration in a lens can only be reduced but cannot be removed completely.

Workout Example:

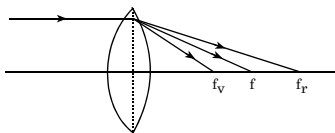
For the light of red colour, the focal length of the convex lens is found to be 26cm. However, using the violet colour, the focal length becomes 25 cm. What is the dispersive power of the lens?

Solution: Here,

$$f_r = 26\text{cm and } f_v = 25\text{cm}$$

$$\text{Where, } f = \frac{f_r + f_v}{2} = \frac{26 + 25}{2}$$

$$\therefore f = 25.5\text{cm}$$



Dispersive power (ω) = ?

Chromatic aberration = ?

We have, $\omega f = f_r - f_v$

$$\text{or, } \omega = \frac{f_r - f_v}{25.5} = \frac{26 - 25}{25.5} = 0.039$$

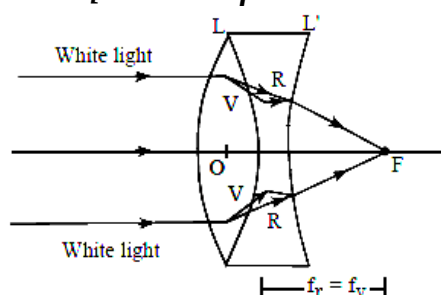
Hence, dispersive power = 0.039.

Project Work:1

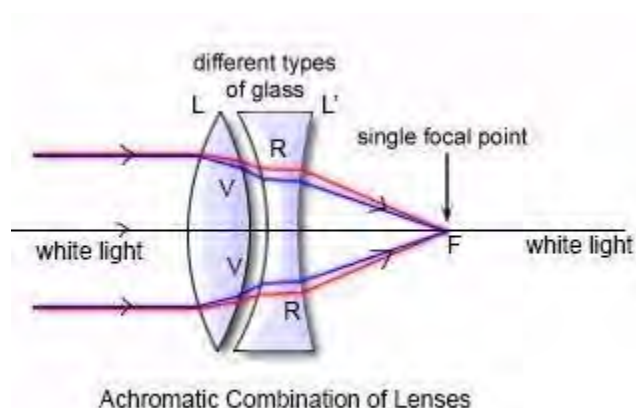
Use any available source, and complete the following table. Discuss the result among your friends and interact with your teacher about conclusions.

S N	Device	Combinations of lenses	results	Conclusions
1.	Microscope			
2.	Camera lenses			
3.	Telescope			
4.	Astronomical instruments			
5.	Optical instruments			
6.	Laser			
7.	Biomedical imaging			
8.	Fiber optics			
9.	Projector system			

18.8 Achromatic Combination [Removal of chromatic aberration] of Lenses



[Fig.18.11, Achromatic combination of lenses]



All the colours of the white light can be focused at a single point on the principal axis by making a suitable combination of lenses. "The combination of two thin lenses such that it is free from chromatic aberration is called the achromatic combination of lenses or simply achromatism."

Suppose two lenses L and L' having corresponding dispersive power ω and ω' are combined together as shown in figure 18.11. Let, μ_v, μ, μ_r and f_v, f, f_r be the refractive indices and focal lengths of lens L for violet, mean and red colour of light respectively, and μ_v', μ', μ_r' and f_v', f', f_r' be the corresponding parts for lens L'

For lens L,

Focal length for mean colour

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\text{or, } \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{f(\mu - 1)} \dots (1) \quad [R_1 \text{ and } R_2 \text{ are radii of curvature of two surfaces}]$$

Focal length for red colour

$$\frac{1}{f_r} = (\mu_r - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \dots (2)$$

From equation (1) and (2),

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{f(\mu - 1)} \dots (3)$$

Similarly, for lens L'

Focal length for red colour

$$\frac{1}{f'_r} = \frac{(\mu'_r - 1)}{f'(\mu' - 1)} \quad \dots (4)$$

Let, F_r be the combined focal length of two lenses for red colour then'

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f'_r} \quad \dots (5)$$

From equation (3), (4) and (5)

$$\frac{1}{F_r} = \frac{\mu_r - 1}{f(\mu - 1)} + \frac{\mu'_r - 1}{f'(\mu' - 1)} \quad \dots (6)$$

Similarly, for violet colour,

$$\frac{1}{F_v} = \frac{\mu_v - 1}{f(\mu - 1)} + \frac{\mu'_v - 1}{f'(\mu' - 1)} \quad \dots (7)$$

For achromatic combination of lenses,

$$F_r = F_v$$

or, $\frac{1}{F_r} = \frac{1}{F_v} \quad \dots (8)$

From equation (6), (7) and (8),

$$\frac{\mu_r - 1}{f(\mu - 1)} + \frac{\mu'_r - 1}{f'(\mu' - 1)} = \frac{\mu_v - 1}{f(\mu - 1)} + \frac{\mu'_v - 1}{f'(\mu' - 1)}$$

or, $\frac{\mu_r - 1}{f(\mu - 1)} - \frac{\mu_v - 1}{f(\mu - 1)} = \frac{\mu'_v - 1}{f'(\mu' - 1)} - \frac{\mu'_r - 1}{f'(\mu' - 1)}$

or, $\frac{\mu_r - \mu_v}{f(\mu - 1)} = \frac{\mu'_v - \mu'_r}{f'(\mu' - 1)}$

or, $-\frac{(\mu_v - \mu_r)}{f(\mu - 1)} = \frac{\mu'_v - \mu'_r}{f'(\mu' - 1)}$

or, $-\frac{\omega}{f} = \frac{\omega'}{f'}$ [Since $\omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)}$ and $\omega' = \frac{\mu'_v - \mu'_r}{(\mu' - 1)}$]

or, $\frac{\omega}{f} + \frac{\omega'}{f'} = 0 \quad \dots (9)$

Hence, equation (9) is the condition for the achromatic combination of two lenses.

Applications of Achromatism in Lens

Achromatism refers to the ability of the lens system to minimize or eliminate chromatic aberration. Achromatic lenses are designed for the correction of chromatic aberration, and so that it ensures different colors of light to converge at a single focal point. Achromatic lenses are essential in a wide range of applications such as photography to astronomy for accurate color correction and high-quality imaging. By minimizing chromatic aberration, achromatic lenses can improve image clarity, color accuracy, and precision and reliability of optical performance. Some applications of achromatism in lenses are explained below.

- i) **Camera Lenses in photography:** Achromatic lenses are widely used in camera to produce high-quality images with minimal chromatic aberration. The combination helps to converge all colours of light at the same focal point so that sharp and clear images are obtained.
- ii) **Optical Systems:** Achromatic lenses are in many optical systems such as binoculars, spotting scopes, range finders, surveying instruments, spectrometers to obtain accurate and color-corrected images. As a result, we achieve precise optical performance.
- iii) **Optical Fiber Systems:** Achromatic lenses are used in optical fiber systems to improve the transmission of signals by maintaining the integrity of the optical signal travelling through optical fiber. This makes improvement in performance and reliability of the communication system.
- iv) **Projector Systems:** Achromatic lenses are used in projector systems such as overhead projectors, digital projectors, etc. to ensure the accurate focus and color reproduction to obtain sharp images.

Workout Example:

An achromatic lens of focal length +30cm is to be constructed by combining a crown glass lens and a flint glass lens. What must be the focal lengths of the component lenses if the dispersive powers of the crown glass and the flint glass are 0.018 and 0.027 respectively?

Solution: Here,

Meanfocal length (F) = 30cm

Dispersive power of crown glass (ω_1) = 0.018

Dispersive power of flint glass (ω_2) = 0.027

Focal length of crown glass (f_1) = ?

Focal length of flint glass (f_2) = ?

For achromatic combination of lenses,

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} = -\frac{0.018}{0.027}$$

$$\text{or, } \frac{f_1}{f_2} = -\frac{2}{3}$$

$$\text{or, } f_1 = -\frac{2}{3}f_2 \quad \dots (1)$$

Again, we have the relation of combined focal length.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\text{or, } \frac{1}{F} = \frac{-3}{2f_2} + \frac{1}{f_2}$$

$$\text{or, } \frac{1}{30} = \frac{-3}{2f_2} + \frac{1}{f_2}$$

$$\text{or, } \frac{1}{30} = \frac{-3 + 2}{2f_2}$$

$$\therefore f_2 = -15\text{cm.}$$

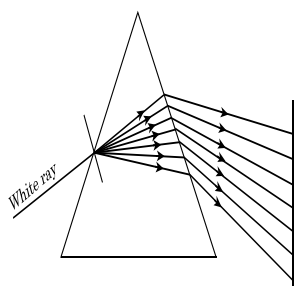
Using the value of f_2 , equation (1) will be,

$$\text{or, } f_1 = -\frac{2}{3} \times (-15) = 10\text{cm.}$$

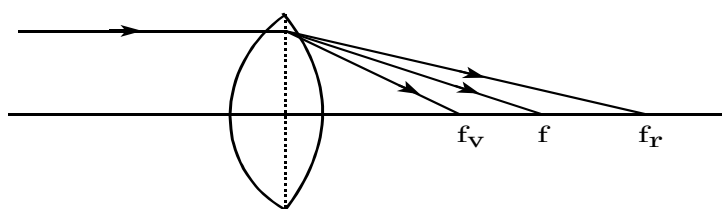
Hence, the focal lengths of the component lenses are 10 cm and – 15 cm respectively.

Exercise

- Which of the followings characterizes the colour of light?
a. frequency b. wave length c. velocity d. amplitude
- A chromatic aberration of a lens forms the image of a white object. Which of the followings is true about the image?
a. colored b. black c. white d. yellow
- Which of the followings must be valid for the combination of lenses to remove a chromatic aberration?
a. $f_r + f_v = 0$ b. $f_r > f_v$ c. $f_r < f_v$ d. $f_r - f_v = 0$
- What information can you draw from the given figure?



- refraction of light
 - dispersion of white light into seven colours
 - deviation produced for white light
 - chromatic aberration of light by prism
- Dispersive power 0.03 and refractive index 1.53 of a prism which forms an achromatic combination with another prism of angle 5° , dispersive power 0.045 and refractive index 1.66. What is the value of angle of prism?
a. 8.34° b. 9.34° c. 10.23° d. 11.54°
 - In the given figure, f_v and f_r of convex lens are 25cm and 26cm respectively. What is the value of the dispersive power?



- 0.39
 - 0.039
 - 0.049
 - 0.39
- An achromatic converging lens of the mean focal length 40 cm is made by combining two lenses of different materials, if the dispersive powers of the two lenses are in the ratio 1:3. What is the focal length of each lens?
a. 24cm, -80 cm b. 26cm, -80 cm

c. $26\frac{2}{3}$ cm, -80 cm

d. $26\frac{2}{3}$ cm, -90 cm

8. The dispersive powers of crown and flint glasses are 0.015 and 0.03 respectively. What is the focal length of the lenses which form an achromatic doublet of focal length 100cm placed in contact?

a. 50 cm

b. 90 cm

c. 100 cm

d. 200 cm

QN	1	2	3	4	5	6	7	8
Ans	b	c	d	b	b	b	c	c

9. (a) What are the causes of dispersion?

(b) Why is there no dispersion of monochromatic light?

(c) Explain why a mirror cannot give rise to chromatic aberration.

10. (a) Does the refractive index depend on the wavelength of light?

(b) What do you know about angular dispersion and dispersive power?

(c) The dispersive power of the material of a lens is 0.026, and it produces a chromatic aberration of 0.351 cm. Calculate the focal length of the mean colour. [13.5 cm]

11. (a) When a white light passes through a rectangular glass slab, no colour is seen, why?

(b) Which one; blue or red light will be deviated more by a prism?

(c) For red colour of light, the focal length of convex lens is found to be 26cm. However, using the violet colour, focal length becomes 25 cm. Calculate the dispersive power of the lens? Also, find the chromatic aberration produced by the lens. [0.039, 1 cm]

12. (a) How is dispersive power related to the refractive index of a material?

(b) Why does the red light appear at the top of the spectrum whereas the violet at the bottom during the dispersion of white light through a prism?

(c) The dispersive powers of crown and flint are 0.015 and 0.03 respectively. Calculate the focal lengths of the lenses which form an achromatic doublet of focal length 100cm placed in contact. [50cm, 100cm]

13. (a) What do you mean by chromatic aberration?

(b) Explain the meaning of achromatism in a lens.

(c) Find the angle of prism for a prism for which dispersive power is 0.03 and the refractive index is 1.53 to form an achromatic combination with another prism of angle 5° , dispersive power 0.045 and refractive index 1.66. [9.34°]

14. (a) Explain, how is it possible to produce dispersion without deviation and deviation without dispersion with two prisms.

(b) On what factors do the angular dispersion and the dispersive power depend?

(c) When white light is dispersed by a prism, the red light appears at the top of the spectrum whereas violet is at the bottom, why?

- (d) The dispersive power of a crown glass is 0.013 and that of a flint glass is 0.026. Calculate the focal length of the lenses which form an achromatic doublet of focal length 100cm placed in contact. [40cm, -80cm]
15. (a) How will you combine two lenses of different materials so that there is no chromatic aberration?
- (b) An achromatic converging lens of focal length 30 cm is to be constructed by combining a crown glass lens and a flint glass lens. What must be the focal lengths of the component lenses, if the dispersive power of the crown glass is 0.018 and that of the flint glass is 0.027? [-15cm, 10cm]
16. A concave lens made of crown glass has radii of curvature 25cm and 30 cm respectively. Compute the longitudinal chromatic aberration, given that $\mu_v = 1.523$ and $\mu_r = 1.5204$. [0.13 cm]
17. An achromatic combination of a convex lens of crown glass and a concave lens of flint glass is formed. If its power is +5 dioptre, calculate the focal length of two lenses. The dispersive power of crown glass is 0.115 and that of flint glass is 0.030. [10 cm, -20 cm]
18. The dispersive power of crown glass is 0.015 and that of flint glass is 0.030. Calculate, the focal lengths of lenses (made of crown and flint glass), which form an achromatic doublet of focal length 80 cm when placed in contact. [40 cm, -80 cm]
19. An achromatic lens combination of focal length 50 cm consists of a crown glass convex lens and a flint glass concave lens. Find the focal length of each lens from the data given below: For crown glass, $\mu_v = 1.523, \mu_r = 1.517$, for flint glass, $\mu_v = 1.664, \mu_r = 1.650$. [22.97 cm, -42.5 cm]
20. Calculate the angular dispersion between violet and red colours produced by a small angled prism with vertex angle $A = 5^\circ$, $\mu_v = 1.66$ and $\mu_r = 1.54$. [0.6°]
21. The refractive indices of crown glass and flint glass for violet and red light are 1.523, 1.513, 1.773 and 1.743 respectively. Calculate their dispersive power. [0.0193, 0.0396]

Chapter 19

Electric Charges



Charles Coulomb, French physicist (1736–1806)

Coulomb's major contributions to science were in the areas of electrostatics and magnetism. During his lifetime, he also investigated the strengths of materials, and determined the forces that affect objects on beams, thereby contributing to the field of structural mechanics. In the field of ergonomics, his research provided a fundamental understanding of the ways in which people and animals can work the best.

You can receive a mildly unpleasant shock, and produce a little spark if you touch a metal doorknob after walking across a carpet. Vigorously brushing your freshly washed hair makes all the hairs fly apart. A plastic comb that you've run through your hair will pick up bits of paper and other small objects, but a metal comb won't. The common factor in these observations is that two objects are rubbed together. Why should rubbing an object cause forces and sparks? What kind of forces are these? Why do metallic objects behave differently from nonmetallic? These are the questions with which we begin our study of electricity.

19.1 Introduction

Everyday objects contain an enormous number of charged particles, and they usually have an equal number of positive charges (protons) and negative charges (electrons), so that they have no overall charge, and are therefore described as being neutral. Negatively charged electrons are the outermost particles in atoms and some of them are not tightly bound to atoms. If electrons can be added to, or removed from, a neutral object, it will then have an overall charge and we describe the object as being charged, which can then result in electrostatic effects. Protons, unlike electrons, are located in the nuclei of atoms and cannot be separated or moved from their positions, so they are not involved in producing electrostatic effects.

If a neutral object is given excess electrons, it becomes negatively charged. If the number of electrons is reduced, the object becomes positively charged.

19.2 Charging by friction

One common example is seen in Figure 19.1, when brushing dry hair with a plastic brush, electrons can be transferred in the process, one object (the hair brush) gains electrons, becoming negatively charged, while removing electrons from the other object (the hair), leaving it with a positive charge.

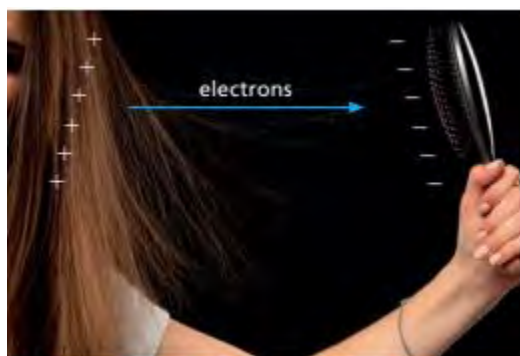


Fig. 19.1

The two objects will then attract each other. In this example, individual hairs with similar charge can also be seen to be repelled from each other.

In school experiments, in order to produce electrostatics effects, insulating rods are often rubbed with cloths. Depending on the materials, one will become positively charged, and the other negatively charged, as electrons are transferred between the rod and the cloth. These insulating materials have much fewer mobile charge carriers (free electrons) than metals, but if metals were used, the charges would not stay in the same place.

When friction occurs between substances, electrons are often transferred between their surfaces.

19.3 Charging by Induction

If you bring a charged object near a conducting surface, electrons are made to move in the surface material, even without physical contact.

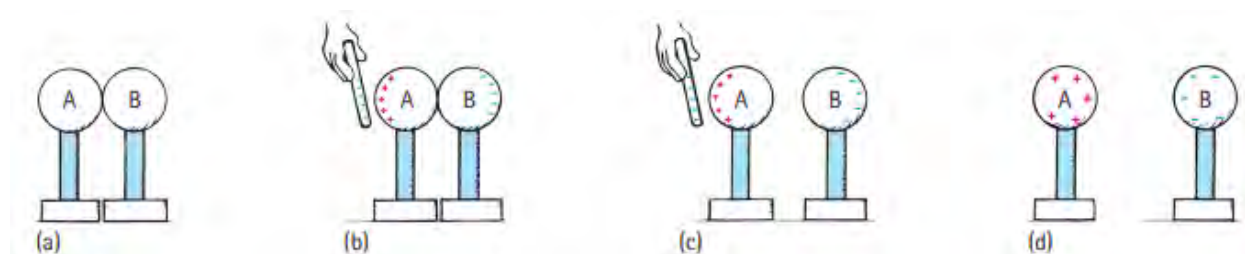


Fig. 19.2

Consider the two insulated metal spheres, A and B, in the figures above. In part 19.2 (a), the spheres touch each other, so in effect they form a single uncharged conductor. In part (b), when a negatively charged rod is brought near A, electrons in the metal, being free to move,

are repelled as far as possible until their mutual repulsion is strong enough to balance the influence of the rod: The charge is redistributed. In parts (c) and (d), if the spheres are separated while the rod is still present, each will be equally and oppositely charged. This is charging by induction. The charged rod has never touched the spheres, and the rod retains the same charge it had initially.

Activities:

1. Charge a plastic rod by rubbing it with wool. Touch a neutral metal sphere with the rubbed area of the rod. The metal sphere then picks up small pieces of paper and repels a charged, hanging plastic rod. The metal sphere appears to have acquired “plastic charge.”

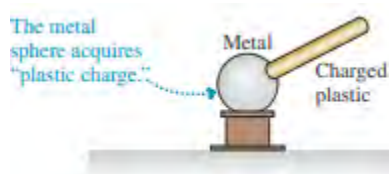


Fig. 19.3

2. Charge a plastic rod, then run your finger along it. After you’ve done so, the rod no longer picks up small pieces of paper or repels a charged, hanging plastic rod. Similarly, the metal sphere of Experiment 1 no longer repels the plastic rod after you touch it with your finger.

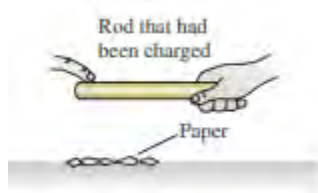


Fig. 19.4

3. Place two metal spheres close together with a plastic rod connecting them. Charge a second plastic rod, by rubbing, and touch it to one of the metal spheres. Afterward, the metal sphere that was touched picks up small pieces of paper and repels a charged, hanging plastic rod. The other metal sphere does neither.

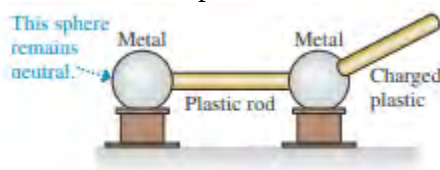


Fig. 19.5

Our experiments have shown that charge can be transferred from one object to another, but only when the objects touch. Contact is required. Removing charge from an object, which you can do by touching it, is called discharging.

There are two types or classes of materials with very different electric properties. We call these conductors and insulators.

19.4 Coulomb's law for the electric force

In physics, fields are regions in which an object will experience a force at a distance. An electric (or electrostatic) field is created by charged objects. Other charged objects in this electric field will experience force.

The electric force between two electric charges, Q_1 and Q_2 , was investigated in 1785 by Charles Augustin Coulomb (1736–1806).

Coulomb discovered that the electric force between two point charges is inversely proportional to the square of the separation of the charges, and is proportional to the product of the two charges. It is attractive for charges of opposite sign, and repulsive for charges of the same sign.

Mathematically,

$$F \propto Q_1 Q_2 \text{ and } F \propto \frac{1}{r^2}$$

where r is the separation of the two charges (Figure 19.6).

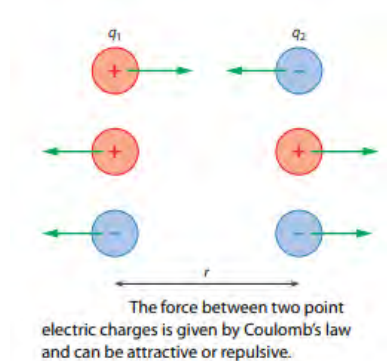


Fig. 19.6

Combining both,

$$F \propto \frac{Q_1 Q_2}{r^2}$$

The magnitude of the force F between

two-point charges of charge Q_1 and Q_2 separated by distance r in a vacuum is given by

$$F = k \frac{Q_1 Q_2}{r^2}$$

where k is the constant of proportionality and is known as Coulomb's constant.

The constant k is also written as $\frac{1}{4\pi\epsilon_0}$,

Hence, the Coulomb's law can be written as:

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

The direction of the electric force is always along the line joining the two charges. If the two charges have the same sign, the force on either charge is directed away from the other (they repel each other). If the two charges have opposite signs, the force on one is directed toward the other (they attract). See Fig. Fig. 19.7. Notice that the force one charge exerts on the second is equal but opposite to that exerted by the second on the first, in accord with Newton's third law.

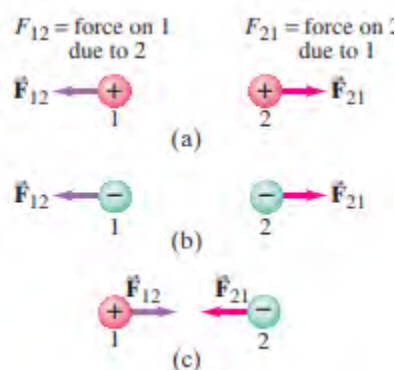


Fig. 19.7

The numerical value of the factor $\frac{1}{4\pi\epsilon_0}$ or k is $8.99 \times 10^9 \approx 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ in a vacuum. The constant ϵ_0 is called the electric permittivity of vacuum and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$.

The equation applies only for charges that are in a vacuum. If the charges are immersed in a different medium (say, air or water) then the value of the permittivity is different. It is usual to amend the equation slightly too, k becomes $\frac{1}{4\pi\epsilon}$ as the “0” subscript in ϵ_0 should only be used for the vacuum case. For example, the permittivity of water is $7.8 \times 10^{-10} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ and the permittivity of air is $8.8549 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$. The value for air is so close to the free-space value that we normally use $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ for both.

If we have a positive and a negative charge, then the force F is negative. We interpret this as an attraction. Positive forces, as between two like charges, are repulsive.

So far, we have considered point charges. If we are considering uniformly charged spheres we measure the distance from the centre of one to the centre of the other – they behave as if their charge was all concentrated at the centre. Hence, we can apply the equation for Coulomb's law for both point charges (e.g. protons, electrons, etc.) and uniformly charged spheres, as long as we use the centre to-centre distance between the objects.

The direction of the electric field is toward the charge Q , to the left as shown in Fig. a, since we defined the direction as that of the force on a positive test charge which here would be attractive. If Q had been positive, the electric field would have pointed away, as in Fig. b. NOTE There is no electric charge at point P. But there is an electric field there. The only real charge is Q .

Worked out:

Electric force on electron by proton.

Determine the magnitude and direction of the electric force on the electron of a hydrogen atom exerted by the single proton that is the atom's nucleus. Assume the average distance between the revolving electron and the proton is $0.53 \times 10^{-10} \text{ m}$.

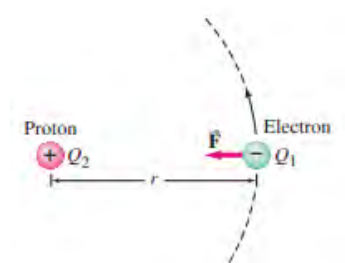


Fig. 19.8

The magnitude of the force is

$$F = k \frac{Q_1 Q_2}{r^2} = \frac{(9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times (1.6 \times 10^{-19}) \times (1.6 \times 10^{-19})}{(0.53 \times 10^{-10} \text{ m})^2} = 8.2 \times 10^{-8} \text{ N}$$

The direction of the force on the electron is toward the proton, because the charges have opposite signs so the force is attractive.

Workedout Examples

1. Electric field of a single point charge.

Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge.

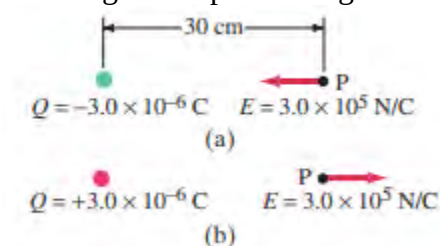


Fig. 19.9

Solution:

The magnitude of the electric field is:

$$F = k \frac{Q}{r^2} = \frac{(9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times (3.0 \times 10^{-6})}{(0.30 \text{ m})^2} = 3.0 \times 10^5 \text{ N/C}$$

2. Electric field at a point between two charges.

Two point charges are separated by a distance of 10.0 cm. One has a charge of $-25 \mu\text{C}$ and the other $+50 \mu\text{C}$. (a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative

charge (Fig.19.10 a). (b) If an electron (mass= 9.11×10^{-31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?

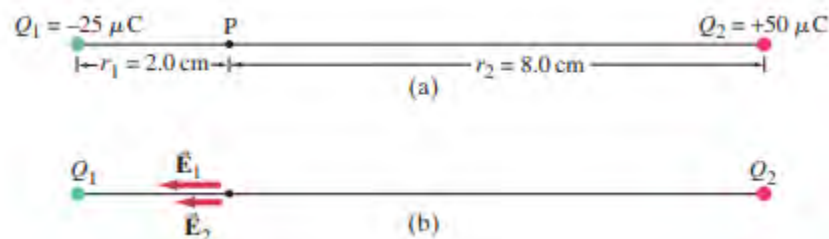


Fig. 19.10

The electric field at P will be the vector sum of the fields created separately by Q_1 and Q_2 . The field due to the negative charge Q_1 points toward Q_1 and the field due to the positive charge Q_2 points away from Q_2 . Thus, both fields point to the left as shown in Fig.19.10 b, and we can add the magnitudes of the two fields together algebraically. The total field points to the left and has magnitude

$$E = E_1 + E_2 = k \frac{Q_1}{r_1^2} + k \frac{Q_2}{r_2^2} = k \left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2} \right)$$

$$(9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \left(\frac{25 \times 10^{-6} \text{ C}}{(2.0 \times 10^{-2} \text{ m})^2} + \frac{50 \times 10^{-6} \text{ C}}{(8.0 \times 10^{-2} \text{ m})^2} \right) = 6.3 \times 10^8 \text{ N/C}$$

The electric field points to the left, so the electron will feel a force to the right since it is negatively charged. Therefore, the acceleration will be to the right.

Hence the magnitude of the electron's initial acceleration is

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(6.3 \times 10^8 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 1.1 \times 10^{20} \text{ m s}^{-2}$$

Exercise

- Figure below shows two charged particles at a distance d apart.



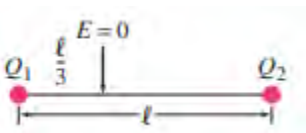
The two charged particles exert an attractive force of F .

The charge of each particle is changed by $+Q$. The distance between the two charges is increased to $2d$. What is the force between the two charges now?

- attractive force of $\frac{F}{4}$
 - repulsive force of $\frac{F}{4}$
 - attractive force of $\frac{F}{2}$
 - repulsive force of $\frac{F}{2}$
- Two point charges $Q_1 = -32\mu\text{C}$, and $Q_2 = +45\mu\text{C}$ are separated by a distance of 12 cm. The electric field at the point P (see Fig.) is zero. How far from is P?



- You are given two unknown point charges, Q_1 and Q_2 . At a point on the line joining them, one-third of the way from to the electric field is zero (Fig.). What is the ratio Q_1/Q_2 ?



- Two point charges, of $+30\text{ nC}$ and -30 nC , form an electric dipole of length 0.25 m .
 - Calculate the force between the charges.
 - Calculate the electric field strength at the midpoint between the charges.
 - Sketch the shape of the electric field in the region of the dipole.
- Figure shows two light polystyrene spheres, which have been coated in a conducting metallic paint. Each has been charged positively by a high-voltage supply. They are

suspended by pieces of cotton 15cm long, and they are pushed apart by the repulsive electrostatic force between them. The mass of each sphere is 0.08g. Use the information in the diagram to calculate the charge on each sphere.

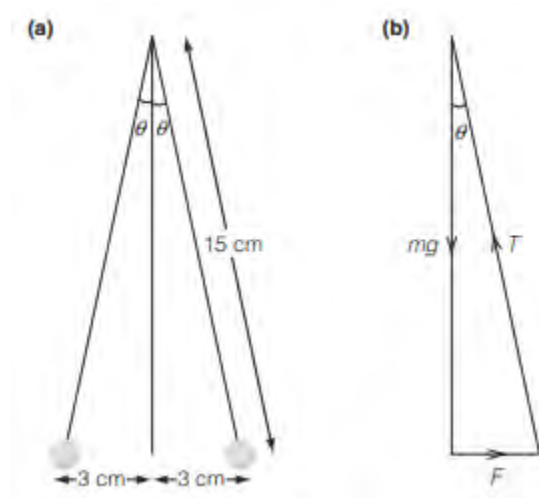


Fig. 19.11

Chapter 20

Electric field

Karl Friedrich Gauss, German mathematician and astronomer (1777–1855)



German mathematician and astronomer (1777–1855) Gauss received a doctoral degree in mathematics from the University of Helmstedt in 1799. In addition to his work in electromagnetism, he made contributions to mathematics and science in number theory, statistics, non-Euclidean geometry, and cometary orbital mechanics. He was the founder of the German Magnetic Union, which studies the Earth's magnetic field continually.

20.1 Introduction

In physics, fields are regions in which an object will experience a force at a distance. An electric (or electrostatic) field is created by charged objects. Other charged objects in this electric field will experience force. You can create an electric field very easily by rubbing a glass rod with a silk cloth. Friction transfers electrons from the glass to the silk, making the cloth negative, and the rod positive. The rod is then surrounded by an electric field — it will attract small pieces of paper or water from a dripping tap.

Nature creates its own electric fields. The strong electric field between the base of a thundercloud and the ground helps to channel the electric current of lightning.

1. Electric charge, Q , is measured in coulombs (C), and can be either positive or negative.
2. Oppositely charged particles attract each other. Like charges repel.
3. If a charged object is placed in an electric field, then it will experience a force.
4. If the charged object is a sphere, you can assume that all of its charges is at its centre.
5. Just like gravitational fields, electric fields can be represented by field lines

20.2 Electric field lines

Sometimes, the origin of a force between two objects is obvious, an example is the friction pad in a brake rubbing on the rim of a bicycle wheel to slow the cycle down. In other cases, there is no physical contact between two objects yet a force exists between them. Examples include the magnetic force between two magnets and the electrostatic force between two charged objects. Such forces are said to “act at a distance”.

The term field is used in physics for cases where two separated objects exert forces on each other. We say that in the case of the comb picking up the paper, the paper is sitting in the electric field due to the comb. The concept of the field is an extremely powerful one in physics not least because there are many ideas common to all fields.

Any two charged objects exert equal and opposite forces on each other without being directly in contact. An electric field is said to surround each charge. Suppose a small positive charge is placed as a test charge near a body with a much bigger charge which is also positive. If the test charge is free to move, it will follow a path away from the body with the bigger charge. The path a free positive test charge follows is called a line of force or a field line.

The field lines of an electric field are the lines which the positive test charges follow. The direction of an electric field line is the direction a positive test charge would move along.

The idea of field lines was first introduced by Michael Faraday. Although the field lines are imaginary, they are useful for illustrating and understanding the nature of a particular field. There are some conventions for drawing these electric field patterns.

The lines start and end on charges of opposite signs.

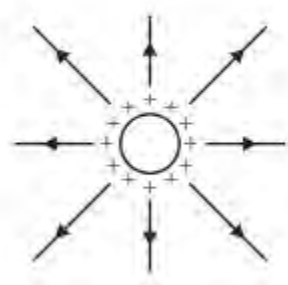
An arrow is essential to show the direction in which a positive charge would move (i.e. away from the positive charge and towards the negative charge).

Where the field is strong, the lines are close together. The lines act to repel each other.

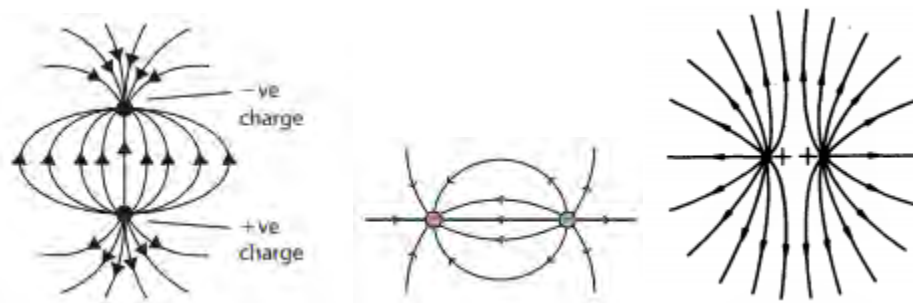
The lines never cross.

The lines meet a conducting surface at 90° .

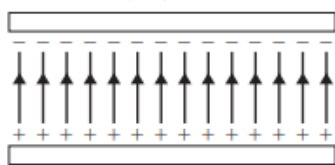
Figure 20.1 shows the patterns of the field around different charged objects.



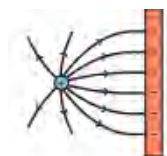
Field lines due to a charged sphere



Field lines due to dipole



Field lines due to a uniform field



Field lines due to a positive charge close to a conductor.

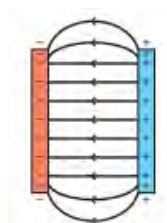


Fig. 20.1

Field lines between oppositely charged parallel plates.

Experiments suggest that the field between two parallel plates is uniform in the region between the plates and becomes weaker at the edges (these are known as edge effects).

At the edge, the field lines curve outwards as the field gradually weakens from the large value between the plates to the much weaker field well away from them.

Radial electric fields

The electric field between two parallel charged plates is uniform; what shape of electric field does a small 'point' charge like a single proton produce?

An isolated positive charge P produces an electric field like that shown in Figure 20.2. The radial lines of force show the direction of the field around P.

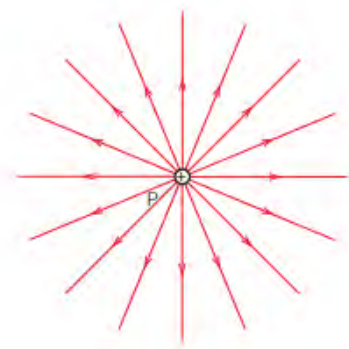


Fig. 20.2

Of course, the electric field lines spread out in three dimensions from the charge – the diagram only shows them in two dimensions.

The arrows on the lines of force would be reversed if the charge were negative. Equipotential lines in this case will be drawn as circles, but in three dimensions they will be spherical surfaces.

A charged sphere, or even an almost-spherical charged head, also produces a radial field. Figure 20.3 confirms this as the boy's hair shows clearly the direction of the electric field around his head.



Fig. 20.3

20.3 Electric field strength

The electric field strength is defined using the concept of a positive test charge.

The electric field strength, E , at a point in the field, is defined as the force per unit charge on a positive test charge placed at that point.

Imagine an isolated charge Q sitting in space. We want to know what the strength of the field is at point P , a distance r away from the isolated charge. We put another charge, a positive test charge of size q , at P and measure the force F that acts on the test charge due to Q . Then the magnitude of the electric field strength is defined to be

$$E = \frac{F}{q}$$

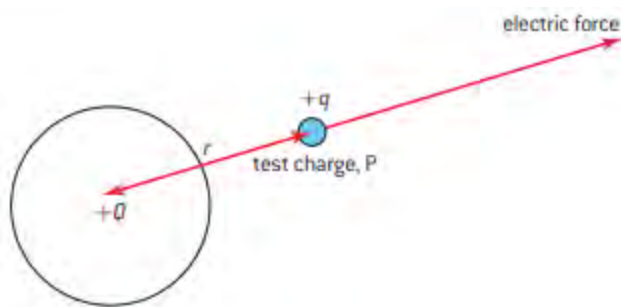


Fig. 20.4

The units of electric field strength are N C^{-1} . (Alternative units, that have the same meaning, are V m^{-1} and will be discussed later.) Electric field strength is a vector, it has the same direction as the force F (This is because the charge is a scalar which only “scales” the value of F up or down).

A formal definition for electric field strength at a point is the force per unit charge experienced by a small positive point charge placed at that point.

Coulomb’s law can be used to find how the electric field strength varies with distance for a point charge. Q is the isolated point charge and q is the test charge, so;

$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$$

$$E = \frac{F}{q}$$

Therefore,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \times \frac{1}{q}$$

So,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = \frac{kQ}{r^2}$$

If Q is a positive charge, then E is also positive. Applying the rule that r is measured from the charge to the test charge, then if E is positive, it acts outwards away from the charge. This is what we expect as both charge and test charge are positive. When Q is negative, E acts towards the charge Q . The field shape for a point charge is known as a radial field. The field lines radiate away (positive) or towards (negative) the point charge as shown in Fig. 20.5.

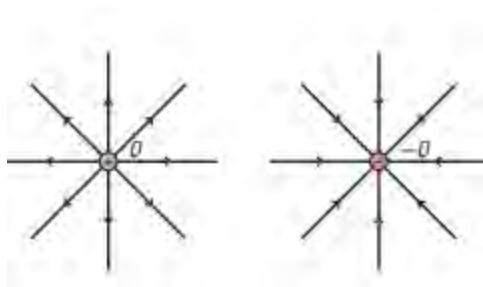


Fig. 20.5

The electric field between two parallel plates (Uniform electric field)

The field pattern for two oppositely charged flat plates is as shown in Fig. 20.6.

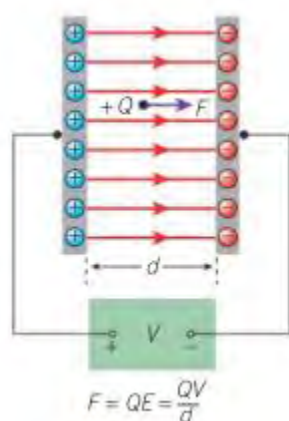


Fig. 20.6

The field lines parallel to each other, are at right angles to the plates, and are from the positive plate to the negative plate.

20.4 The field between the plates is uniform.

This is because the electric field strength has the same magnitude and direction everywhere between the plates. The electric field strength E can be calculated from the potential difference V between the plates and their separation d using the equation,

$$E = \frac{V}{d}$$

To prove this equation, consider a small charge Q between the plates, as in Figure.

1. The force F on a small charge Q in the field is given by $F = QE$, where E is the electric field strength between the plates.
2. If the charge is moved from the positive to the negative plate, the work done W by the field on Q is given by $W = \text{force } F \times \text{distance moved } d$, so $W = QEd$
3. By definition, the potential difference between the plates, V , is the work done per unit charge when a small charge is moved through potential difference V .

$$\text{Therefore, } V = \frac{W}{Q} = \frac{QEd}{Q} = Ed$$

Rearranging $V = Ed$ gives $E = \frac{V}{d}$

A beam of charged particles can be deflected by an electric field, as shown in the figure. By adjusting the strength of the field, the extent of the deflection can be controlled. The electric field used to deflect the beam is produced by applying a constant p.d. between the metal deflecting plates A and B.

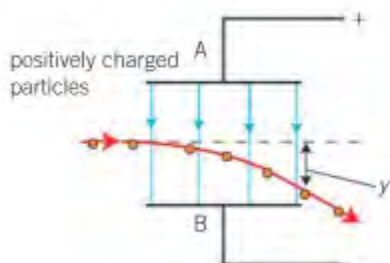


Fig. 20.7

The beam in the figure consists of identical positively charged particles which are deflected towards the negative plate. If the particles had been negatively charged, they would have been deflected towards the positive plate. The force on each charged particle in the field is constant in magnitude and direction because the field is uniform so the beam curves in a parabolic path (just as a projectile projected horizontally does). The screen in the tube enables the path of the beam to be observed and measured.

If the p.d. between plates A and B is V_P , the force F on each charged particle is given by the equation;

$$F = qE = \frac{qV_P}{d}$$

where d is the perpendicular distance between plates A and B, q is the charge of each charged particle, and $E = \frac{V_P}{d}$ is the electric field strength between the plates.

Notes:

1. The acceleration of each charged particle towards the negative plate; $a = \frac{F}{m} = \frac{qE}{m} = \frac{qV_P}{md}$, where m is the mass of the particle.
2. The time taken, t , by each charged particle to cross the field $= \frac{L}{v}$, where L is the length of each plate and v is the initial speed of the particle on entry to the field. This is the horizontal component of the particle's velocity in the field as it has no horizontal acceleration.
3. The deflection, y , of the charged particle on leaving the field is given by the equation
 $y = \frac{1}{2} at^2$. Using the above equations, it can be shown that y is directly proportional to the plate p.d., V_P .
4. The vertical deflection, y , of the charged particle on leaving the field is given by the equation

$$y = \frac{1}{2} at^2 = \frac{1}{2} \frac{qE}{m} t^2 \dots \dots \dots (i)$$

In a horizontal direction, the displacement,

$$x = vt \dots \dots \dots (ii)$$

Eliminating t between (i) and (ii), we obtain,

$$y = \frac{1}{2} \left(\frac{qE}{m} \right) \left(\frac{x}{v} \right)^2 = \frac{qE}{2m_e v^2} x^2$$

The path is therefore a parabola.

If the moving charge is an electron then,

$$y = \frac{1}{2} \left(\frac{eE}{m_e} \right) \left(\frac{x}{v} \right)^2 = \frac{Ee}{2m_e v^2} x^2$$

20.5 Electric Flux

The concept of electric field lines was described qualitatively earlier. We now treat electric field lines more quantitatively. Consider an electric field that is uniform in both magnitude and direction as shown in figure 20.8.

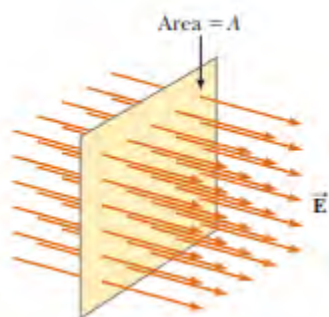


Fig. 20.8

In the field lines representing a uniform electric field penetrating a plane of area A perpendicular to the field, the electric flux ϕ_E through this area is equal to EA .

The field lines penetrate a rectangular surface of area A , whose plane is oriented perpendicular to the field. The number of lines per unit area (in other words, the line density) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product EA . This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the electric flux ϕ (uppercase Greek letter phi):

$$\phi = EA$$

From the SI units of E and A , we see that ϕ has units of newton meters squared per coulomb (Nm^2C^{-1}). Electric flux is proportional to the number of electric field lines penetrating some surface.

If the surface under consideration is not perpendicular to the field, the flux through it must be less than EA . Consider the figure, where the normal to the surface of area A is at an angle θ to the uniform electric field.

Field lines representing a uniform electric field penetrating an area A are at an angle θ to the field. Because the number of lines that go through the area $A_{\perp}$ is the same as the number of lines that go through A , the flux through $A_{\perp}$ is equal to the flux through A and is given by,

$$\phi = EA \cos\theta$$

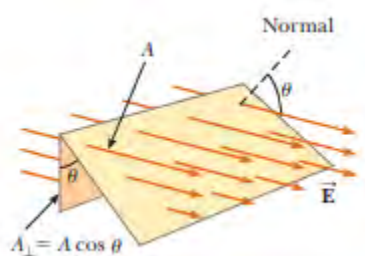


Fig. 20.9

Notice that the number of lines that cross this area A is equal to the number of lines that cross the area $A_{\perp}$, which is a projection of area A onto a plane oriented perpendicular to the field. The figure shows that the two areas are denoted by $A_{\perp} = A \cos \theta$. Because the flux through A is equal to the flux through $A_{\perp}$, the flux through A is;

$$\phi = EA_{\perp} = EA \cos \theta$$

From this result, we see that the flux through a surface of fixed area A has a maximum value EA when the surface is perpendicular to the field (when the normal to the surface is parallel to the field, that is, when $\theta = 0^\circ$ in Fig.); the flux is zero when the surface is parallel to the field (when the normal to the surface is perpendicular to the field, that is, when $\theta = 90^\circ$).

Test yourself:

The figure 20.10 shows a cube of side s in a uniform electric field. What's the flux through each of the three cube faces and C with the cube oriented as in (a)? Repeat for the orientation in (b), with the cube rotated at 45° .

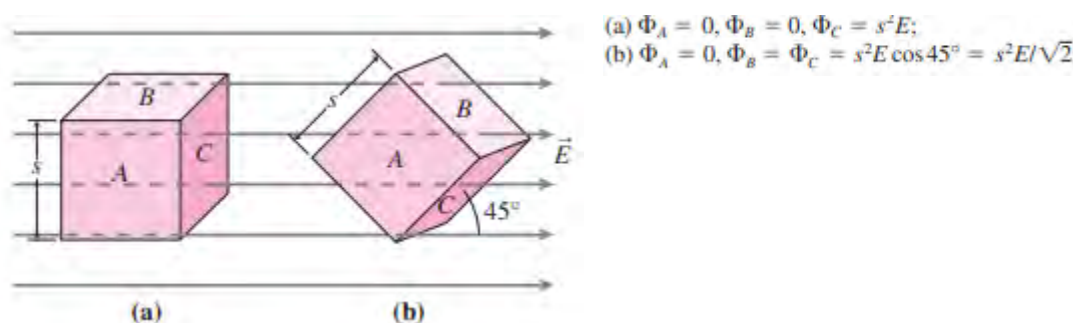


Fig. 20.11

20.6 Gauss's Law

Gauss's law is based on the inverse-square behavior of the electric force between point charges. Although Gauss's law is a consequence of Coulomb's law, it is more convenient for calculating the electric fields of highly symmetric charge distributions, and makes it possible to deal with complicated problems using qualitative reasoning.

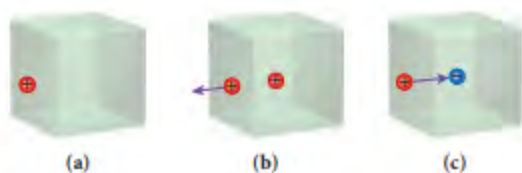


Fig. 20.12: Three imaginary boxes are constructed of a material that does not affect electric fields. A positive test charge is brought up to the box from the left toward: (a) an empty box; (b) a box with a positive charge inside; (c) a box with a negative charge inside.

To begin our discussion of Gauss's Law, let's imagine a box in the shape of a cube (Figure a), which is constructed of a material that does not affect electric fields. A positive test charge brought close to any surface of the box will experience no force. Now, suppose a positive charge is inside the box and the positive test charge is brought close to the surface of the box (Figure b). The positive test charge experiences an outward force due to the positive charge inside the box. If the test charge is close to any surface of the box, it experiences the outward force. If twice as much positive charge is inside the box, the positive test charge experiences twice the outward force when brought close to any surface of the box.

Now, suppose there is a negative charge inside the box (Figure c). When the positive test charge is brought close to one surface of the box, the charge experiences an inward force. If the positive test charge is close to any surface of the box, it experiences an inward force. Doubling the negative charge in the box doubles the inward force on the test charge when it is close to any surface of the box.

In an analogy with flowing water, the electric field lines seem to be flowing out of the box containing positive charge and into the box containing negative charge.

Now, let's imagine an empty box in a uniform electric field (Figure 20.13).



Fig 20.13: Imaginary empty box in a uniform electric field.

If a positive test charge is brought close to side 1, it experiences an inward force. If the charge is close to side 2, it experiences an outward force. The electric field is parallel to the other four sides, so the positive test charge does not experience any inward or outward force when brought close to those sides. Thus, in the analogy with flowing water, the net amount of electric field that seems to be flowing in and out of the box is zero. Whenever a charge is inside the box, the electric field lines seem to be flowing in or out of the box. When there is no charge in the box, the net flow of electric field lines in or out of the box is zero.

These observations and the definition of electric flux, which quantifies the concept of the flow of the electric field lines, lead to Gauss's Law:

$$\phi = \frac{q}{\epsilon_0}$$

Here, q is the net charge inside a closed surface, called a Gaussian surface. The closed surface could be a box like that we have been discussing or any arbitrarily shaped closed surface.

Usually, the shape of the Gaussian surface is chosen so as to reflect the symmetries of the problem situation.

Gauss's law can be stated as 'the electric flux ϕ through any closed surface is equal to the net charge inside the surface, q inside, divided by ϵ_0 .'

Gauss's law, is one of four fundamental relations that govern the behavior of electromagnetic fields throughout the universe. Whether you journey into a star in some remote galaxy, down among the strands of a DNA molecule, or into the microprocessor chip at the heart of your computer, you'll find that the flux of the electric field through any closed surface depends only on the enclosed charge. In nearly 200 years of experiments, no electric field has ever been found to violate Gauss's law.

Consider a point charge q surrounded by a spherical surface of radius r centered on the charge, as in figure 20.14.

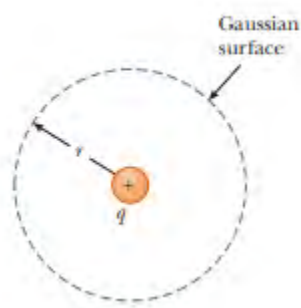


Fig.20.15: The flux through a spherical surface of radius r surrounding a point charge q is $\phi = \frac{q}{\epsilon_0}$

The magnitude of the electric field everywhere on the surface of the sphere is,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Now, the electric field is perpendicular to the spherical surface at all points on the surface. The electric flux through the surface is therefore EA , where $A = 4\pi r^2$ is the surface area of the sphere:

$$\phi = EA = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

The flux is independent of the radius r of the sphere. It depends only on the charge q enclosed by the sphere.

20.8 Gauss's Law: A Uniformly Charged Sphere

An insulating solid sphere of radius a has a uniform volume charge density ρ (charge per unit volume) and carries a total positive charge Q (Fig. 20.18).

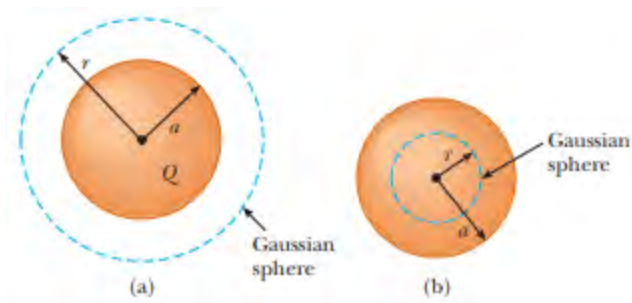


Fig. 20.18

The magnitude of the electric field at a point outside the sphere

The charge is distributed uniformly throughout the sphere so we can apply Gauss's law to find the electric field. Let's choose a spherical Gaussian surface of radius r , concentric with the sphere, as shown in figure a.

Using Gauss's law,

$$\phi = EA = \frac{Q}{\epsilon_0}$$

$$\text{As area of Gaussian surface } (A) = 4\pi r^2$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Therefore, the electric field due to a uniformly charged sphere in the region external to the sphere is equivalent to that of a point charge located at the center of the sphere.

The magnitude of the electric field at a point inside the sphere.

Let's choose a spherical Gaussian surface having radius $r < a$, concentric with the insulating sphere (Fig. b). Let V' be the volume of this smaller sphere. To apply Gauss's law in this situation, consider that the charge q_{in} within the Gaussian surface of volume V' is less than Q .

$$\text{As volume charge density is } \rho, q_{in} = \rho V' = \rho \times \frac{4}{3}\pi r^3$$

Using Gauss's law,

$$\phi = EA = \frac{q_{in}}{\epsilon_0}$$

$$\text{As area of Gaussian surface } (A) = 4\pi r^2$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{in}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\rho \times \frac{4}{3}\pi r^3}{r^2} = \frac{\rho}{3\epsilon_0} r$$

The total charge in the volume of sphere $\frac{4}{3}\pi a^3$ is Q and volume charge density is ρ .

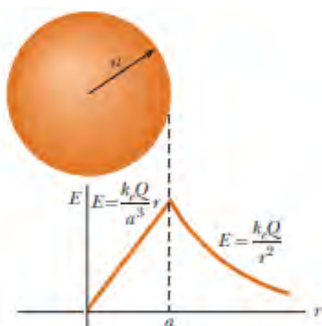
$$\therefore \rho = \frac{Q}{\frac{4}{3}\pi a^3} \text{ So}$$

$$E = \frac{\rho}{3\epsilon_0} r = \frac{\frac{Q}{\frac{4}{3}\pi a^3}}{3\epsilon_0} r = \frac{Q}{4\pi\epsilon_0 a^3} r$$

The magnitude of the electric field at a point inside the sphere

At the surface of the sphere $r = a$

$$E = \frac{Q}{4\pi\epsilon_0 a^3} a = \frac{Q}{4\pi\epsilon_0 a^2}$$



20.9 Gauss's law to find the electric field of an infinite line charge carrying charge density λ in coulombs per meter.

The line of charge is infinitely long. Therefore, the field is the same at all points equidistant from the line, regardless of the vertical position of the point in figure 20.19 a.

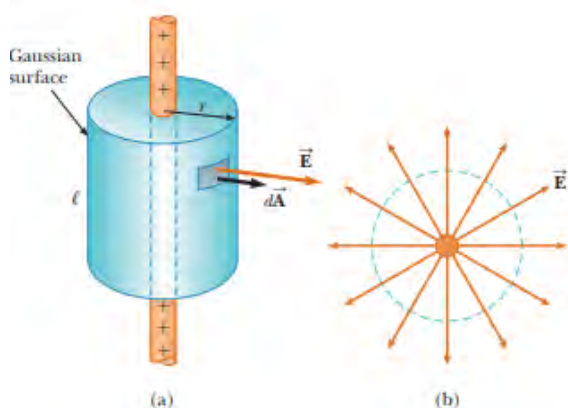


Fig. 20.19

Let's choose a cylindrical Gaussian surface of radius r and length l that is coaxial with the line charge. The flux through the ends of the Gaussian cylinder is zero because E is parallel to these surfaces. We must take the surface integral in Gauss's law over the entire Gaussian surface. The charge enclosed in the curved surface is $q_{in} = \lambda l$

Applying Gauss's law,

$$\phi = EA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

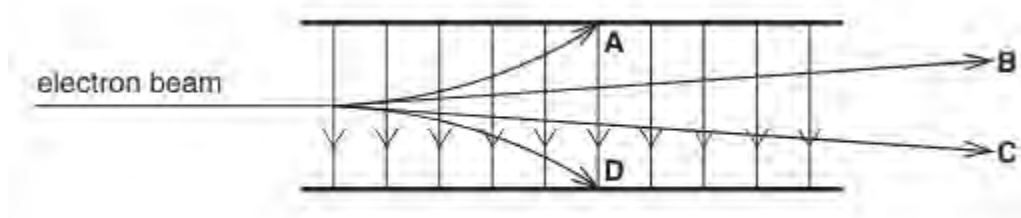
As the area of the curved surface is $A = 2\pi rl$,

$$E(2\pi rl) = \frac{\lambda l}{\epsilon_0}$$

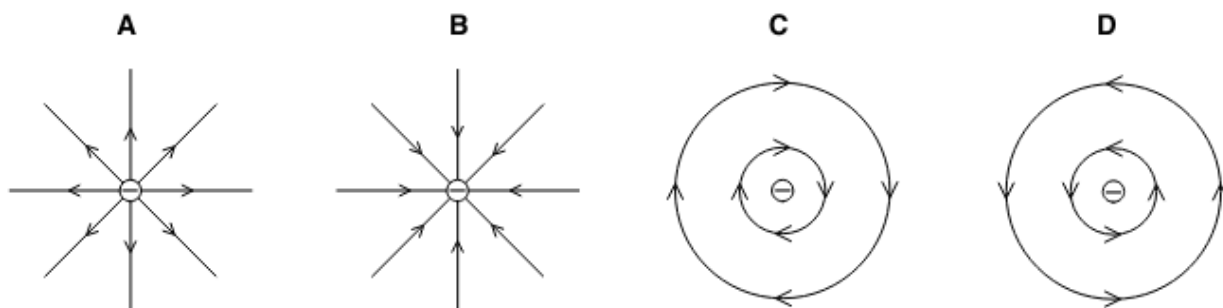
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Exercise

- Which path shows a possible movement of an electron in the electric field shown?



- Which diagram shows the electric field pattern of an isolated negative point charge?



- Which one of the following correctly describes the electric field strength?

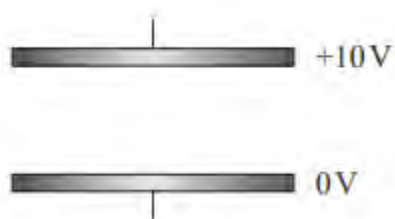
- It is defined as charge per unit force.
- It is defined as force per unit charge.
- It is defined as the energy per unit charge.
- It is defined as the charge per unit energy.

- The electric field between two charged plates is 650 N C^{-1} .

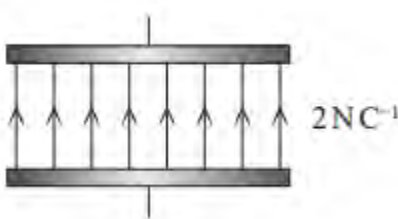
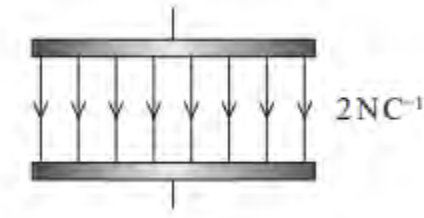
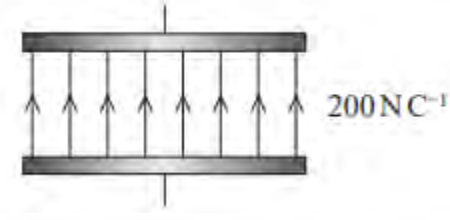
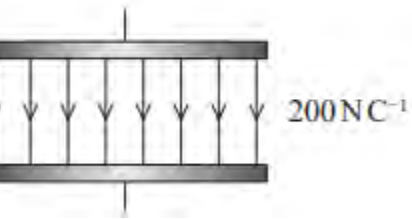
If the distance between the plates is 12 cm, what should be the potential difference between the plates?

- 54 V
- 78 V
- 4500 V
- 7800 V

5. Two conducting plates are 5 cm apart. The potential difference between the plates is 10 V as shown.



Which diagram correctly shows the electric field between the plates?

- A. 
- B. 
- C. 
- D. 

6. This question is about electric field. The figure below shows the electric field pattern drawn by a student for two oppositely charged plates.

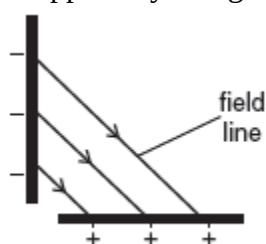


Fig.

State two errors made by the student in this drawing of the filed pattern. 7. At a distance r from the centre of a radioactive nucleus the electric field strength is E . The figure below shows the graph of electric field strength E against $\frac{1}{r^2}$.

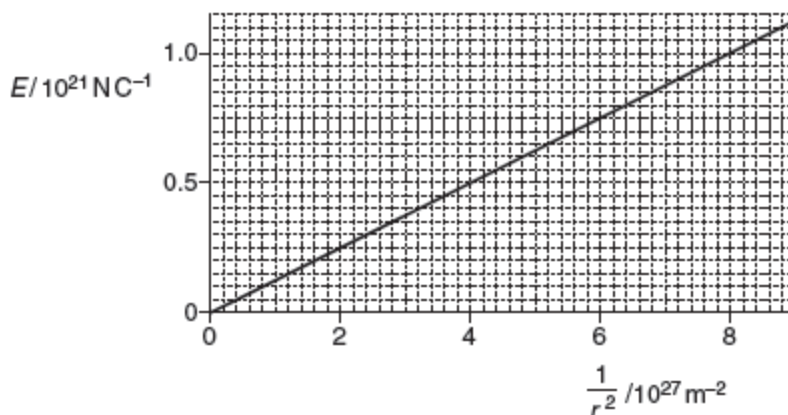
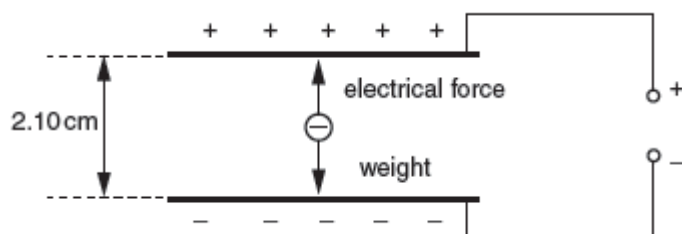


Fig.

- Determine the gradient of the graph and hence find the charge in the nucleus.
- The radioactive nucleus emits an alpha particle. State the change, if any, to the graph shown in for the resultant (daughter) nucleus. Explain your answer.

8. A negatively charged droplet of oil is held stationary between two horizontal plates.

The potential difference between the plates is 1.50 kV. The figure below shows the two forces acting on this oil droplet.



The droplet is spherical and has a radius of $1.27 \times 10^{-6} \text{ m}$. The density of oil is 950 kg m^{-3} . The separation between the plates is 2.10 cm.

- Show that the magnitude of charge on the droplet is about $1.1 \times 10^{-18} \text{ C}$.
- Calculate the number of electrons causing the charge on the droplet.

Chapter 21

Electric Potential, Potential difference and Potential energy

21.1 Electrical Potential

The electric potential at a point is the work done in bringing a unit positive test charge from infinity to the point.

$$V = \frac{\text{work done in moving charge between two points}}{\text{magnitude of charge}} = \frac{W}{Q}$$

The energy stored in the system per unit charge is what we call the potential of the system.

If we have two charges of opposite sign, then they attract each other, and the situation is exactly the same as in gravitation. In other words, when the charges are some finite distance apart, energy is required to move them to infinite separation. As the charges separate, the electric potential will gradually climb to zero.

The situation changes when the two charges have the same sign: now both charges repel each other.

Imagine two negative charges held at some finite distance apart. Energy was stored in the system when the charges were brought together from infinity. If they are released, the two charges will fly apart (to infinity where the repulsive force becomes zero). The system does work on the charges. There must have been a positive amount of energy stored in the system before the charges were allowed to separate. The potential is still defined to be zero at infinity, but this time as one charge is moved from infinity towards the other charge, the overall value of the potential rises above the zero level to become positive. Work has been done on the charge.

21.2 The Potential due to a Point Charge

The point charge at A exerts a repulsive force F on the charge at B. (from Coulomb's law we have])

$$F = \frac{1}{4\pi\epsilon_0} \frac{QQ'}{x^2}$$

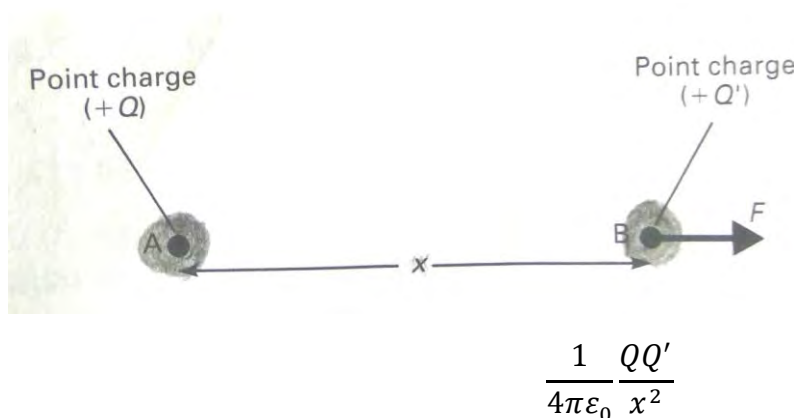


Fig. 21.1

If the charge at B is moved by some external agent a small distance Δx towards A where Δx is so small that F can be considered constant, the work done ΔW by the external agent is given by,

$$\Delta W = -F\Delta x$$

The minus sign is necessary because the motion is in the opposite direction to F.)

Substituting for F gives,

$$\Delta W = -\frac{1}{4\pi\epsilon_0} \frac{QQ'}{x^2} \Delta x$$

The total work done W in bringing the charge at B from infinity (where $x = \infty$) to some point a distance r from A (where $x = r$) is therefore given by,

$$W = -\frac{QQ'}{4\pi\epsilon_0} \int_{\infty}^r \frac{1}{x^2} dx = -\frac{QQ'}{4\pi\epsilon_0} \left[-\frac{1}{x} \right] = \frac{QQ'}{4\pi\epsilon_0 r}$$

If Q' is a unit charge, the work done is $\frac{Q}{4\pi\epsilon_0 r}$, and therefore by the definition of potential the potential V at a distance r from a point charge Q in free space is given by,

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Notes

- i. Potential is a scalar quantity and therefore the potential at a point due to a number of point charges is the algebraic sum of the (separate) potentials due to each charge.
- ii. It should be clear from equation that the potential due to a positive charge is positive; that due to a negative charge is negative.
- iii. It is clear from equation that all points which are equidistant from a point charge are at the same potential. (A surface over which potential is constant is called an equipotential surface. If a point lies on an equipotential surface, the electric field at that point is perpendicular to the surface.)

- iv. The potential energy of a charge Q' at a distance r from a point charge Q is given by,

$$PE = \frac{1}{4\pi\epsilon_0} \frac{QQ'}{r}$$

21.3 Relation between E and V

Suppose that a charge, $+Q$ is moved by a force, F , from some point A, in an electric field of strength, E , to some other point, B. Suppose also, that the distance, Δx , from A to B is so small that the force can be considered to be constant between A and B. The work done, ΔW , by the force is given by the equation,

$$W = F\Delta x$$

It follows from the definition of electric field intensity that,

$$F = -EQ$$

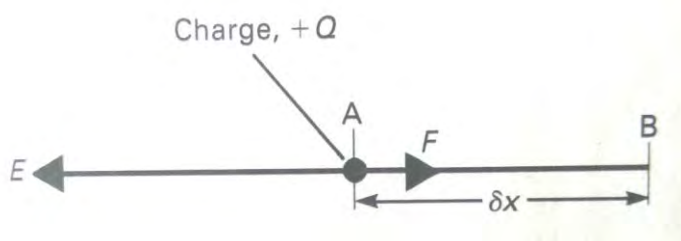


Fig. 21.2

The minus sign is necessary because E and F are oppositely directed, i.e., we are considering the work done by some external agency against the influence of the field.

From equations above,

$$\Delta W = -EQ\Delta x$$

If the potential difference between A and B is ΔV (B is at the higher potential), then

$$\Delta W = Q\Delta V$$

$$\therefore -EQ\Delta x = Q\Delta V$$

$$\text{Hence } E = -\frac{\Delta V}{\Delta x}$$

We Know,

- i) $\frac{\Delta V}{\Delta x}$ is the rate of change of potential with distance at a point in an electric field, and is called the potential gradient at that point. The minus sign expresses the fact that the field is in the negative direction of x if the potential increases in the positive direction of x . (V and x increase in going from A to B, E is directed from B to A.)

- ii) It should now be obvious why we were able to give V m^{-1} as an alternative to N C^{-1} as a unit of field strength.

Workedout Example: Two charges $3 \times 10^{-8} \text{ C}$ and $-2 \times 10^{-8} \text{ C}$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero?



Solution

Let P be the required point where the potential is zero.

Let P be at distance x from the positive charge.

- (i) If x lies between O and A, we have

$$\frac{1}{4\pi\epsilon_0} \left[\frac{3 \times 10^{-8}}{x \times 10^{-2}} - \frac{2 \times 10^{-8}}{(15 - x) \times 10^{-2}} \right] = 0$$

where x is in cm. That is,

$$\frac{3}{x} - \frac{2}{15 - x} = 0$$

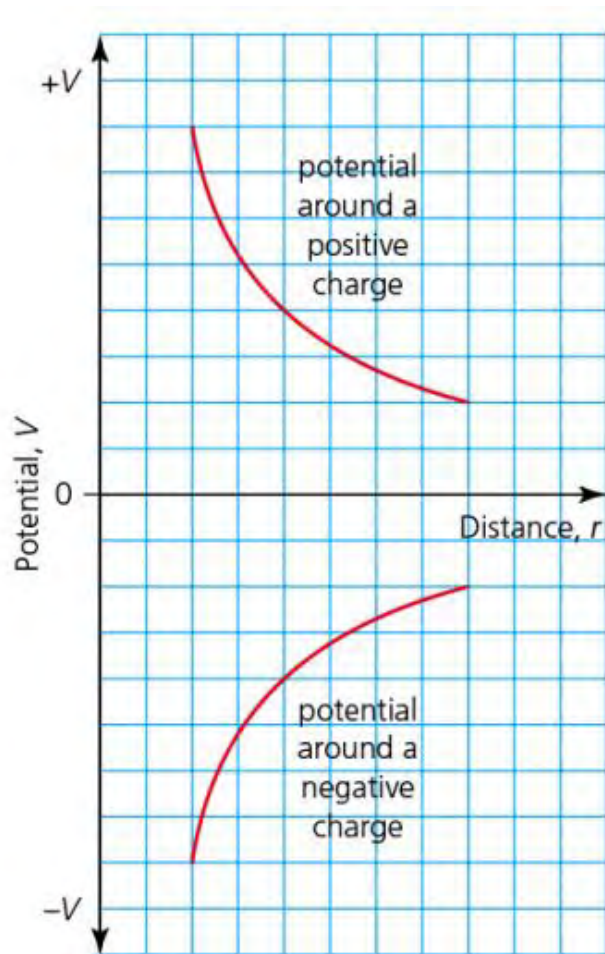
which gives $x = 9 \text{ cm}$.

- (ii) If x lies on the extended line OA, the required condition is,

$$\frac{3}{x} - \frac{2}{x - 15} = 0$$

Which gives $x = 45 \text{ cm}$

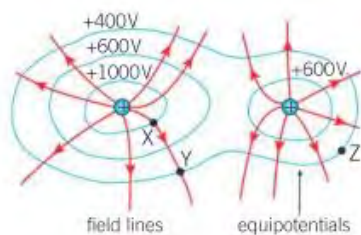
Thus, the electric potential is zero at 9 cm and 45 cm away from positive charge.



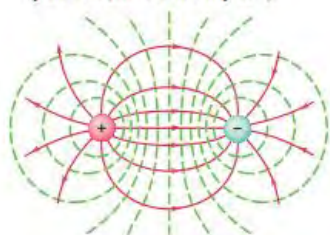
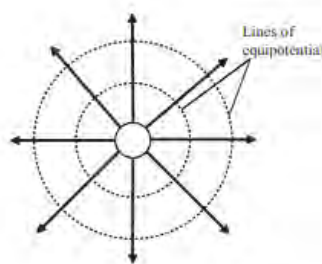
21.5 Potential Gradients

The electric potential can be represented by drawing equipotential lines or, in three dimensions, equipotential surfaces. An equipotential surface is one on which all points are at the same potential. That is, the potential difference between any two points on the surface is zero, so no work is required to move a charge from one point on the surface to the other. An equipotential surface must be perpendicular to the electric field at any point.

Equipotentials are surfaces of constant potential. A test charge moving along a line on an equipotential surface has constant potential energy. No work is done by the electric field on the test charge because the force due to the field is at right angles to the equipotential. In other words, the lines of force of the electric field cross an equipotential surface at right angles (ie. along the normal). The equipotentials for an electric field are like equipotentials for a gravitational field. Both are lines of constant potential energy for the appropriate test object, in one case a test charge, and in the other case a test mass.

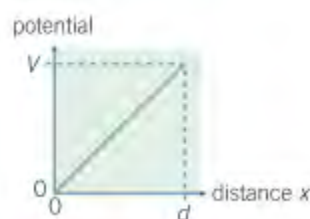
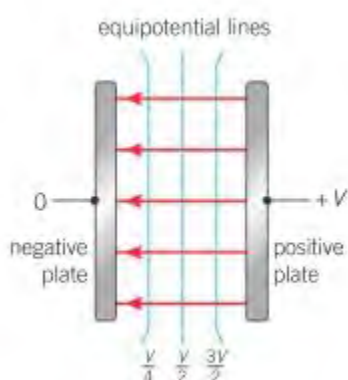


Equipotential lines (green, dashed) are always perpendicular to the electric field lines (solid red), shown here for two equal but oppositely charged particles (an "electric dipole").



The potential gradient at any position in an electric field is the change of potential per unit change of distance in a given direction.

1. If the field is non-uniform, as in the figure above, the potential gradient varies according to position and direction. The closer the equipotentials are, the greater the potential gradient is at right angles to the equipotentials.
2. If the field is uniform, such as the field between the two oppositely charged parallel plates shown in the figure below, the equipotential between the plates are equally spaced lines parallel to the plates. The figure below also shows how the potential relative to the negative plate changes with perpendicular distance x from the negative plate.



The graph shows that the potential relative to the negative plate is proportional to distance x . In other words, the potential gradient is constant. It is such that the potential increases in the opposite direction to the electric field and it is equal to $\frac{V}{d}$.

As we know, the electric field strength E between the plates is equal to $\frac{V}{d}$ and is directed from the +ve to the -ve plate.

In other words, the electric field strength is equal to the negative of the potential gradient.

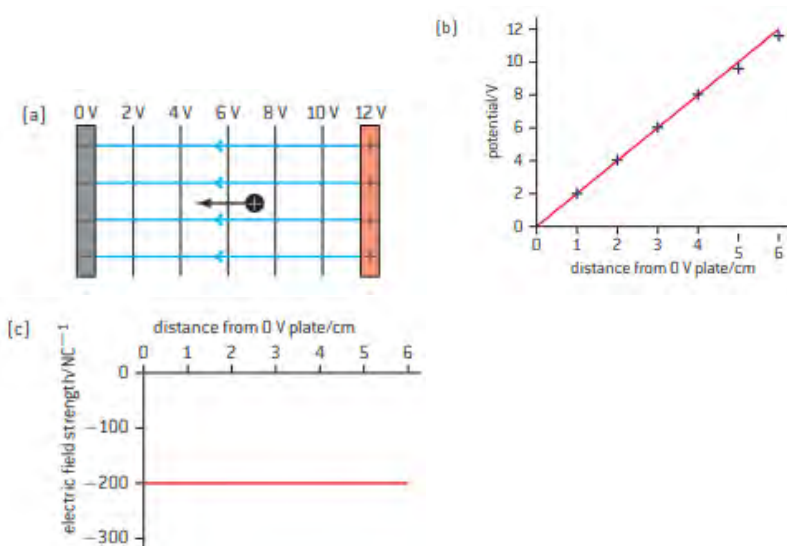
If the field is non-uniform, this statement still holds. However, the potential gradient at any position in an electric field is given by $\frac{\Delta V}{\Delta x}$ (instead of $\frac{V}{d}$), where ΔV is the change of potential between two closely spaced points at distance Δx apart.

Because the potential gradient is in the opposite direction to the lines of force of the electric field, then, in general, electric,

$$\text{field strength } E = -\frac{\Delta V}{\Delta x}$$

21.6 Graphical Interpretations of Electric Field Strength and Potential

Figure (a) is a reminder of the field pattern between charged parallel plates. The equipotentials are equally spaced. Figure (b) shows the data from measurements of the potential at a series of points between the plates plotted as a graph of potential against distance from the zero potential plate. It is a straight line because the electric field is uniform. A graph of electric field strength against distance from the 0V plate is a straight line parallel to the distance axis (Figure (c)).



The gradient of the potential–distance graph is $\frac{\text{change in } V}{\text{change in } x}$, and because it is a straight line, this is equal to $\frac{V}{d}$, in other words, E .

This leads to:

$$\text{electric field strength}(E) = -\text{potential gradient} = -\frac{\Delta V}{\Delta x}$$

(Δ , as usual, stands for “change in”).

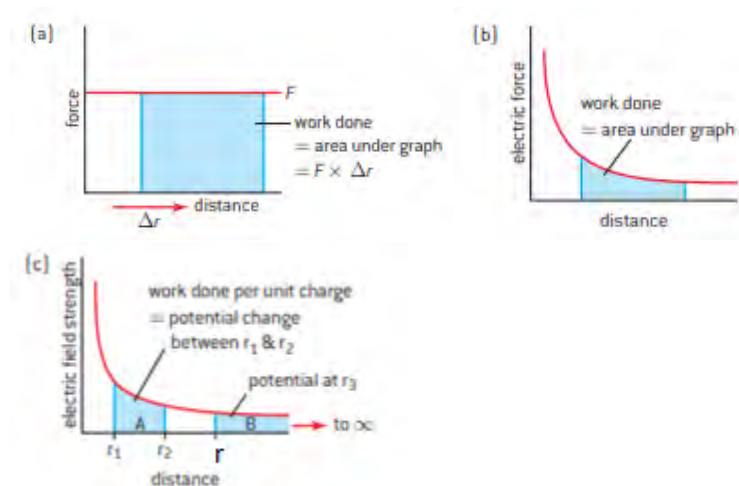
The minus sign that appears in the equation needs an explanation. It means that the direction of the vector electric field is always opposite to the variation of the potential (ΔV) of a positive charge. So, travel in the opposite direction to that of the field means moving to a position of higher potential and thus a positive gain in potential. In figure (a) the motion of the positive charge is to the left, ΔV is negative (final state minus initial state: 0 V - 12 V); so, according to

the equation, E will be positive and this tells us that the motion is in the direction of the field. In other words, going upstream in the field (against the field) means going to higher potential so a gain in potential (positive change in ΔV). Going downstream, ΔV is negative and E is positive (product of two minus signs) indicating the motion is in the direction of field.

If an electron moves from 0 V towards the 12 V plate (a position of higher potential) its potential energy is reduced because the potential energy change ΔE_p is given by $e\Delta V$ as usual, but here e is negative and ΔV is positive. The electron will accelerate in this direction if free to do so gaining kinetic energy from the field at the expense of the electrical potential energy.

A graph of force against distance for a constant force is shown in figure (a). The work done when this force moves an object through a distance Δr is equal to the area under the graph. In symbols,

$$F \times \Delta r = W$$



When an electric force varies with distance then the work done is still the area under the graph (Figure (b)). Electric field strength (Figure (c)) is the force per unit charge, and so the area under a graph of electric field strength against distance is equal to the work done per unit charge – in other words the change in potential.

Figure (c) shows the variation of electric field strength against distance for a single point positive charge – this is an inverse-square variation. Two areas are shown on this graph, one area (A) that shows the potential change between r_1 and r_2 , the other (B) shows the potential at r (as it is change of potential between ∞ and r : potential at ∞ is zero)– in this case the area included goes all the way to infinity.

$$E = -\frac{\Delta V}{\Delta r} = -\frac{dV}{dr}$$

$$dV = -E dr$$

$$\text{And } E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = \frac{kQ}{r^2}$$

Integrating from ∞ to d distance r from the charge,

$$\int_{\infty}^r dV = V_r - V_{\infty} = V_r = V(\text{say}) = \int_{\infty}^r -E dr = \int_{\infty}^r -k \frac{Q}{r^2} dr = -kQ \int_{\infty}^r \frac{dr}{r^2} = kQ \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$= \frac{kQ}{r}$$

$$\therefore V = \frac{kQ}{r^2} r = \frac{kQ}{r} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

This is the expression for the potential V at a distance r from a point charge. It predicts that the closer we are to a (positive) charge, the greater (more positive) is the potential. Conversely, the closer we are to a negative charge, the more negative is the potential.

Potential is the work done in moving a positive unit charge (from infinity) to a particular point, so the work required to move a charge of size q from infinity to the point will be: $V \times q$. This is the potential energy that charge q possesses due to its position in the field that is giving rise to the potential.

If a charge q is in a field that arises from another single point charge Q then its potential energy $E_p = qV$ or

$$E_p = \frac{kqQ}{r} = \frac{qQ}{4\pi\epsilon_0 r}$$

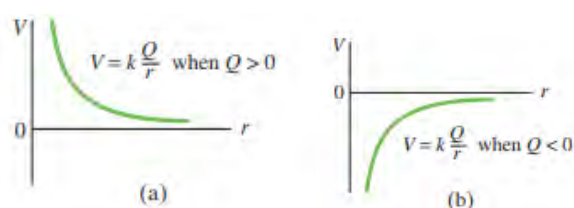
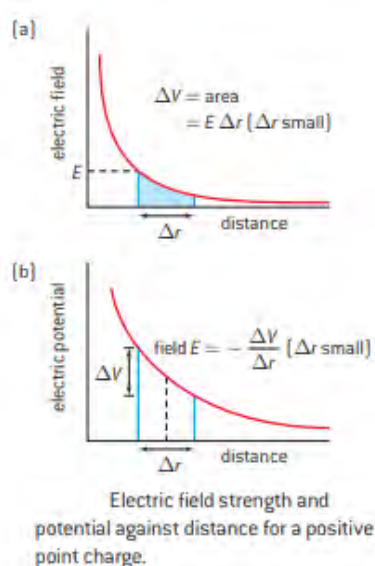


Fig : Potential V as a function of distance r from a single point charge Q when the charge is (a) positive, (b) negative.

Graphs of field strength and potential against distance for the field due to a single positive point charge are shown in figure below.



21.7 Similarities between Electric and Gravitation Fields

A comparison between Newton's law of gravitation and Coulomb's law shows that there are many similarities between the actions of the two types of force. There are also some important differences.

The main similarities are as follows:

- i) Both electric and gravitational forces are non-contact; forces are exerted over a distance without direct contact.
- ii) Both forces are of infinite range.
- iii) Both forces obey inverse square laws.

The main differences are as follows:

- i) Gravitational forces between masses are always attractive; electric forces can be attractive or repulsive.
- ii) An electric force is much stronger than the gravitational force.
- iii) It is possible to shield an electric force, but the gravitational force acts on all objects.
- iv) An electric force only acts on charged objects; the gravitational force acts on all objects.

Table below shows a useful summary of the similarities and differences.

Characteristic	Gravitation	Electric
Acts on	Mass (positive only)	Charge (positive or negative)
Force (N)	$F = G \frac{Mm}{r^2}$ attractive only infinite range	$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$ attractive or repulsive infinite range
relative strength	1	10^{36}
Field strength	$g = \frac{F}{m}$	$F = \frac{F}{Q}$
Units	N kg ⁻¹	N C ⁻¹ or V m ⁻¹
Radial field	$g = \frac{GM}{r^2}$	$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = \frac{kQ}{r^2}$
Potential difference	$\Delta V = \frac{\Delta W}{m}$	$\Delta V = \frac{\Delta W}{Q}$
units	J kg ⁻¹	J C ⁻¹
Potential gradient	$g = -\frac{\Delta V}{\Delta r}$	$E = -\frac{\Delta V}{\Delta r}$
Potential in radial fields	$V = -\frac{GM}{r}$ $V = 0 \text{ at } \infty$	$V = \frac{Q}{4\pi\epsilon_0 r}$ $V = 0 \text{ at } \infty$
Potential energy	$E_p = -\frac{GMm}{r}$	$E_p = \frac{Qq}{4\pi\epsilon_0 r}$

Exercise

1. The electric field between two charged plates is known to be 650 N C^{-1} . If the distance between the plates is 12 cm, what is the potential difference between the plates?

A. 54 V B. 78 V C. 4500 V D. 7800 V

2. The field in figure,



- A. could only be a gravitational field
B. could only be a magnetic field
C. could only be an electric field
D. could be any one of gravitational, magnetic or electric fields
3. Explain why the test charges we use when measuring electric fields must be small.
4. When determining an electric field, must we use a positive test charge, or would a negative one do as well? Explain.
5. Why can electric field lines never cross?
6. Given two point charges, Q and $2Q$, a distance apart, is there a point along the straight line that passes through them where their signs are (a) opposite, (b) the same? If yes, state roughly where this point will be.
7. If two points are at the same potential, does this mean that no net work is done in moving a test charge from one point to the other? Does this imply that no force is exerted? Explain.
8. If a negative charge is initially at rest in an electric field, will it move toward a region of higher potential or lower potential? What about a positive charge? How does the potential energy of the charge change in each instance? Explain.
9. State clearly the difference (a) between electric potential and electric field, (b) between electric potential and electric potential energy.
10. An electron is accelerated from rest by a potential difference of 0.10 V. How much greater would its final speed be if it is accelerated with four times as much as the voltage? Explain.
11. Is there a point along the line joining two equal positive charges where the electric field is zero? Where the electric potential is zero? Explain.
12. Can a particle ever move from a region of low electric potential to one of high potential and yet have its electric potential energy decrease? Explain.
13. If $V=0$ at a point in space, must $E=0$ there? If $E=0$ at some point, must $V=0$ at that point? Explain. Give examples for each.
14. Is the electric potential energy of two isolated unlike charges positive or negative? What about two like charges? What is the significance of the sign of the potential energy in each case?
Choose correct option with explanation.

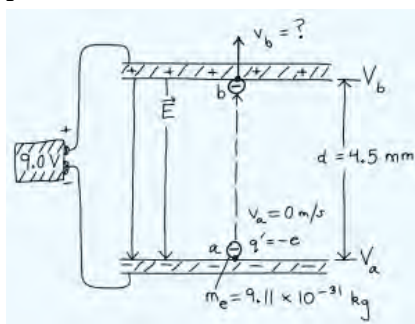
15. A $+0.2\mu\text{C}$ charge is in an electric field. What happens if that charge is replaced by a $+0.4\mu\text{C}$ charge?
- The electric potential doubles, but the electric potential energy stays the same.
 - The electric potential stays the same, but the electric potential energy doubles.
 - Both the electric potential and electric potential energy double.
 - Both the electric potential and electric potential energy stay the same.
16. Which of the following statements is valid?
- If the potential at a particular point is zero, the field at that point must be zero.
 - If the field at a particular point is zero, the potential at that point must be zero.
 - If the field throughout a particular region is constant, the potential throughout that region must be zero.
 - If the potential throughout a particular region is constant, the field throughout that region must be zero.
17. Two identical positive charges are placed near each other. At the point halfway between the two charges,
- the electric field is zero and the potential is positive.
 - the electric field is zero and the potential is zero.
 - the electric field is not zero and the potential is positive.
 - the electric field is not zero and the potential is zero.
 - None of these statements is true.
18. A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from near the positive plate and the electron from near the negative plate. Which feels the larger electric force?
- The proton.
 - The electron.
 - Neither—there is no force.
 - The magnitude of the force is the same for both and in the same direction.
 - The magnitude of the force is the same for both but in opposite directions.
19. How much work does the electric field do in moving a $-7.7\mu\text{C}$ charge from ground to a point whose potential is $+65\text{V}$ higher?
20. How strong is the electric field between two parallel plates 6.8 mm apart if the potential difference between them is 220 V ?
21. What potential difference is needed to stop an electron that has an initial velocity $6.0 \times 10^5\text{ m/s}$?
- Energy is conserved, so the change in potential energy is the opposite of the change in kinetic energy. The change in potential energy is related to the change in potential.
22. A point charge Q creates an electric potential of $+165\text{V}$ at a distance of 15 cm . What is Q ?
23. In the Bohr model of the hydrogen atom, an electron orbits a proton (the nucleus) in a circular orbit of radius $0.53 \times 10^{-10}\text{ m}$. (a) What is the electric potential at the

electron's orbit due to the proton? (b) What is the kinetic energy of the electron? (c) What is the total energy of the electron in its orbit? (d) What is the ionization energy— that is, the energy required to remove the electron from the atom, and take it to at rest?

24. How much voltage must be used to accelerate a proton (radius $1.2 \times 10^{-15} \text{ m}$) so that it has sufficient energy to just “touch” a silicon nucleus? A silicon nucleus has a charge of $+14e$ and its radius is about $2.6 \times 10^{-15} \text{ m}$. Assume the potential is that for point charges.
25. An electron is liberated from a hot filament, and attracted by an anode, of potential 1200 volts positive with respect to the filament? What is the speed of the electron when it strikes the anode? ($m_e = 9.1 \times 10^{-31} \text{ kg}$)

The energy which the electron gains from the field = $QV = eV$

26. A 9.0 V battery is connected across two large parallel plates that are separated by 4.5 mm of air, creating a potential difference of 9.0 V between the plates. (a) What is the electric field in the region between the plates? (b) An electron is released from rest at the negative plate. If the only force on the electron is the electric force exerted by the electric field of the plates, what is the speed of the electron as it reaches the positive plate? The mass of an electron is ($m_e = 9.11 \times 10^{-31} \text{ kg}$).



Chapter 22

Capacitors

Look at the pictures. Have you ever noticed these electrical components in any electrical/electronic devices? Why are they so important in electronics?

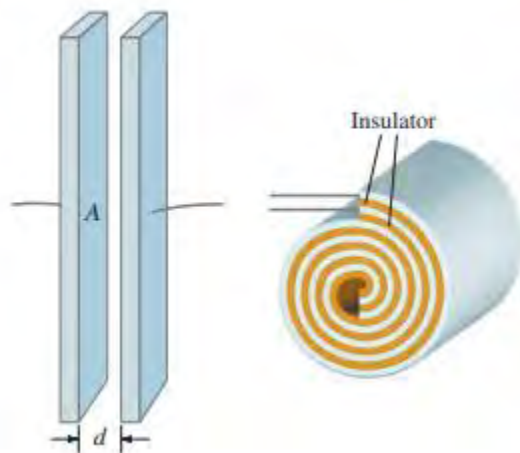


Fig. 22.1

22.1 Introduction

Capacitors are components of electrical circuits that temporarily store electric charge. They are electrical components in which charge is separated. They come in a variety of shapes and sizes. However, their basic construction is the same.

A capacitor consists of two metallic plates separated from each other by an insulator (often known as a dielectric) such as air, paper, ceramic, or mica. The circuit symbol for a capacitor shows two lines separated by a gap). You can easily make your own capacitor from two sheets of aluminium foil separated by newspaper.



Fig. 22.2

22.2 Storing charge

Figure 22.3 shows a capacitor connected to a cell of e.m.f. E .

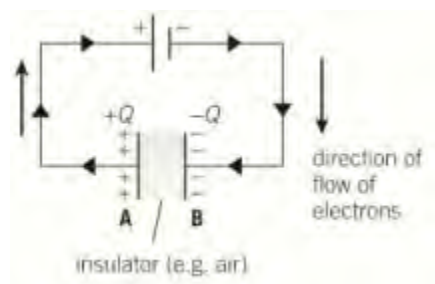


Fig. 22.3

When the capacitor is connected to the cell, electrons flow from the cell for a very short time. They cannot travel between the plates because of the insulation. The very brief current means electrons are removed from plate A of the capacitor, and at the same time electrons are deposited onto the other plate B. Plate A becomes deficient in electrons, that is, it acquires a net positive charge. Plate B gains electrons and hence acquires a negative charge. The current in the circuit must be the same at all points and charge must be conserved, so the two plates have an equal but opposite charge of magnitude Q . Therefore, there is a potential difference (p.d.) across the plates. The current in the circuit falls to zero when the p.d. across the plates is equal to the e.m.f. E of the cell. The capacitor is then fully charged. The net charge on the capacitor plates is zero.

The capacitor is therefore really a device that separates electrical charge into $-Q$ and $+Q$.

23.3 Capacitance

If a voltage is applied across a capacitor by connecting the capacitor to a battery with conducting wires as in figure 22.6, the two plates quickly become charged: one plate acquires a negative charge, the other an equal amount of positive charge. Each battery terminal and the capacitor plate connected to it are at the same potential. Hence the full battery voltage appears across the capacitor.

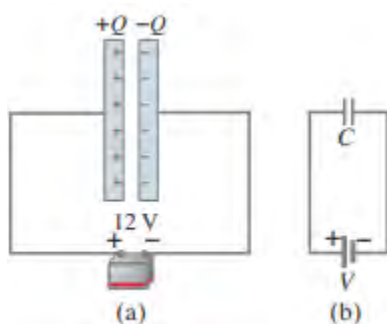


Fig. 22.6

For a given capacitor, it is found that the amount of charge Q acquired by each plate is proportional to the magnitude of the potential difference V between the plates:

$$Q = CV$$

The constant of proportionality, C , in Eq. is called the capacitance of the capacitor.

The ability of any object to store charge is called capacitance. Capacitance is given the symbol C , and the SI unit is the farad (F). Commercial capacitors are usually marked with their capacitance value, which indicates the amount of charge Q that the capacitor can store for a given p.d. V .

The capacitance of a capacitor is defined as the charge stored per unit p.d. across it. That is;

$$\text{capacitance, } C = \frac{\text{charge stored on one plate}}{\text{potential difference between plates}} = \frac{Q}{V}$$

For any capacitor, the greater the amount of positive and negative charge stored on the two plates, the greater the p.d. across them, so the charge on the capacitor is always proportional to the p.d.

The unit of capacitance is the coulomb per volt, or farad (symbol: F) and is named after Michael Faraday, the English physicist. In fundamental units the farad is equivalent to $\text{A}^2 \text{S}^4 \text{kg}^{-1} \text{m}^{-2}$. One farad is a very large unit of capacitance since 1 C of charge is a very large amount of charge and you will usually use capacitances measured in mill farads (mF), microfarads (μF), and Pico farads (pF).

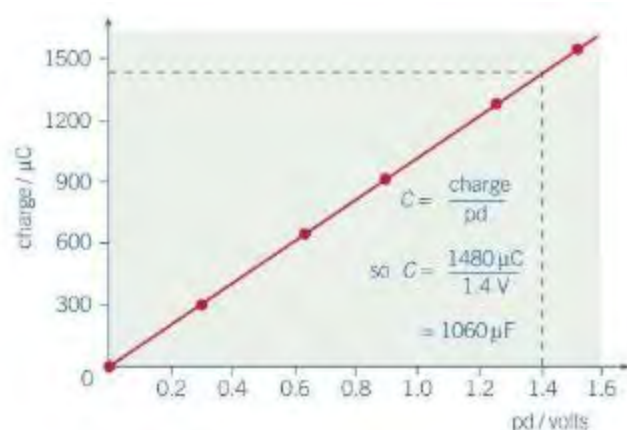


Fig. 22.7

A graph of charge stored, Q , against pd, V from measurements, is shown in figure 22.7. A straight line is passing through the origin. Therefore, the charge stored, Q , is proportional to the pd, V . In other words, the charge stored per volt is constant.

Workedout Example:

A pair of parallel plates store $2.5 \times 10^{-6} \text{ C}$ of charge at a potential difference of 15 V.

Calculate the capacitance of this capacitor.

Solution:

$$C = \frac{Q}{V} = \frac{2.5 \times 10^{-6} \text{ C}}{15 \text{ V}} = 1.7 \times 10^{-7} \text{ F} = 170 \text{ nF}$$

22.4 Parallel-Plate Capacitor

The arrangement of parallel plates in which the two plates are separated by an insulator is called a parallel plate capacitor. The insulator might be a vacuum, it could also be air or another gas or it could also be a non-conducting material such as a plastic.

The capacitance of a parallel-plate capacitor depends on the area of the plates, their distance apart and the ability of the insulating material between the plates to separate the charge, a property known as permittivity.

The permittivity of a material is the resistance of the material to an electric field passing through it. If the permittivity is high, then a larger charge can be stored on the plates for any given potential difference across them. The permittivity of capacitor insulating materials is always measured relative to the permittivity of free space (vacuum), ϵ_0 , using a relative permittivity, ϵ_r , sometimes called the dielectric constant of the material. The total absolute permittivity of an insulator is therefore given by the product $\epsilon_r \epsilon_0$. The permittivity of free space is $\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$.

Materials used for the dielectric include; paper, mica (a mineral that can be cut into thin layers), Teflon, plastics, ceramics, and the oxides of various metals such as aluminium.

From Gauss's law, the electric field between two parallel plates is given by,

$$E = \frac{Q}{\epsilon_0 A}$$

As,

$$E = \frac{V}{d}$$

$$\frac{V}{d} = \frac{Q}{\epsilon_0 A}$$

$$\therefore V = \left(\frac{Q}{\epsilon_0 A} \right) d$$

From the definition of capacitance,

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Q}{\epsilon_0 A} \right) d} = \epsilon_0 \frac{A}{d}$$

We see that C depends only on geometric factors, A and d, and not on Q or V.

Workedout Example

Two parallel plates both have an area of 0.015 m^2 , and are placed $2.0 \times 10^{-3} \text{ m}$ apart. Calculate the capacitance of this arrangement.

Solution:

$$C = \epsilon_0 \frac{A}{d} = 8.9 \times 10^{-12} \text{ Fm}^{-1} \times \frac{0.015 \text{ m}^2}{2.0 \times 10^{-3} \text{ m}} = 6.7 \times 10^{-11} \text{ F}$$

22.5 Dielectric Strength of Dielectrics

If the potential difference across a capacitor is increased, a maximum field strength will be reached after which the dielectric becomes a conductor. A spark will then jump between the plates. This maximum field is called the dielectric strength of the dielectric. For air this is about 1 kV mm^{-1} (the exact value depends on temperature, pressure, and how dry the air is); for Teflon the dielectric strength is 60 kVmm^{-1} . If during thunderstorms the electric field strength in air between charged clouds or between a charged cloud and the ground exceeds this critical value, then a lightning strike may occur.

If a material between the plates replaces the air (or vacuum), the new material can do two things for the capacitor designer: it can be used to separate the plates by a fixed amount, and also change the properties of the capacitor at the same time.

As we know materials have their own permittivity which is greater than that of a vacuum. To treat this mathematically, when a material of permittivity ϵ is present we replace ϵ_0 in the equations where it occurs with ϵ alone. The values for ϵ are different for different materials.

A useful thing happens when a dielectric material is inserted between capacitor plates to fill all the space between them. A dielectric is an electrical insulator that is polarized when placed in an electric field. The origin of the polarization is in the molecules of the dielectric substance. Each molecule is slightly more positive at one end than the other. This means that when a molecule is in an electric field, it responds either by moving slightly or by rotating so that the more positive end of the molecule moves in the direction of the electric field. If the molecules are in a solid and the solid is packed between the plates of a capacitor then this reduces the electric field strength between the plates.

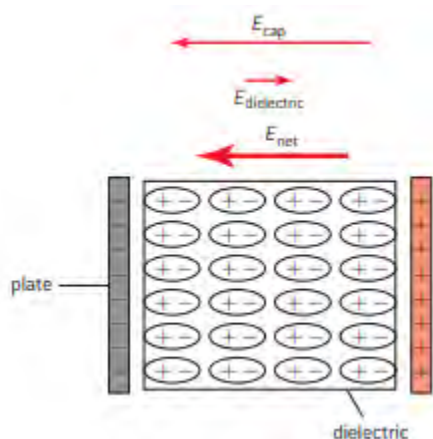


Fig. 22.8

When the dielectric is in place (Fig. 22.8), the molecules inside it respond to the field produced by the capacitor. Notice the field directions carefully. In the diagram, the original field E_{cap} of the capacitor is from right to left, but the dielectric field $E_{dielectric}$ (indicated by the charges at the surface of the dielectric) is from left to right. Therefore, the net field between the plates is equal to the capacitor field minus the dielectric field. The overall field E_{net} in the dielectric is reduced, and because $E_{net} = E_{cap} - E_{dielectric} = \frac{V}{d}$ (V is the potential difference) V also decreases as d is fixed. The insertion of the dielectric reduces the potential difference between the plates because some of the stored energy of the capacitor has been used to align the dielectric molecules. The overall charge stored is unchanged, so, $\frac{Q}{V}$ is increased, and hence so is the capacitance of the capacitor. Dielectrics increase the capacitance of a capacitor. Another way to describe the action of the dielectric is by saying that the presence of the dielectric raises the potential of the negative plate and lowers the potential of the positive plate; this reduces the potential difference between the plates.

This explanation of the dielectric effect is a simplified one. There are other reasons for the increase in capacitance, but in many cases the explanation can be given in terms of potential change as here.

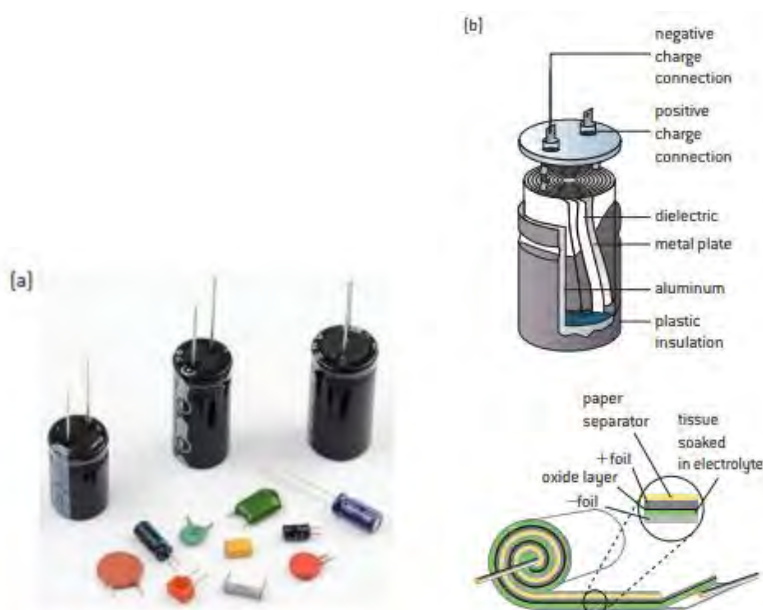


Fig. 22.9

Figure 22.9 (a) shows some typical designs used for practical capacitors. A very common type of capacitor is the electrolytic capacitor shown as a cutaway in figure 22.9 (b). The designer here uses the dielectric advantageously by using it to space the two metal foils apart. The capacitor layers are then rolled up together (rather like rolling up three carpets initially on top of each other). This type of capacitor must be connected into a circuit correctly. The dielectric material is a chemical (the electrolyte in the diagram) that is a good dielectric when the electric field direction is correct. In this design the layers are very thin (good because d is small) and the plate area is large (even better) giving some of the largest values of capacitance possible in a given volume.

Materials used for the dielectric include; paper, mica (a mineral that can be cut into thin layers), Teflon, plastics, ceramics, and the oxides of various metals such as aluminium.

Workedout Example

In a laboratory experiment, two parallel plates, each of area 100 cm^2 , are separated by 1.5 cm . Calculate the capacitance of the arrangement if the gap between the plates is filled with:

- (a) air
- (b) polystyrene.

Solution:

- (a) Convert the cm units to m: area = $100 \times 10^{-4} = 10^{-2} \text{ m}^2$; separation = 0.015 m

$$C = \epsilon_0 \frac{A}{d} = 8.9 \times 10^{-12} \text{ Fm}^{-1} \times \frac{10^{-2} \text{ m}^2}{0.015 \text{ m}} = 5.9 \times 10^{-12} \text{ F}$$

- (b) Polystyrene has a dielectric constant of 3, which means that

$$\frac{C_2}{C_1} = \frac{\epsilon_0 \frac{A}{d}}{\epsilon \frac{A}{d}} = \frac{\epsilon_0}{\epsilon} \therefore C_2 = C_1 \times \frac{\epsilon_0}{\epsilon} \text{ (dielectric constant)}$$

$$C = 5.9 \times 10^{-12} \text{ F} \times 3 = 1.8 \times 10^{-11} \text{ F}$$

22.6 Combination of Capacitors

Super capacitors are compact specialist capacitors with capacitance values in the thousands of farads. They are used as alternatives to battery packs, memory backup devices, and emergency lighting. Unlike rechargeable batteries, which degrade over time, super capacitors can be charged over and over again. We can connect the capacitors available in order to get a desired value. We need an enormous number of capacitors if we want thousands of farads.

2.6.1 Capacitors in Parallel

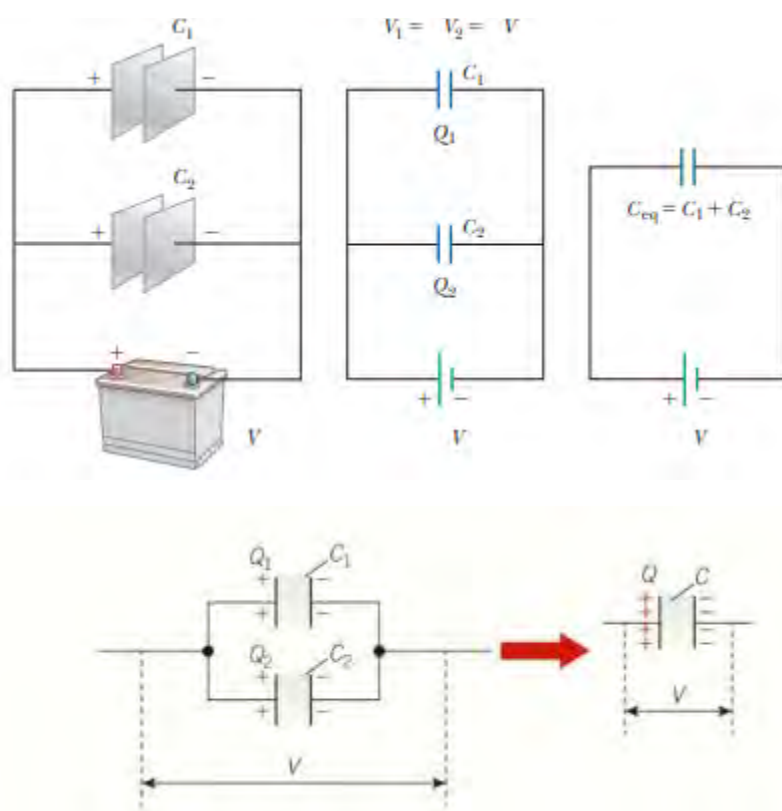


Fig. 22.10

Figure 22.10 shows two capacitors of capacitances C_1 and C_2 connected in parallel. The combined capacitance is greater than their individual capacitances, so the combination will store more charge for a given potential difference (p.d.).

For two or more capacitors in parallel:

The p.d. V across each capacitor is the same.

Electrical charge is conserved. Therefore, the total charge stored Q is equal to the sum of the individual charges stored by the capacitors, $Q = Q_1 + Q_2 + \dots$

The potential difference between each capacitor is the same. If the total capacitance is C , then,

$$CV = C_1V + C_2V$$

$$\therefore C = C_1 + C_2$$

The total capacitance C is the sum of the individual capacitances of the capacitors.

2.6.2 Capacitors in series

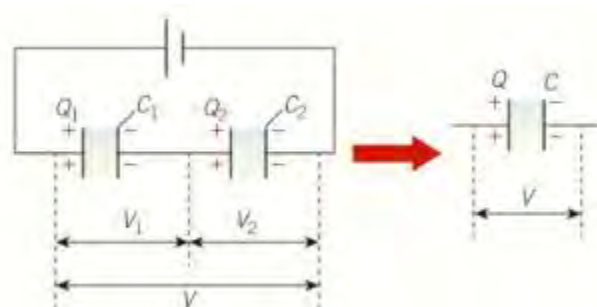
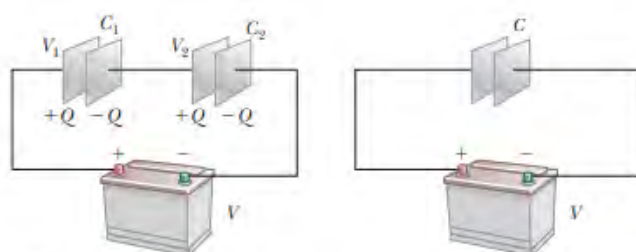


Fig. 22.11

Figure 22.11 shows two capacitors of capacitances C_1 and C_2 connected in series. Their combined capacitance is less than their individual capacitances, so this combination will store less charge for a given p.d.

All the capacitors in the series store the same charge. This is even true when they have different capacitances. The cell is connected to the left-hand plate of the capacitor of capacitance C_1 and to the right-hand plate of the capacitor of capacitance C_2 . These plates acquire equal and opposite charges as electrons flow from and to these plates. The middle two plates are not connected to the cell because of the presence of the dielectric layers, but transfer of electrons between these plates ensures that they too acquire charge Q of the same magnitude. The overall charge of each capacitor is zero, but the magnitude of the charge on each plate is Q .

For two or more capacitors in series:

The total p.d. V across the combination is the sum of the individual p.d.s across the capacitors,

$$V = V_1 + V_2$$

The charge Q stored by each capacitor is the same, i.e., $Q = Q_1 = Q_2$

If the combined capacitance is C , then,

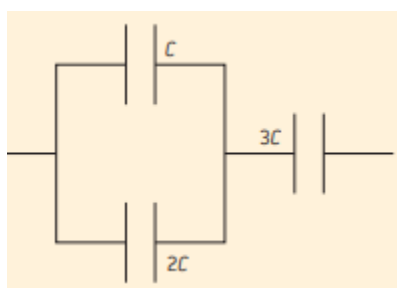
$$\begin{aligned} \frac{Q}{C} &= \frac{Q}{C_1} + \frac{Q}{C_2} \\ \therefore \frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} \end{aligned}$$

Mixed Capacitor Combinations

1. Combine capacitors that are in series or in parallel, following the derived formulas.
2. Redraw the circuit after every combination.
3. Repeat the first two steps until there is only a single equivalent capacitor.
4. Find the charge on the single equivalent capacitor, using $C = \frac{Q}{V}$
5. Work backwards through the diagrams to the original one, finding the charge and voltage drop across each capacitor along the way. To do this, use the following collection of facts:
 - A. The capacitor equation: $C = \frac{Q}{V}$
 - B. Capacitors in parallel: $C = C_1 + C_2$
 - C. Capacitors in parallel all have the same voltage difference, V , as does their equivalent capacitor.
 - D. Capacitors in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$
 - E. Capacitors in series all have the same charge, Q , as does their equivalent capacitor.

Workedout Example

Calculate the capacitance of the network below.



Solution:

The capacitors in parallel have capacitance of $C + 2C = 3C$

Combining the parallel capacitors and $3C$ gives:

$$\frac{1}{C_{total}} = \frac{1}{3C} + \frac{1}{3C} = \frac{2}{3C} \therefore C_{total} = 1.5C$$

22.7 Energy Stored in a Capacitor

If we charge a capacitor of capacitance 1000 pF by connecting it to the terminals of a 3.0V battery and then the capacitor is removed from the circuit, and is connected it to a small filament lamp, we will see a brief flash of light. This is the evidence that the capacitor stores energy. Mobile phones and camera flashes rely on capacitors to store and release the energy they need. The amount of energy stored in a capacitor depends on the value of the capacitance and the initial potential difference (p.d.) across it.

Figure 22.12 shows an electron moving towards the negative plate of a capacitor that is being charged. This electron will experience a repulsive electrostatic force from all the electrons already on the plate. External work has to be done to push this electron onto the negative plate. Similarly, work is done to cause an electron to leave the positive plate of the capacitor. The external work is provided by the battery or power supply connected to the capacitor. In short, the energy stored in a capacitor comes from the energy of the battery or power supply.

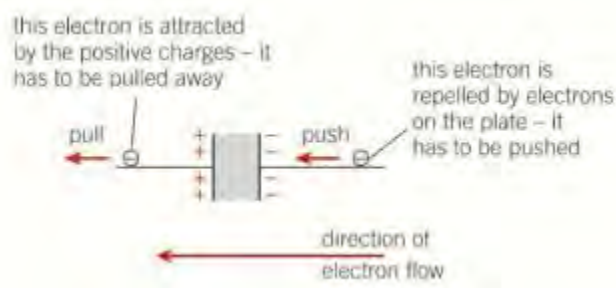


Fig. 22.12

When a capacitor is charged, energy is stored in it, because electrons are forced onto one of its plates and taken off the other plate. The energy is stored in the capacitor as electric potential energy.

How much energy is stored in a charged capacitor? The charge is forced onto the plates by the battery.

The charge on a capacitor is directly proportional to the p.d. across it (i.e. $Q = CV$). A graph of potential difference against the charge for a capacitor is shown in figure 22.13.

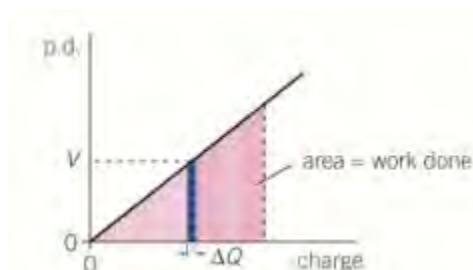


Fig. 22.13

From the definition of the potential difference, we know that $V = \frac{W}{Q}$

$$\text{Work done} = p.d \times \text{charge}$$

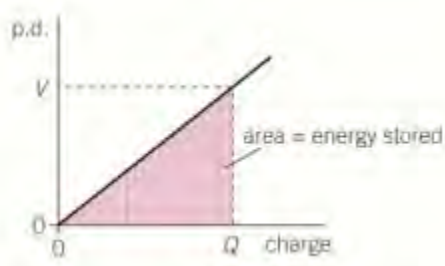
The small amount of work done, ΔW , to increase the charge stored in the capacitor by a small amount ΔQ is given by the equation,

$$\Delta W = V \times \Delta Q$$

where the p.d. V does not change significantly. If you look at the graph in figure 22.13, this is the area of the thin rectangular strip shown shaded under the graph. Hence ΔW is equal to the area of this strip. If you add all the similar strips together, then the area under the graph is the total work done on the charges for a charging capacitor.

The area under a p.d.–charge graph = work done

The work done on the charges is the same as the energy stored in the capacitor. Since the area under a p.d.—charge graph is equal to the work done, we can derive an equation for the energy stored in a capacitor, W , from that area.



$W = \text{area under graph} = \text{area of shaded triangle}$

$$W = \frac{1}{2} \text{base} \times \text{height} = \frac{1}{2} QV$$

where V is the p.d. when the charge is Q .

The charge stored by the capacitor is also given by the equation $Q = CV$, where C is the capacitance of the capacitor. Substituting this equation into $W = \frac{1}{2} QV$ gives us another very useful equation for the stored energy.

$$W = \frac{1}{2} QV = \frac{1}{2} V \times (CV) = \frac{1}{2} CV^2$$

For a given capacitor, W is directly proportional to V^2 , so doubling the p.d. quadruples the energy stored.

If we substitute $V = \frac{Q}{C}$ into $W = \frac{1}{2} QV$, we get a third equation for the stored energy.

$$W = \frac{1}{2} Q^2 C$$

So, we now have three equations for stored energy —

$$W = \frac{1}{2} QV \qquad W = \frac{1}{2} CV^2 \qquad W = \frac{1}{2} Q^2 C$$

Workedout Example

Calculate the energy that can be stored on a $5000 \mu\text{F}$ capacitor that is charged to a potential difference of 25 V .

Solution:

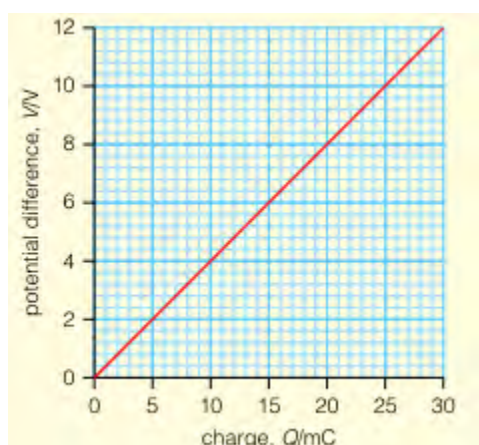
$$\text{Energy stored} = \frac{1}{2} CV^2 = \frac{1}{2} 5 \times 10^{-3} \text{ F} \times (25 \text{ V})^2 = 1.6 \text{ J}$$

Exercise

- What is the charge stored on a $220\mu\text{F}$ capacitor with a 6V p.d. across it?
A. 36.6mC B. 1.32mC C. 2.70mC D. 3.96mC
- Two capacitors, P and Q, are both charged by the same 12V battery. The capacitance of P is $4200\mu\text{F}$, and the capacitance of Q is $420\mu\text{F}$. Which row of the table gives the correct ratios of energy and charge stored by each capacitor?

	energy stored in P energy stored in Q	charge stored in P charge stored in Q
A	1	1
B	10	1
C	10	10
D	1	10

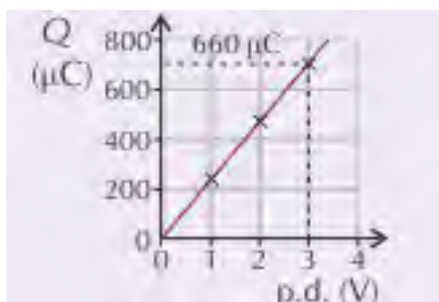
- The figure below shows how the p.d. across a capacitor varies with the charge stored on it.



The table shows possible values of the capacitance and the energy stored by the capacitor when the p.d. across it is 12V . Which row gives the correct values?

	Capacitance, C/mF	Energy stored, E/mJ
A	2.5	90
B	2.5	180
C	0.4	90
D	0.4	180

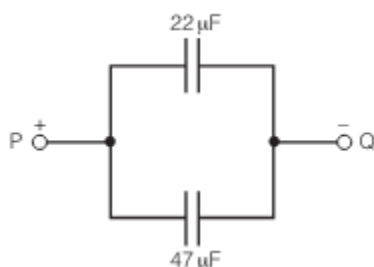
- The potential difference of a test circuit was measured as a capacitor was charged. The graph below was plotted from the recorded data.



(a) Explain what is meant by the term ‘capacitance’.

(b) Calculate the capacitance of the capacitor.

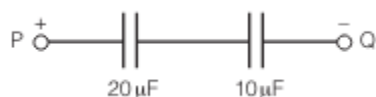
5. In the figure below, a potential difference of 12 V is applied between P and Q.



(a) Calculate the charges on each capacitor.

(b) What single capacitor placed between P and Q would be equivalent to the two shown? Comment on your answer.

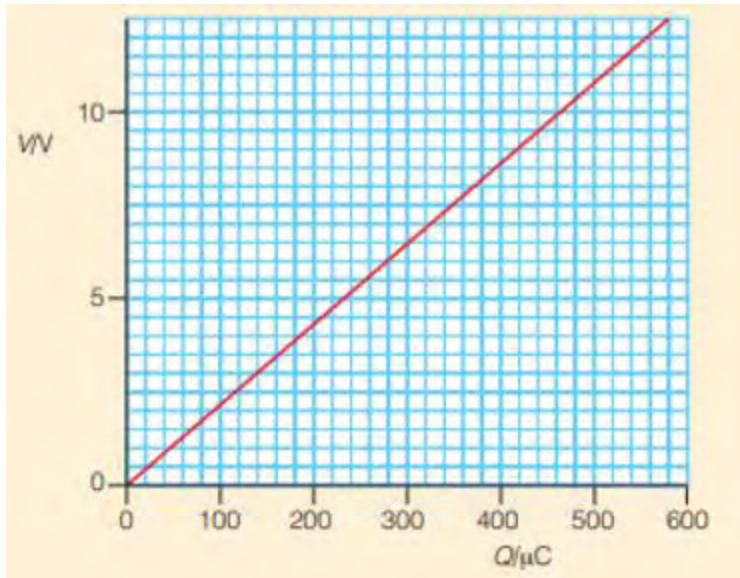
6. In the figure below, a potential difference of 12 V is applied between P and Q.



(a) What is the p.d. across each capacitor? Explain your answer.

(b) What single capacitor placed between P and Q would be equivalent to the two shown? Comment on your answer.

7. The figure below shows how the potential difference across a capacitor changes as it is charged.



- (a) What is the capacitance of the capacitor?
 - (b) Calculate the energy stored in the capacitor when it has a charge of $100\mu\text{C}$, $200\mu\text{C}$, $500\mu\text{C}$.
 - (c) Sketch a graph of energy stored against charge.
8. A capacitor with a capacitance of 137 pF is charged until it stores a charge of 2.47 nC , and is then disconnected. It has a dielectric with a relative permittivity of 3.1 . The dielectric is removed, so there is now a vacuum between the plates, but nothing else is changed. Calculate the change in potential difference across the capacitor.
 9. An $8.0\mu\text{F}$ capacitor is fully charged from a 12 V supply. Calculate the energy stored by the capacitor.

Chapter 23

D.C. Circuits



Georg Simon Ohm (1787–1854)

His father was a locksmith, and his mother was the daughter of a tailor. Neither had formal education. Ohm's self-educated father gave his sons an excellent home education, and his brother Martin became a well-known mathematician. A high school teacher in Cologne and later a professor at Munich, Ohm formulated the concept of resistance and developed a relationship between the potential difference and current through a conductor popularly called Ohm's law.

23.1 Electric Current

Electric current is defined as the rate of flow of charge – that is, the quantity of charge flowing past a given point in one second. Mathematically, we can express this as:

$$I = \frac{\Delta Q}{\Delta t}$$

where I is the current and ΔQ is the amount of charge flowing in time Δt .

Current is measured in amperes (symbol A). The ampere is the base unit of electric current, and is defined in terms of the force between two parallel wires each carrying a current of 1 A. In laboratory practice, an ampere is quite a large current, so we often use mA (milliampere = 10^{-3} A) and μ A (microampere = 10^{-6} A).

We can arrange,

$$I = \frac{\Delta Q}{\Delta t}$$

to give:

$$\Delta Q = I \Delta t$$

This tells us that the amount of charge flowing in a certain time is given by multiplying the current by the time. If 1 A current flows for 1 s, the quantity of charge flowing is said to be 1

coulomb (symbol C). The coulomb is a relatively large amount of charge, so we often use μC or even nC ('nano' = 10^{-9}); indeed, the charge on a single electron is only $1.6 \times 10^{-19} \text{ C}$.

Workedout Example:

1. How much charge flows through the filament of an electric lamp in 1 hour when the current in it is 250 mA?
2. The electron charge is $1.6 \times 10^{-19} \text{ C}$. How many electrons flow through the filament during this time?

Solution

$$1. \quad \Delta Q = I \Delta t = 250 \times 10^{-3} \text{ A} \times (60 \times 60) \text{ s} = 900 \text{ C}$$

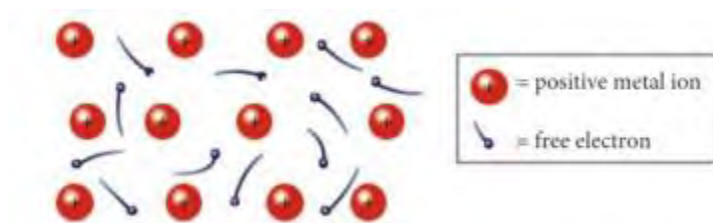
$$2. \quad \text{number of electrons} = \frac{\text{total charge}}{\text{charge of an electron}} = \frac{900 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 5.6 \times 10^{21} \text{ electrons}$$

23.2 Conventional Current - Drift Velocity

People often think that conventional current means the normal, or usual current, but the word 'convention' refers here to an agreement. The scientists who discovered and worked on electricity agreed to define the direction of current as from the positive terminal of a power supply to the negative terminal before the electron was discovered in 1897. Unfortunately, this means that conventional current is in the opposite direction to electron flow. However, not all currents are due to a flow of electrons. In a solution, or an ionised gas, positive ions flow in the direction of conventional current.

Imagine a battery connected in series with a switch and a lamp. When we close the switch, we complete the circuit and the lamp lights up instantaneously, so far as our senses can tell us. How fast do you think electrons flow through the wires?

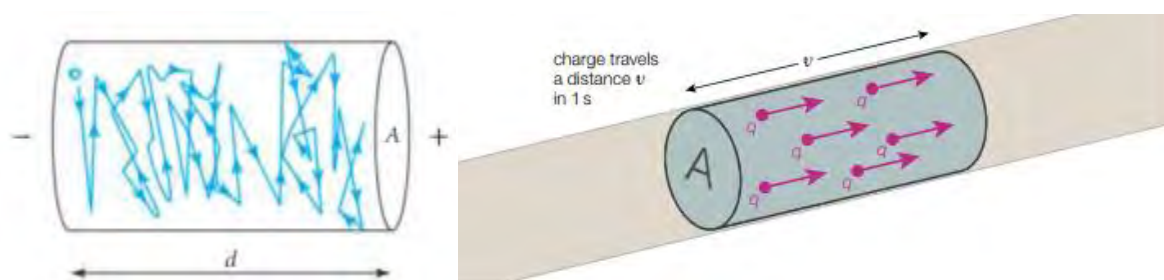
The outer electrons in metal atoms are not attracted as strongly to the nucleus as those in atoms of non-metals. These electrons are free to move between all the atoms (or rather ions, because they have lost their outermost electrons) in the metal. The metal ions sit like islands in a 'sea' of free electrons, also called delocalised or conduction electrons.



The free electrons in a metal move around randomly due to their thermal energy with no overall direction to their movement. However, if a cell or battery is connected across a metal wire, the free electrons will also be pushed away from its negative terminal and pulled towards its positive terminal. The average speed at which they move in this direction is called their mean drift velocity. Mean drift velocity is small compared to the electrons' thermal velocities, but as the thermal velocities are random, they produce no overall motion of the electrons.

In a metallic conductor, the charge carriers are loosely bound to outer electrons – so-called ‘free’ or ‘delocalised’ electrons. On average, there is approximately one ‘free’ electron per atom. These ‘free’ electrons move randomly to and fro within the crystal lattice of the metal at speeds approaching one-thousandth the speed of light and are also mainly responsible for metals being good conductors of heat. When a potential difference is applied to a circuit, an electric field is created and exerts a force on the ‘free’ electrons, which causes them to ‘drift’ in the direction of the force.

Consider a wire of length d and cross-sectional area A . This wire contains n free electrons per unit volume, where n is called the number density of charge carriers (here, free electrons), or the number of charge carriers per cubic metre.



The total number of free electrons in this volume, V , of the wire is simply $n \times V$, or $n \times A \times d$. The size of the charge on an electron is e , so the total amount of charge that is free to move is given by, $Q = e \times n \times A \times d$.

Now

$$I = \frac{Q}{t} = \frac{e \times n \times A \times d}{t}$$

But the mean drift velocity of an electron, v , is equal to the distance it travels along the wire divided by the time taken, i.e.,

$$v = \frac{d}{t}$$

So,

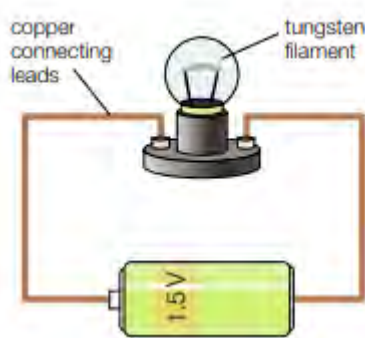
$$I = enAv$$

Note that n also applies to other charge carriers such as ions in a solution or a plasma (ionised gas). In this case, the charge on each charge carrier would be q , as an ion can have several times the electronic charge. This gives us:

$$I = nAvq$$

Workedout Example

Consider the circuit in Figure.



The diameter of the tungsten filament is 0.025 mm and the diameter of the copper connecting leads is 0.72 mm.

Calculate the drift velocity of the electrons in:

- (a) the filament
- (b) the connecting leads

when the circuit current is 160 mA.

You may take the number of charge carriers per cubic metre to be $4.0 \times 10^{28} \text{ m}^{-3}$ for tungsten and $8.0 \times 10^{28} \text{ m}^{-3}$ for copper.

Solution:

- (a) For the filament: $I = nAvq$

Rearranging: $v = \frac{I}{nAq}$

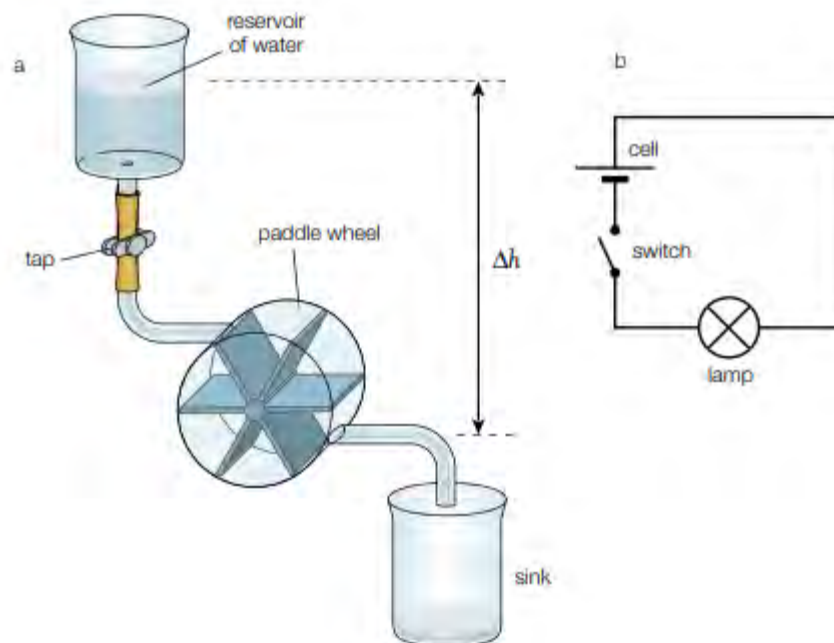
$$= \frac{160 \times 10^{-3} \text{ A}}{4.0 \times 10^{28} \text{ m}^{-3} \times \pi \times (0.5 \times 0.025 \times 10^{-3} \text{ m})^2 \times 1.60 \times 10^{-19} \text{ C}}$$
$$= 0.051 \text{ ms}^{-1} = 51 \text{ mm s}^{-1}$$

- (b) For the leads:

$$v = \frac{I}{nAq}$$
$$= \frac{160 \times 10^{-3} \text{ A}}{8.0 \times 10^{28} \text{ m}^{-3} \times \pi \times (0.5 \times 0.72 \times 10^{-3} \text{ m})^2 \times 1.60 \times 10^{-19} \text{ C}}$$
$$= 3.1 \times 10^{-5} \text{ ms}^{-1} = 0.031 \text{ mm s}^{-1}$$

23.3 Potential difference

To understand how electrons can be made to flow around a circuit and thereby create an electric current, a comparison with the flow of water in a pipe is often helpful. Such an analogy is shown in the figure.



In figure a, there is a pressure difference between the water level in the reservoir and that in the horizontal pipe. When the tap is opened, this pressure difference causes the water to flow through the pipe and turn the paddle wheel. The gravitational potential energy stored in the reservoir is converted into kinetic energy of rotation of the paddle wheel.

In figure b, a chemical reaction in the cell creates an electrical potential difference (symbol V), which exerts a force on the electrons in the wires. When the switch is closed, this force causes the electrons to move around the circuit and through the lamp. Here, the chemical potential energy stored in the cell is converted into vibrational kinetic energy of the atoms in the filament of the lamp, thereby causing its temperature to rise.

In an electric circuit, electrical energy is converted into other forms of energy – for example in the circuit of figure b, the lamp converts electrical energy into thermal energy and light. Conversely, an electric motor converts electrical energy into mechanical energy. In our analogy, if the paddle wheel in figure a were to be connected to an electric motor, the water could be pumped up again from the sink to the reservoir.

These energy conversions in electrical circuits form the basis of the definition of potential difference (often abbreviated to p.d.). The potential difference between two points in a circuit is the amount of electrical energy converted into other forms of energy (thermal, light, mechanical, etc) when a unit charge passes from one point to the other. This can be expressed as:

$$\text{potential difference} = \frac{\text{electrical energy converted into other forms}}{\text{charge passing}}$$

or, as energy is the result of doing work, W :

$$\text{potential difference} = \frac{\text{work done}}{\text{charge passing}}$$

From $V = \frac{W}{Q}$ it follows that the unit of p.d. is joules per coulomb (J C^{-1}).

This unit is called the volt. Thus, a p.d. of 1 V exists between two points in a circuit if 1 J of energy is converted when 1 C of charge passes between the points. Because the potential difference is measured in volts, the potential difference is often called the voltage between two points in a circuit.

Workedout Example

The element of an electric kettle that takes a current of 12.5 A produces 540 kJ of thermal energy in three minutes.

1. How much charge passes through the element in these three minutes?
2. What is the potential difference across the ends of the element?

Solution:

$$1. \quad \Delta Q = I \Delta t = 12.5 \text{ A} \times (3.0 \times 60) \text{ s} = 2250 \text{ C}$$

$$2. \quad V = \frac{W}{Q} = \frac{540 \times 10^3 \text{ J}}{2250 \text{ C}} = 240 \text{ V}$$

23.4 Potential Difference and Electromotive Force

Electric current only measures the rate at which the charged particles (usually electrons) flow around a circuit. It tells us nothing about the electrical energy involved with circuits. The quantity used to describe the electrical energy in circuits is potential difference (p.d.), symbol V , measured in volts, V.

A potential difference across an electrical component is measured by putting a voltmeter across the component, in parallel with it.

Electrical energy in circuits is defined in terms of the electrical work done by the electric charge flowing through the circuit. Potential difference (p.d.) is defined as the electrical work done per unit (coulomb) of charge flowing through components such as bulbs, motors, resistors, etc. This electrical energy is transferred into heat, light and other more useful forms of energy by the components.

$$\text{potential difference, } V \text{ (V)} = \frac{\text{electrical work done by the charge, } W \text{ (J)}}{\text{charge flow, } Q \text{ (C)}}$$

$$V = \frac{W}{Q}$$

$$1\text{V} = 1\text{J C}^{-1}$$

The potential difference, however, cannot be used to describe the energy changes involved with power supplies such as cells, generators and mains power supply units. These devices transfer other forms of energy, such as chemical energy into electrical energy. To make a distinction between these different energy transfers, we define another quantity, electromotive force (emf), symbol E . Both emf and p.d. are measured in volts, symbol V , using a voltmeter.

Electromotive force (emf) is defined as the electrical work done per unit (coulomb) of charge as it flows through a source of electrical energy such as a cell, generator or power supply unit (psu). The sources of electrical energy transfer other forms of energy such as kinetic or light energy into electrical energy.

$$\text{electromotive force, } E \text{ (V)} = \frac{\text{electrical work done on the charge, } W \text{ (J)}}{\text{charge flow, } Q \text{ (C)}}$$

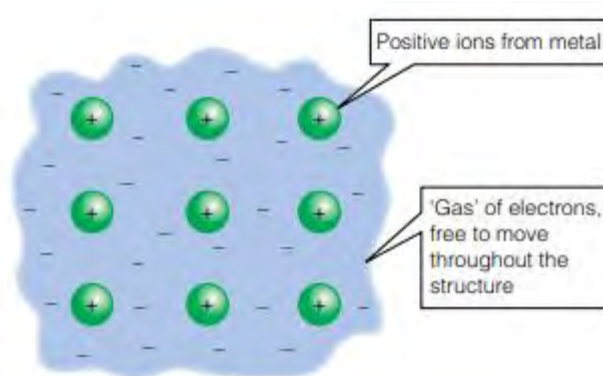
$$E = \frac{W}{Q}$$

The law of conservation of energy can now be written in terms of emf and p.d. In a series circuit, where the components are connected one after another in a complete loop, the total electrical energy per coulomb transferring into the circuit (the sum of the emfs in the circuit) must equal the energy per coulomb transferring into other forms of energy (the sum of the p.ds).

23.5 Resistance

When current flows through the material of a circuit, such as the metal of the connecting wires, the material of the circuit gets in the way of the flow of the charge. On a microscopic level, as the electrons flow through the metal they collide with the vibrating positive ion cores of the metal structure. The collisions between the electrons and the positive ion cores transfer electrical energy from the electrons to the structure of the metal, causing the metal ion cores to vibrate more, thus heating up the wire.

The opposition of a component to the flow of electric current through it is called resistance, the symbol is R , and the unit is ohms, Ω .



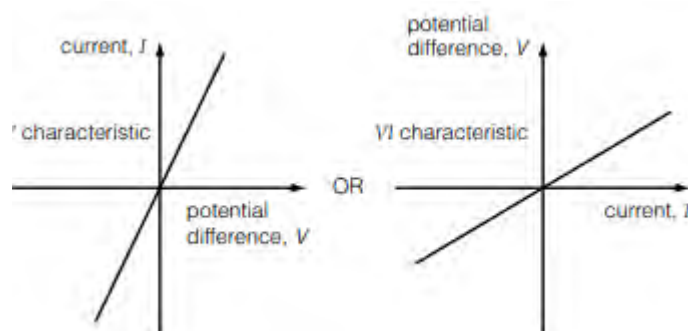
The model of a metal shown above explains what happens to the resistance of the metal when it is heated. The resistance increases as the temperature increases. While vibrating the positively charged ion cores move around more as the temperature increases, thus getting in the way of electron flow. This opposes the flow of the gas of electrons moving through the structure. Components with very high resistance let very little current through them, and are considered to be electrical insulators. Some materials, at very low temperatures, have zero resistance. These materials are called superconductors.

23.6 Current/Potential Difference Characteristics and Ohm's Law

Most of the properties of an electrical component can be determined by plotting the current – potential difference graph of the component. This is called an electrical characteristic.

Electrical characteristics are graphs that show how the potential difference and current vary when the component is connected in both forward and reverse bias (when the current passes one way and then in the other direction). The most straightforward electrical characteristic is that of a fixed resistor.

You can see that the graph can be drawn with each variable on either axis.



23.6.1 Ohm's Law

In 1827, the German physicist Georg Ohm performed potential difference–current experiments on metal wires at constant temperature. Ohm discovered that the current, I , flowing through the wire was proportional to the potential difference, V , across the wire, provided that the temperature (and other physical variables) remained constant. If this is shown on an electrical characteristic graph, then the graph will be linear. The relationship between the current and the potential difference became known as Ohm's law, which can be written mathematically by:

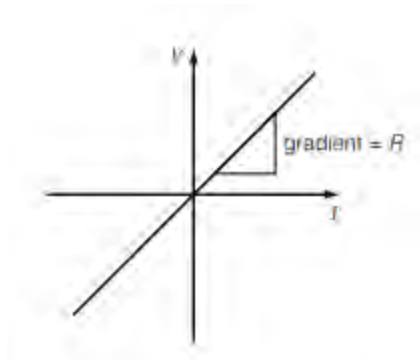
$$\text{current } I \propto \text{potential difference, } V$$

Ohm's law applies to some conductors under some circumstances. It is generally a special case applied to metal wires at constant temperature. But, using the concept of resistance, a convenient, familiar mathematical equation can be written that uses the relationship between I and V to define the unit of resistance, the ohm (Ω):

$$R = \frac{V}{I}$$

In other words, if a potential difference of 1V produces a current of 1A flowing through a component, then its resistance must be 1 Ω . This defines resistance in ohms. This equation applies in all cases, and also makes it clear that for a fixed p.d., V , a small measured current, I , implies that the component has a large resistance, R , and vice versa. Similarly, for a fixed current, I , we need a large p.d., V , to drive the current through a large resistance, R .

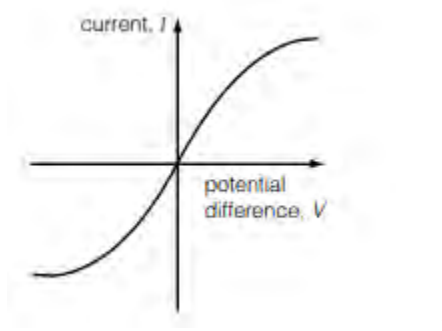
One use of the electrical characteristic graph is that if the current is plotted on the x-axis, for components following Ohm's law then the gradient of the line is the same value as the resistance of the component. As fixed resistors and metal wires at constant temperature have a fixed resistance, then the electrical characteristic graph will be a straight line.



In this type of graph, high-resistance components have large, steep gradients, and low-resistance components have small, shallow gradients. Remember though, the electrical characteristics can be drawn with either variable on either axis. Fixed resistors and metal wires at constant temperatures obey Ohm's law across their current range and their electrical characteristics are linear. Components like this are said to be ohmic conductors, i.e., they obey Ohm's law.

23.6.2 Other Electrical Characteristics

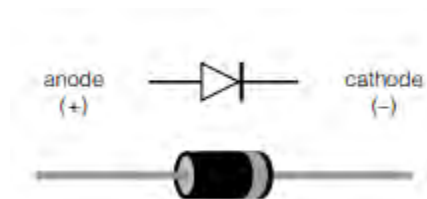
A standard tungsten filament lamp transfers electrical energy into light and heat as the current flows through it. As the current increases, the frequency of electron collisions with the positive ion cores of the tungsten lattice increases, transferring more kinetic energy. The positive ion cores vibrate with greater amplitude and so the resistance increases. A higher current leads to a higher temperature, which in turn leads to a higher resistance. The electrical characteristic of a filament lamp is shown in the figure below.



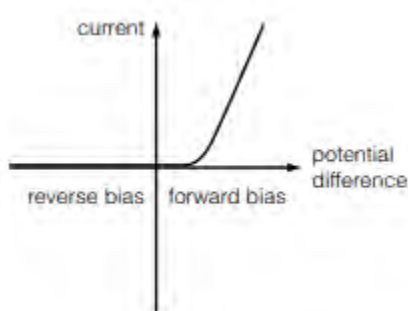
The electrical characteristic shows the ratio $\frac{V}{I}$ increasing and therefore the resistance increasing. Components like this, whose electrical characteristics are non-linear, are said to be non-ohmic conductors.

Components such as fixed resistors and filament lamps have the same characteristics independent of the direction of the current flowing through them. Their V–I graphs produce the same shapes in forward and reverse bias. Components such as semiconductor diodes do not behave in this way. Diodes are generally found in electronic circuits where they act as one-way gates, preventing the current from flowing back through the circuit. They are particularly useful in mains power supplies where they can be used in circuits to convert alternating current (ac) into direct current (dc).

Diodes only conduct in forward bias (in the direction of the arrow on the symbol); their circuit symbol shows this direction using the arrow, and the component itself normally has a different coloured ring at the forward bias end.



Diodes do not conduct in reverse bias; this means that the resistance of the diode in reverse bias is infinite. Diodes have very low resistance in forward bias. The electrical characteristics of diodes are usually drawn in current (y-axis)–potential difference (x-axis) format:



23.6.3 Thermistors

Thermistors are a type of resistor that change their resistance with temperature. Thermistors have the following circuit symbol:



Most thermistors are negative temperature coefficient (ntc) components. This means that their resistance decreases with increasing temperature. The vast majority of ntc thermistors also have a non-linear response to changing temperature; in other words, a graph of resistance against temperature is a curve.

Thermistors are widely used in circuits to sense temperature changes, and then to control devices; the change in the resistance of the thermistor affects a current, which can be used to switch devices on or off. They can be found in thermal cut-out circuits to prevent devices from overheating (such as a hairdryer), central heating circuits, digital thermometers and engine-management circuits.

23.6.4 Strain Gauge

A strain gauge is made by sealing a length of very fine wire in a small rectangle of thin plastic, as shown in the figure below.

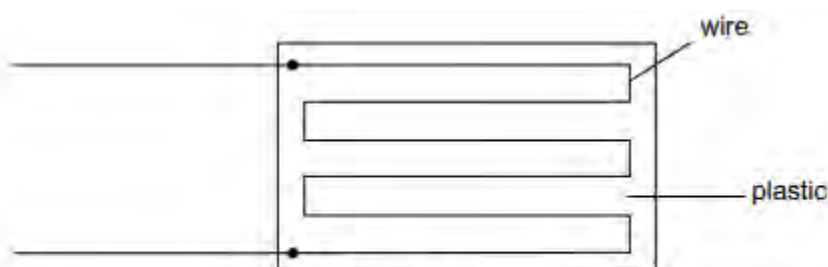


Fig. 1.5

When the plastic is stretched (the plastic experiences a strain), the wire will also be stretched. This causes the wire's length to increase and its cross-sectional area to decrease slightly. Both these changes cause the resistance of the wire to increase. Strain gauges are usually glued very securely to the material that is under test.

The resistance R of a wire of length L and of uniform cross-sectional area A is given by the expression

$$R = \rho L / A,$$

where ρ is the resistivity of the material of the wire.

Assuming that, when the wire extends by a small amount ΔL , the change in the cross-sectional area is negligible, the new resistance will be given by,

$$(R + \Delta R) = \rho(L + \Delta L) / A,$$

where ΔR is the change in the resistance. Subtracting these two expressions,

$$\Delta R = \rho \Delta L / A$$

or,

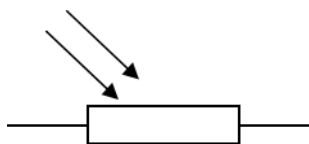
$$\Delta R \propto \Delta L.$$

Thus, the strain which is proportional to the extension ΔL is also proportional to the change in resistance

ΔR . Note that the cross-sectional area A is assumed to be constant.

23.6.5 The Change in Resistance with Light Intensity of a Light-dependent Resistor (LDR)

A light-dependent resistor (LDR) may be made by sandwiching a film of cadmium sulphide between two metal electrodes. Typically, in moonlight, its resistance is about $1 \text{ M}\Omega$ and in sunlight, 100Ω . The symbol for an LDR is shown in the figure below.

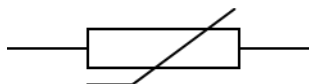


The resistance of an LDR is constant at constant light intensity.

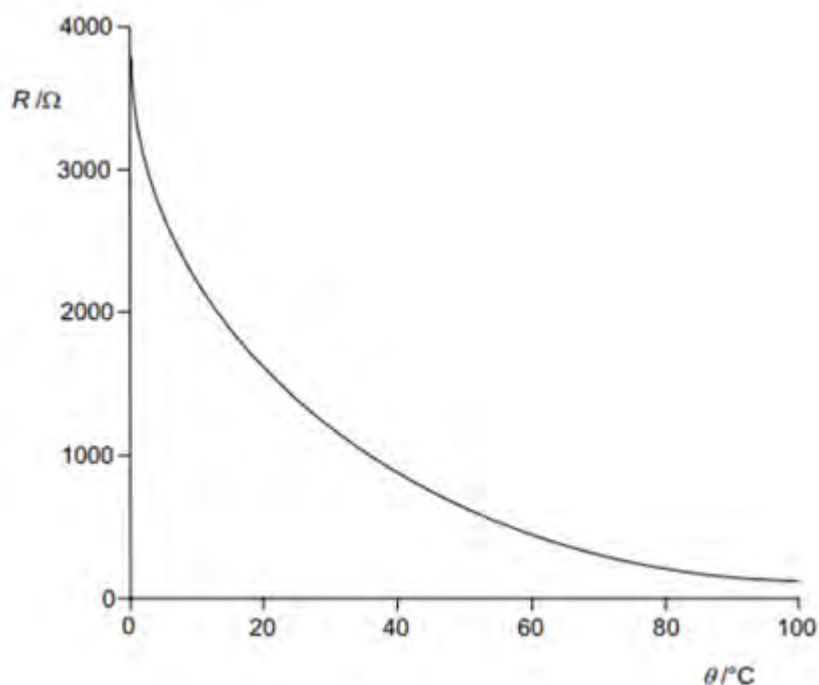
An LDR is sensitive to changes in light intensity. Note that the change in resistance with change in light intensity is not linear.

23.5.6 The Temperature Characteristic of a Negative Temperature Coefficient Thermistor

The resistance of most substances does change slightly with a change in temperature. However, a thermistor is a device that is manufactured in various shapes and sizes using the oxides of different metals so that there is a significant change in resistance with temperature. The symbol for a thermistor is shown in the figure below.



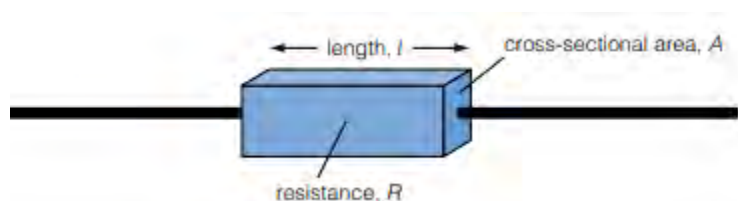
Negative temperature coefficient thermistors have a resistance that becomes less as the temperature of the thermistor rises. The change in resistance R with temperature θ for a typical thermistor is illustrated in the figure below.



It can be seen that there is a comparatively larger change in resistance with temperature but this change is non-linear.

23.7 Resistivity

How is the intrinsic ‘resistance’ of different materials compared? How do you decide which material would be best for the internal connections of a mains plug, or for using as an insulator on a high-voltage line? The resistance of a metal wire, for example, depends on the length and area of the wire, as well as the material that the wire is made from. Consider figure 13.28, showing a metal conductor.



The resistance of the conductor, R , increases with increasing length, l (There are more positive ion cores in the way of the gas of electrons moving through the conductor.). In fact, if the length, l , doubles, then the resistance, R , doubles. This means that $R \propto l$.

The resistance of the conductor decreases with increasing cross-sectional area, A . (There are more conducting pathways through the conductor for the electrons to move through.) In this case, if the cross-sectional area, A , doubles, then the resistance, R , halves. This means that $R \propto \frac{1}{A}$. Combining both of these proportionality statements together:

$$R \propto \frac{l}{A}$$

and replacing the proportionality sign and adding a constant of proportionality, ρ , we have:

$$R = \frac{\rho \times l}{A}$$

where ρ (‘rho’) is called the electrical resistivity of the material. Resistivity is the property that gives the intrinsic resistance of the material independent of its physical dimensions, such as length and cross-sectional area. Resistivity has the units of ohm metres, Ωm , and is defined by the rearranged form of the equation:

$$\rho = \frac{RA}{l}$$

where R is the resistance (measured in ohms, Ω), A is the cross-sectional area (measured in metres squared, m^2) and l is the length (measured in metres, m).

The resistivity of a material depends on some intrinsic properties of the material. In particular, it relates directly to the number of free, conducting electrons that can flow through the structure and the mobility of these electrons to flow through the structure. The arrangement of the atoms in the conductor and any distribution of impurities affects this mobility, as does the temperature of the material.

At room temperature (20°C), good insulating materials, such as ABS plastic (the material that most mains plugs are now made from), have extremely high resistivity (ABS has an electrical resistivity of $1 \times 10^{15} \Omega \text{m}$ at 20°C).

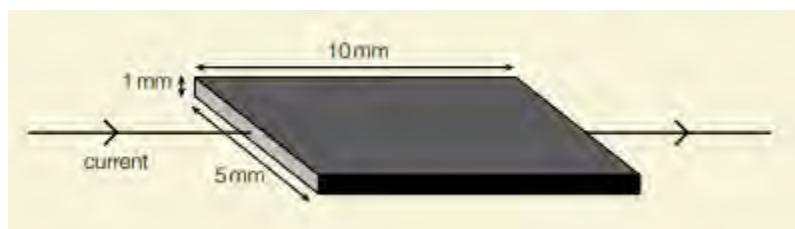
Good metallic conductors have very low resistivity; the resistivity of copper, for example, is $1.68 \times 10^{-8} \Omega \text{m}$ at 20°C. Superconductors have zero resistivity below their critical temperature.

We can use a water analogy to help understand resistivity. A hose pipe full of sand is like a high resistivity conductor, as water trickles through slowly when pressure is applied. However, that same pressure pushes water quickly through an empty hose pipe, which is like a low resistivity conductor. Remember though that the ability of the water to pass through the hose pipe also depends on the length of the hose pipe and its cross-sectional area.

Resistivity is also dependent on temperature. The resistivity of metals increases with increasing temperature, and the resistivity of many semiconductors, such as silicon and germanium, decreases with increasing temperature. The resistivity of a superconducting material decreases with decreasing temperature above its critical temperature (like a metal), but its resistivity drops to zero below the critical temperature.

Workedout Example:

A carbon chip of resistivity $3.0 \times 10^{-5} \Omega \text{m}$ has the dimensions shown in Figure.



What resistance does the chip have for a current in the direction shown?

Solution:

Using the formula:

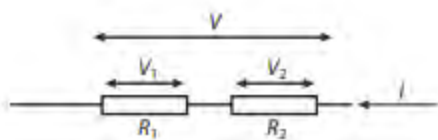
$$R = \frac{\rho \times l}{A}$$

where $l = 10 \text{ mm} = 10 \times 10^{-3} \text{m}$ and $A = 5 \text{ mm} \times 1 \text{ mm} = 5 \times 10^{-3} \text{m} \times 1 \times 10^{-3} \text{m} = 5 \times 10^{-6} \text{m}^2$

$$R = \frac{\rho \times l}{A} = \frac{3.0 \times 10^{-5} \Omega \text{m} \times 10 \times 10^{-3} \text{m}}{5 \times 10^{-6} \text{m}^2} = 0.060 \Omega$$

23.8 Resistors in Series

The figure shows the part of a simple circuit, but now there are two resistors connected in series. Connecting resistors in series means that there are no junctions in the wire connecting any two resistors, and so the current through all of them is the same. Let I be the common current in the two resistors.



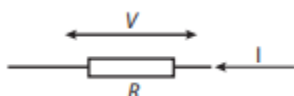
Applying the law of conservation of energy, the p.d. across R_1 plus the p.d. across R_2 must be equal to the p.d. across the combination.

$$V = V_1 + V_2$$

The potential difference across each of the resistors is:

$$V_1 = IR_1 \text{ and } V_2 = IR_2$$

The combination could be replaced by one resistor.



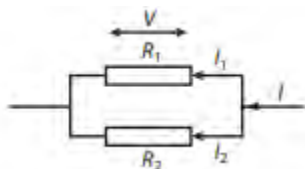
$$IR = IR_1 + IR_2$$

$$\therefore R = R_1 + R_2$$

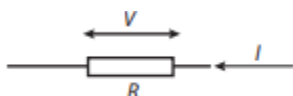
If more than two were present, we would simply add all of them. Adding resistors in series increases the total resistance.

23.9 Resistors in Parallel

In a parallel circuit the current splits in two.



The combination could be replaced by one resistor.



Applying the law of conservation of charge, we know that the current going into a junction must be equal to the current coming out.

$$I = I_1 + I_2$$

Applying Ohm's law to each resistor gives;

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

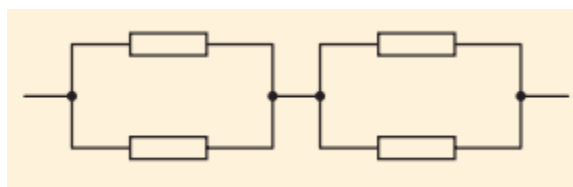
$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

The formula shows that the total resistance is smaller than any of the individual resistances being added.

We have thus learned how to replace resistors that are connected in series or parallel by a single resistor in each case, thus greatly simplifying the circuit.

Workedout Example:

Four resistors each of resistance $1.5\ \Omega$ are connected as shown. Calculate the combined resistance of these resistors.



Solution:

Two $1.5\ \Omega$ resistors in parallel have a resistance given by,

$$\frac{1}{R} = \frac{1}{1.5} + \frac{1}{1.5} = \frac{2}{1.5} \therefore R = 0.75\Omega$$

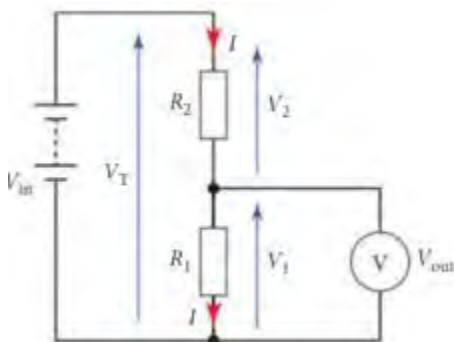
Two $0.75\ \Omega$ resistors in series have a combined resistance of $0.75 + 0.75 = 1.5\ \Omega$

23.10 Potential Dividers

The potential divider is a device that uses two resistances to divide an input potential difference into smaller values. It can provide a set or varying p.d. output, which becomes the input for a further circuit, or it can be used in making measurements of p.d. or resistance.

23.10.1 The Potential Divider Circuit

The circuit in Figure contains two resistors, R_1 and R_2 . The voltmeter has an extremely high resistance, so we can ignore the tiny current that flows through it. R_1 and R_2 are in series. This tells us that we can use the relationship $R = R_1 + R_2$. It also tells us that the same current passes through both R_1 and R_2 . Finally, the p.d.s across R_1 and R_2 add up to give the total p.d. across the circuit, i.e., $V = V_1 + V_2$.



$V = IR$ tells that:

$$V_1 = IR_1 \text{ and } V_2 = IR_2$$

Dividing V_1 by V_2 gives:

$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2}$$
$$\therefore \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

The potential difference has been divided in the ratio of the resistances.

Workedout Example

A potential divider consists of two resistors in series with a battery of 18 V. The resistors have resistances $3.0 \, \Omega$ and $6.0 \, \Omega$. Calculate, for each resistor:

- (a) the pd across it
- (b) the current in it.

Solution:

- (a) The two resistors have the same current in them, and the sum of the pds across the resistors is equal to the source emf. So,

The current I in both resistors is:

$$I = \frac{\text{total pd across both resistors}}{\text{total resistance}} = \frac{V}{R}$$

As usual notation $V_1 = IR_1$ and $V_2 = IR_2$

The resistors are in series, $R = R_1 + R_2$

This leads to equations for the pd across each resistor;

$$V_1 = \frac{R_1}{R_1 + R_2} V_{in} \text{ and } V_2 = \frac{R_2}{R_1 + R_2} V_{in}$$

The pd across the $3.0 \, \Omega$

$$\frac{3.0 \, \Omega}{9.0 \, \Omega} \times 18V = 6V$$

The pd across the $6.0 \, \Omega$

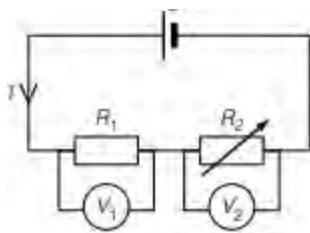
$$\frac{6.0 \, \Omega}{9.0 \, \Omega} \times 18V = 12V$$

- (b) The current is the same in both resistors because they are in series. The total resistance is $9.0 \, \Omega$ and the emf of the battery is 18 V.

$$I = \frac{V}{R} = \frac{18}{9} = 2.0 \, A$$

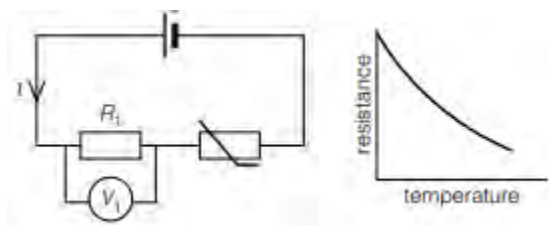
23.10.2 Potential Dividers as Sensors

The use of the potential divider is in sensor circuits. If the variable resistor, R_2 , in the potential divider circuit shown in the figure is replaced with a component whose resistance varies with an external physical variable such as temperature or light intensity, then the potential difference across the fixed resistor, R_1 , will also vary with the external physical variable.



The voltmeter connected across R_1 can then be calibrated in terms of the value of the external physical variable. The circuit can then be made to act as an electronic thermometer or an electronic light sensor.

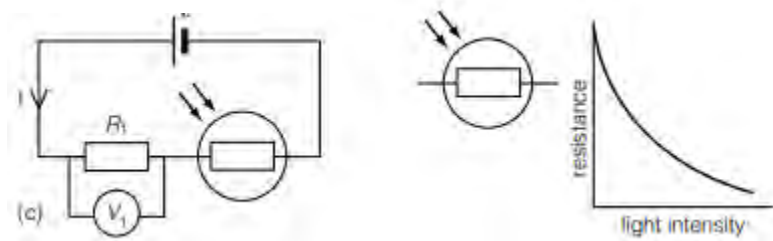
With an electronic thermometer, a thermistor is connected in place of R_2 . Most thermistors are NTC (negative temperature coefficient) thermistors. This means that as the temperature increases, their resistance decreases, as shown in figure, which also shows a circuit diagram showing how the components are connected:



The effect on the potential difference across the fixed resistor R_1 is that as the temperature increases, so the resistance of the thermistor, R_2 , drops and V_1 rises. Thus, increasing temperature produces increasing p.d. If the voltmeter is connected across the thermistor, the opposite will happen: increasing the temperature will produce a decrease in p.d. In most cases, applications require the p.d. to increase with temperature, so the voltmeter is usually connected across the fixed resistor.

A similar effect involves the use of a light-dependent resistor (LDR). These are components that change their resistance with light intensity, which makes them incredibly useful as light sensors. LDRs are made of semi-conducting materials; as the light shines on the material, electrons are freed from the structure and can flow through it, reducing the resistance of the LDR.

In the dark, LDRs can have very high resistance, of the order of megaohms, and yet in the light, their resistance can drop as low as a few hundred ohms. If an LDR is connected to a potential divider in place of R_2 in the potential divider circuit then, as the light intensity increases, then so will the output p.d. V_1 across the fixed resistor.



Workedout Example

A light sensor consists of a 6.0 V battery, an 1800 Ω resistor, and a light-dependent resistor in series. When the LDR is in darkness the pd across the resistor is 1.2 V.

- (a) Calculate the resistance of the LDR when it is in darkness.
- (b) When the sensor is in the light, its resistance falls to 2400 Ω . Calculate the pd across the LDR.

Solution:

- (a) As the pd across the resistor is 1.2 V, the pd across the LDR must be $6 - 1.2 = 4.8$ V. The current in the circuit is,

$$I = \frac{V}{R} = \frac{1.2V}{1800\Omega} = 0.67 \text{ mA}$$

The resistance of the LDR is,

$$\frac{4.8V}{0.67 \times 10^{-3}A} = 7200\Omega$$

- (b) The ratio of $\frac{\text{resistance across LDR}}{\text{resistance across } 1800 \Omega} = \frac{2400\Omega}{1800\Omega} = 1.33$

This is the same value as $\frac{\text{pd across LDR}}{\text{pd across } 1800 \Omega}$ for the ratio of pds to be 1.33, the pds must be 2.6 V and 3.4 V with the 3.4 V across the LDR.

23.11 Internal Resistance and Electromotive Force

The potential difference across the terminals of a cell (the terminal potential difference) depends on the size of the current being drawn from the cell. If no current is being drawn (i.e., if the cell is on an open circuit), the terminal potential difference has its maximum value - known as the electromotive force (EMF) of the cell.

The chemicals within a cell offer a resistance to the flow of the current, known as the internal resistance of the cell. When a cell is connected across an external circuit (a load) some of the EMF is used to drive current through the load and the rest of the EMF drives the same current through the internal resistance.

The internal resistance behaves as if it is in series with the cell and the external circuit.

The chemical action inside a battery produces a surplus of electrons at one of its terminals (the negative), and creates a shortage at the other (the positive). It is then able to maintain a flow of electrons, i.e., an electric current, in any circuit connected across its terminals for as long as the chemical action lasts. A work is done by the battery in moving charge around the circuit. Electromotive force (e.m.f.), symbol E , is defined as the electrical work, W , done by a source in moving a unit charge around a complete circuit:

$$e.m.f. = \frac{\text{work done (by a source)}}{\text{charge}}$$

$$E = \frac{W}{Q}$$

Point to remember:

Electromotive force e.m.f. is the electrical work done by a source in moving unit charge around a complete circuit.

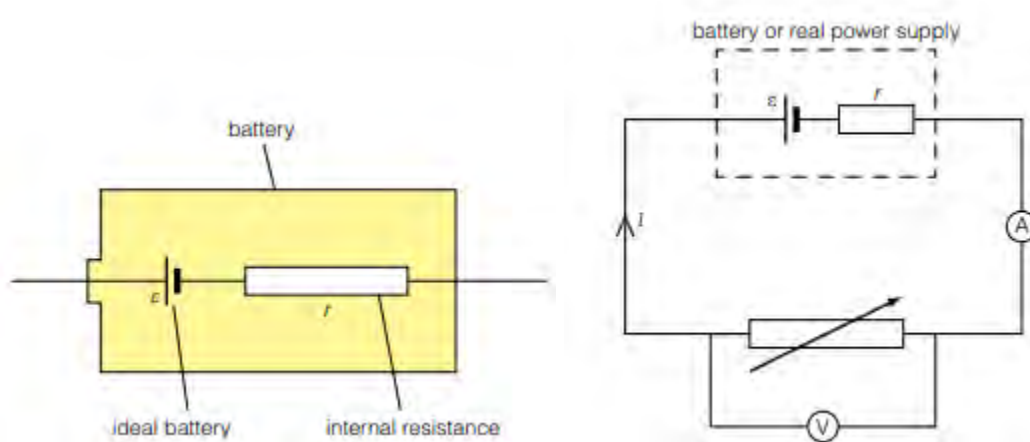
Potential difference p.d. is the work done by a unit charge passing through a component.

In an electric circuit, an electric current transfers energy from an energy store, such as a battery, to components in the circuit which then transfer energy into the surroundings. In the case of a lamp, energy is transferred to the surroundings by light and by heating.

Real power supplies, such as batteries and laboratory power packs, always have an internal resistance, r . As the current flows through the power supply, the internal resistance creates a potential difference that leads to electrical energy being transferred to thermal energy inside the power supply. This is one of the reasons why hand-held devices such as tablets warm up when they are in use for a long period of time.

The internal resistance of most batteries and power supplies is very low: a typical AA battery has an internal resistance of about 0.2Ω and, as such, can effectively be ignored when measuring the electrical properties of most circuits.

The internal resistance of a power supply cannot be measured directly as it is 'inside' the power supply. The only way to measure it is to use its electrical characteristics. A circuit that can be used to do this is shown in the figure below.



Circuit calculations can be used to determine the internal resistance and the electromotive force.

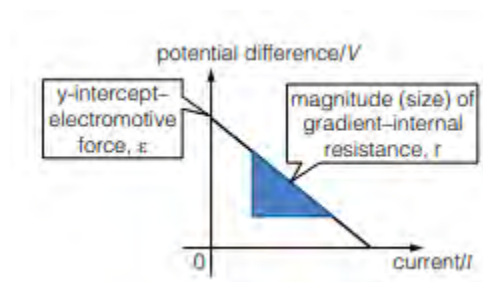
The electromotive force, E , must be the same as the sum of the potential differences in the circuit (Conservation of energy). There are two potential differences in this circuit: one is across the external variable resistor, V , and the other is the p.d. across the internal resistor. This cannot be measured directly, but the p.d. across this resistor is equal to Ir . This means that:

$$E = V + Ir$$

The current, I , can be measured directly using an ammeter; E and r are both constants, so the equation can be rewritten as:

$$V = E - Ir \text{ or } V = -Ir + E,$$

which is the equation of a straight line of the form, $y = mx + c$, where the gradient is negative. If an electrical characteristic is drawn using values of V and I from various values of R (the external load resistance) then the y -intercept of the graph is the electromotive force, E , and the gradient is negative and equal to $-r$, the internal resistance.



An alternative rearrangement of the energy equation that enables us to find the current is:

$$E = IR + Ir \therefore I = \frac{E}{R + r}$$

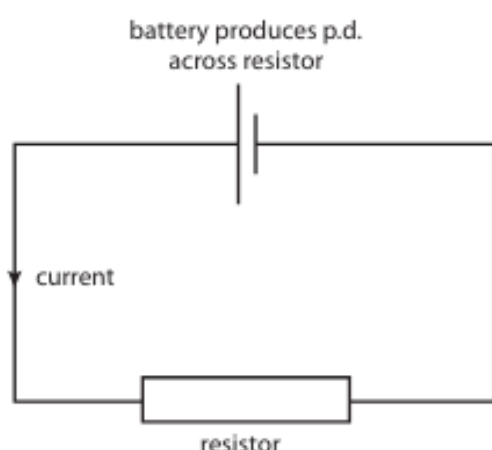
Workedout Example

A torch battery of e.m.f. 4.5 V and internal resistance 0.4Ω is connected across a lamp of resistance 6.4Ω .

What is the current in the lamp?

$$I = \frac{E}{R + r} = \frac{4.5 \text{ V}}{6.4 \Omega + 0.4 \Omega} = 0.66 \text{ A}$$

23.12 Energy and Power in Electrical Circuits **Fig needed**



Consider a simple circuit in which a steady current, I , is flowing through a load. The load may be an electric motor for example, or simply a resistor. As the current flows through the load, it dissipates energy in it. The energy dissipated is equal to the potential energy lost by the charge as it moves through the potential difference that exist between the input and output terminals of the device.

It follows from the definition of potential difference that the energy dissipated is given by;

$$W = QV$$

Where,

W = the energy dissipated in some time t

Q = the charge which flows in time t

V the potential difference across the load

The current is steady, and therefore,

$$Q = It$$

Therefore,

$$W = VIt$$

The rate of dissipation of energy, $\frac{W}{t}$, is known as the power, P , and therefore since I and V are constant;

$$P = \frac{W}{t} = \frac{VIt}{t} = I^2R$$

The unit of power is the watt (W). $1W = 1Js^{-1}$. Note that in the equations above W and P are in joules, and watts respectively when V , I and, t are expressed in volts, amperes and seconds respectively.

Workedout Example:

1. A manufacturer indicates that the maximum safe power for a particular range of resistors is 250 mW. What is the maximum safe current for a 47 k Ω resistor from this range?

Solution

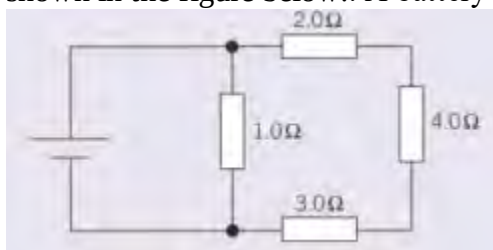
$$P = I^2R$$

$$\text{Rearranging, } I^2 = \frac{P}{R} = \frac{250 \times 10^{-3} W}{47000 \Omega} = 5.32 \times 10^{-6} A^2$$

$$\therefore I = 2.3 \text{ mA}$$

Exercise

- Four resistors of resistances $1.0\ \Omega$, $2.0\ \Omega$, $3.0\ \Omega$, and $4.0\ \Omega$ are connected together as shown in the figure below.. A battery is connected across the $1.0\ \Omega$ resistor.



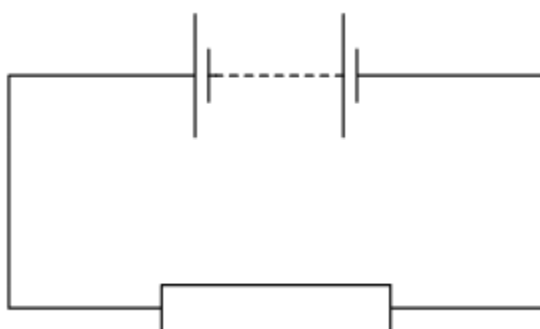
If I_0 is the battery current and I_1 is the current in the $4.0\ \Omega$ resistor, which one of the following gives the ratio $\frac{I_0}{I_1}$?

- 0.1
 - 0.5
 - 1.5
 - 10
- Four different types of lamps produce the same brightness. The power consumption and the current are marked on the packaging for each lamp and shown in the table.

Type of lamp	Power/W	Current/A
filament	60	0.25
fluorescent	12	0.05
halogen	48	4
LED	6	0.5

Which type of lamp has the greatest efficiency?

- filament
 - fluorescent
 - halogen
 - LED
- In the circuit below, the battery converts an amount E of chemical energy to electrical energy when charge Q passes through the resistor in time t .



Which expressions give the e.m.f. of the battery and the current in the resistor?

	e.m.f.	current
A	EQ	Q/t
B	EQ	Qt
C	E/Q	Q/t
D	E/Q	Qt

4. A lightning strike takes a time t . During the lightning strike, the electrons are discharged to the Earth creating a current I .
An expression for the total number of the electrons that are discharged to the Earth during the lightning strike is;

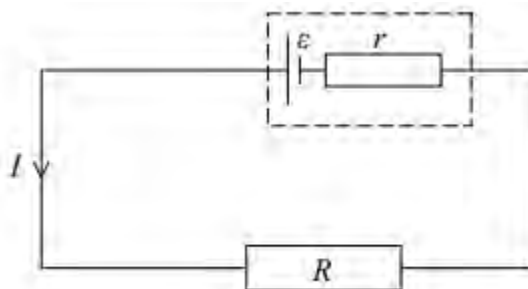
- A. $\frac{It}{e}$
 B. $\frac{I}{te}$
 C. $\frac{e}{It}$
 D. $\frac{te}{I}$

5. The combined resistance R of two resistors of resistances R_1 and R_2 connected in parallel is given by the formula;

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Which statement is used in the derivation of this formula?

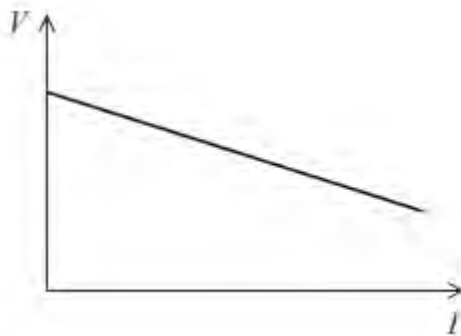
- A. The currents through the two resistors are equal.
 B. The potential difference across each resistor is the same.
 C. The supply current is split between the two resistors in the same ratio as the ratio of their resistances.
 D. The total power dissipated is the sum of the powers dissipated in the two resistors separately.
6. The circuit shows a cell of emf ε with an internal resistance of r connected to a resistor of resistance R . The current in the circuit is I .



Which of the following is an expression for the terminal potential difference V across the cell?

- A. $V = \varepsilon - IR$
 B. $V = \varepsilon + IR$
 C. $V = \varepsilon + Ir$
 D. $V = \varepsilon - Ir$

7. A motor is used to lift a mass m through a height h in a time t . When a current I is drawn from the power supply, the potential difference across the motor is V . Which of the following is a correct expression for the efficiency of the system?
- $\frac{VI}{mgh}$
 - $\frac{mgh}{VI}$
 - $\frac{VI t}{mgh}$
 - $\frac{mgh}{VI t}$
8. A student investigates to determine the e.m.f. $\mathcal{E}$ and internal resistance r of a cell. The potential difference V across the cell and the current I in the cell are measured as the load resistance is increased. The results are plotted on a graph.

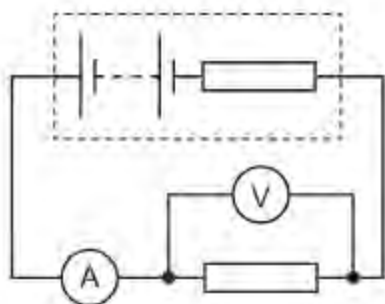


Select the row of the table that correctly identifies how $\mathcal{E}$ and r are determined from the graph.

	$\mathcal{E}$ determined from	r determined from
A	area under graph	$\frac{1}{\text{magnitude of gradient}}$
B	y-intercept	$\frac{1}{\text{magnitude of gradient}}$
C	area under graph	magnitude of gradient
D	y-intercept	magnitude of gradient

Ans: D

9. A battery with internal resistance is connected to a fixed resistor, an ammeter and a voltmeter, as shown.



The battery is replaced by a different battery that has the same electromotive force (e.m.f.) but a greater internal resistance.

What happens to the readings on the ammeter and voltmeter?

	ammeter reading	voltmeter reading
A	decreases	decreases
B	decreases	stays the same
C	stays the same	decreases
D	stays the same	stays the same

10. When four identical lamps P, Q, R and S are connected as shown in diagram 1, they have normal brightness.

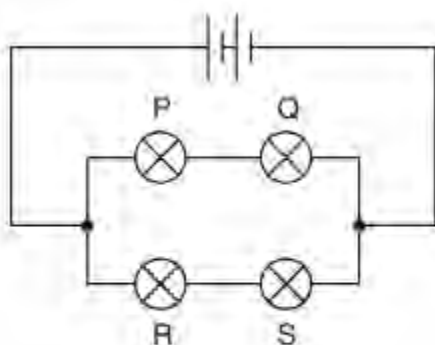


diagram 1

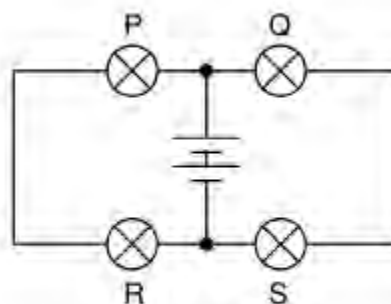


diagram 2

When the four lamps are connected as shown in diagram 2, which statement is correct?

- A. The lamps do not light.
 - B. The lamps are less bright than normal.
 - C. The lamps have normal brightness.
 - D. The lamps are brighter than normal.
11. A nichrome wire is 1 m long and $1 \times 10^{-6} \text{ m}^2$ in cross-sectional area. When connected to a potential difference of 2 V, a current of 4 A exists in the wire. The resistivity of this nichrome is:
- A. $2 \times 10^{-7} \Omega \cdot \text{m}$
 - B. $4 \times 10^{-7} \Omega \cdot \text{m}$
 - C. $5 \times 10^{-7} \Omega \cdot \text{m}$
 - D. $8 \times 10^{-7} \Omega \cdot \text{m}$

12.

- (a) The copper wire supplying the sockets in your house has a cross-sectional area 2.5 mm^2 . A hairdryer can draw a current of 8.2 A . What is the mean drift velocity of the electrons in this situation?

Assume that the electron number density of copper is $8.49 \times 10^{23} \text{ m}^{-3}$.

- (b) A 1 molar solution of copper sulphate contains $6.02 \times 10^{23} \text{ Cu}^{2+}$ ions per cubic metre. A beaker of this solution has two electrodes placed in it, through which a current of 30 mA is passed. What is the mean drift velocity of the copper ions?

Take the cross-sectional area of the solution to be 100 cm^2 .

13. A tungsten lamp filament of diameter 0.046 mm carries a current of 0.50 A . What is the mean drift velocity of electrons in the filament? Assume that the electron number density of tungsten is $1.26 \times 10^{29} \text{ m}^{-3}$.

14. A copper wire of diameter 0.65 mm carries a current of 0.25 A . There are 8.5×10^{28} charge carriers in each cubic metre of copper; the charge on each charge carrier (electron) is $1.6 \times 10^{-19} \text{ C}$. Calculate the drift speed of the charge carriers.

15.

- (a) State what physical quantity is represented by each term in the equation: $I = nAvq$.

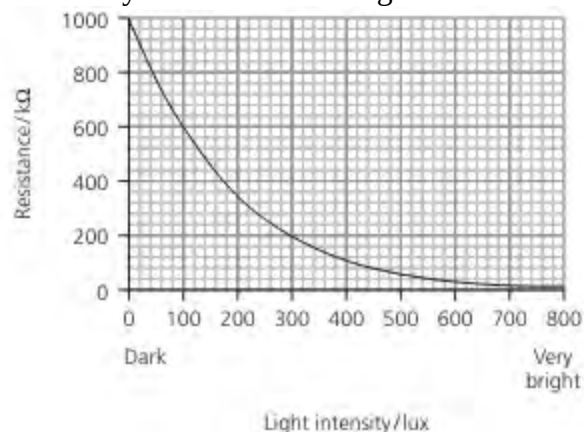
- (b) A semiconducting strip 6.0 mm wide and 0.50 mm thick carries a current of 10 mA , as shown in Figure.



If the value of n for the semiconducting material is $7.0 \times 10^{22} \text{ m}^{-3}$, show that the drift speed of the charge carriers, which carry a charge of $1.6 \times 10^{-19} \text{ C}$, is about 0.3 m s^{-1} .

(c)

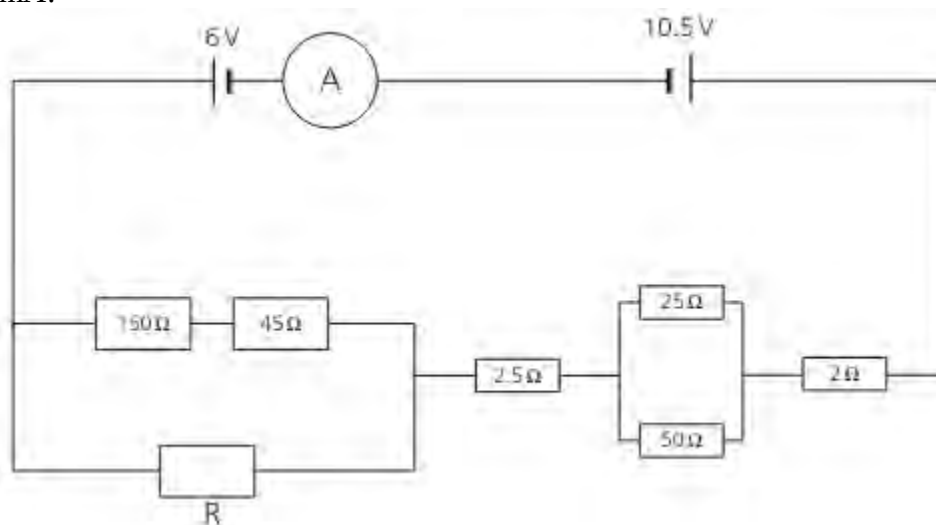
16. The relationship of the resistance of a light-dependent resistor (LDR) as a function of light intensity is shown in the figure below.



- (a) A manufacturer of light-sensitive switches wants to use this LDR to turn a set of garden lights on when the outdoor light intensity drops below 200 lux . Draw a

potential divider that would allow the circuit in the garden lights to turn the lights on when the output potential difference reaches 2.5V or below.

17. The figure below shows a circuit diagram. The current measured by the ammeter is 220mA.



- (a) Calculate the value of the resistance of R.
- (b) Describe how the total current in the circuit would change if resistor R were disconnected from the circuit. Explain why the current would change in the way you suggested.

Chapter 24

Nuclear Physics



Ernest Rutherford (1871–1937)

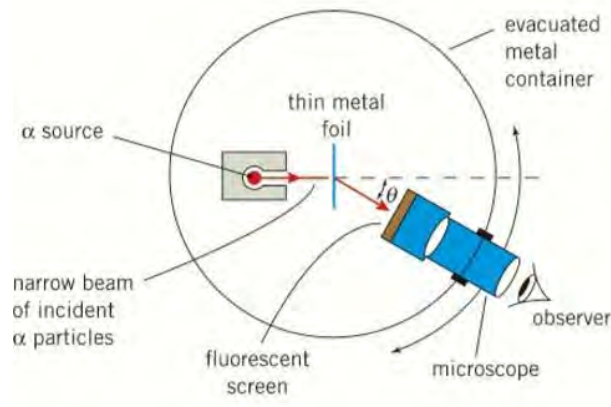
Ernest Rutherford, New Zealand, Physicist (1871–1937)

Rutherford was awarded the 1908 Nobel Prize in Chemistry for studying radioactivity, and for discovering that atoms can be broken apart by alpha rays. “On consideration, I realized that this scattering backward must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greater part of the mass of the atom was concentrated in a minute nucleus. It was then that I had the idea of an atom with a minute massive center carrying a charge,” Says Rutherford.

24.1 The Discovery of Nucleus: Rutherford's alpha-scattering experiment

In 1911, Ernest Rutherford, on the basis of his experiment, proposed that the positive charge of the atom is densely concentrated at the center of the atom, forming its nucleus. The nucleus is responsible for most of the mass of the atom.

Figure shows a simplified version of the arrangement used in Rutherford's experiments.

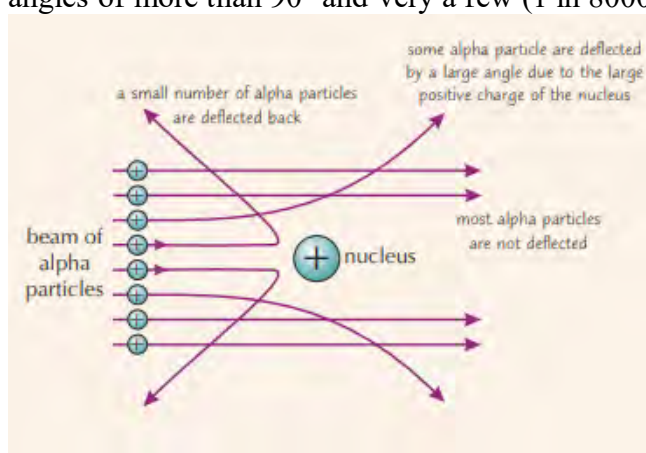


A narrow beam of alpha particles, all of the same kinetic energy, from a radioactive source were targeted at a thin piece of gold foil which was only a few atomic layers thick. The alpha particles were scattered by the foil and detected on a zinc sulfide screen mounted in front of a microscope. Each alpha particle hitting this fluorescent screen produced a tiny speck of light. The microscope was moved around in order to count the number of alpha particles scattered through different values of the angle θ per minute, for θ from zero to almost 180° .

Observations and Conclusions

The scattering experiment led to the following two significant observations.

- Most of the alpha particles passed straight through the thin gold foil with very little scattering. About 1 in every 2000 alpha particles was scattered.
- Very few of the alpha particles — about 1 in every 10000 — were deflected through angles of more than 90° and very a few (1 in 8000) surprisingly ‘bounced’ back.

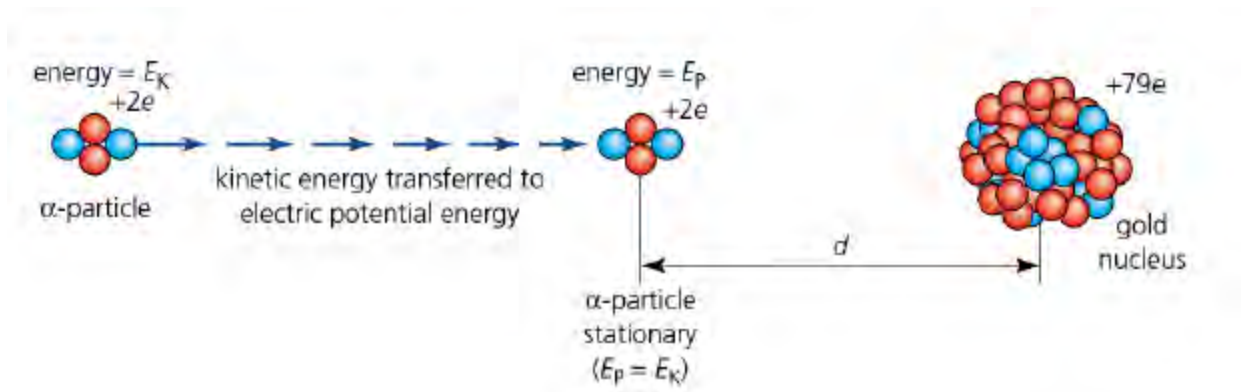


These significant observations can be explained in terms of a new model of the atom — the nuclear model.

- The first observation meant that the most of the atom was empty with most of the mass concentrated in a small region — the nucleus.
- The second observation led to the conclusion that the nucleus has a positive charge, because it repelled the few positive alpha particles that came near it. And, nucleus is massive compared to α - particles.

Sizes of the Atom and the Nucleus

Rutherford was able to estimate the radius of the gold nucleus by calculating how close an alpha particle got to the nucleus during its interaction. At its nearest distance, the alpha particle moving directly towards the nucleus stops, so all its kinetic energy is then in the form of potential energy.



At the closest approach, in a head-on collision, the electric potential energy stored in the electric field is equal to the initial kinetic energy of the alpha particle, $E_P = E_K$.

The equation for the electric potential energy (E_P) depends on the separation, r , of the two charges.

$$E_P = \frac{kq_1q_2}{r}$$

In the case of the alpha particle, at the distance of closest approach to the nucleus, all its kinetic energy, E_K , will have been transferred to potential energy, $E_P = E_K$.

$$E_P = E_K = \frac{kq_1q_2}{r}$$

The charge of the alpha particle is $+2e$ and the charge of the nucleus is $+Ze$ ($Z = 79$, for gold), so if the kinetic energy of alpha particles is known (they are all same), then r can be calculated as follows:

$$r = \frac{kq_1q_2}{E_K} = \frac{k(2e \times Ze)}{E_K}$$

If the target nucleus is gold and the incident alpha particles have kinetic energy of 4.0 MeV (which is typical for alpha particles), the distance of closest approach is:

$$r = \frac{8.99 \times 10^9 \times (2 \times 79) \times (1.6 \times 10^{-19})^2}{(4.0 \times 10^6) \times (1.6 \times 10^{-19})}$$

$$r = 5.7 \times 10^{-14} \text{ m}$$

This distance is assumed to be the upper limit of the nuclear radius. Therefore, the nuclear radius of a gold nucleus is in the order of 10^{-14} m .

From his experiments, Rutherford concluded that the nucleus had a radius of about 10^{-14} m .

The radius of the nucleus is about 10^{-15} m . The radius of most atoms is about 10^{-10} m . So, the nucleus is about 10^5 times smaller than the atom.

An α -particle is travelling in a vacuum towards the centre of a gold nucleus, as illustrated in the figure below.



The gold nucleus has charge of $79e$. The gold nucleus and the α -particle may be assumed to behave as point charges. At a large distance from the gold nucleus, the α -particle has energy $7.7 \times 10^{-13} \text{ J}$.

The α -particle does not collide with the gold nucleus. Show that the radius of the gold nucleus must be less than $4.7 \times 10^{-14} \text{ m}$.

At the distance of closest approach to the nucleus, all its kinetic energy, E_K , will have been transferred to potential energy;

$$E_K = E_P = \frac{kq_1q_2}{r} = \frac{9 \times 10^9 \times 79e \times 2e}{r}$$

$$\therefore r = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{7.7 \times 10^{-13}} = 4.7 \times 10^{-14} \text{ m}$$

r is closest distance of approach so radius less than this.

Determine the acceleration of the α -particle for separation of $4.7 \times 10^{-14} \text{ m}$ between the centres of the gold nucleus and of the α -particle.

$$Force = \frac{kq_1q_2}{r^2} = 4u \times a$$

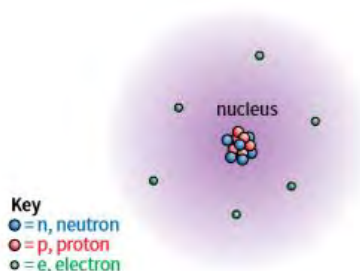
$$a = \frac{9 \times 10^9 \times 79e \times 2e}{4u(4.7 \times 10^{-14})^2} = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{4 \times 1.66 \times 10^{-27} (4.7 \times 10^{-14})^2}$$

$$= 2.5 \times 10^{27} \text{ ms}^{-2}$$

In an α -particle scattering experiment, the beam of α -particles is incident on a very thin gold foil. Suggest why the gold foil must be very thin. So that multiple deflections with the nucleus do not occur.

24.2 A Simple Model of the Atom

After Rutherford had presented his findings, the nuclear model of the atom gained rapid acceptance. Subsequently, the proton was discovered. It had a positive charge, equal and opposite to that of the electron. However, its mass was too small to account for the entire mass of the atom and it was not until the early 1930s that this puzzle was solved by the discovery of the neutron, an uncharged particle with a similar mass to that of the proton. This suggests a model for the atom like the one shown in the figure:



- Protons and neutrons make up the nucleus of the atom.
- The electrons move around the nucleus in a cloud, some closer to and some further from the centre of the nucleus.

24.3 Some Nuclear Terminology

Nuclei are made up of protons and neutrons. Protons and neutrons are called nucleons because they are both components of nuclei. Three numbers describe the composition of a nucleus:

- The number of protons in a nucleus is denoted by Z , and is called the atomic (or proton) number.
- The total number of nucleons (protons + neutrons) is called the mass (or nucleon) number, and is denoted by A .
- The number of neutrons in a nucleus is denoted by N with $N = A - Z$.
- The word nucleon is used to denote a proton or a neutron.
- A nucleus with a specific number of protons and neutrons is also called a nuclide.

We represent nuclides with symbols as,



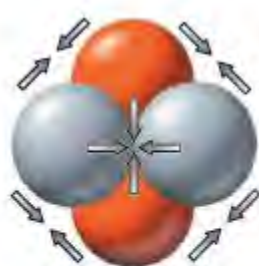
where X is the chemical symbol for the element, A is the nucleon number (the total number of protons and neutrons), Z is the proton number (also known as atomic number) and N is the number of neutrons.

For oxygen,



Particles and Symbols

Particle	Symbol
proton	${}^1_1\text{p}$
neutron	${}^1_0\text{n}$
electron	${}^{-1}_0\text{e}$
positron	${}^{+1}_0\text{e}$
photon	${}^0_0\gamma$ or just γ
alpha particle	${}^4_2\text{He}$ or ${}^4_2\alpha$
neutrino	${}^0_0\nu$ or just ν
anti-neutrino	${}^0_0\bar{\nu}$ or just $\bar{\nu}$



All nucleons, both protons and neutrons, attract one another by the strong nuclear force.



Only protons repel one another by the electrical force.

24.4 Isotopes

Isotopes are nuclei of the same element that have the same number of protons, but different numbers of neutrons.

For example, ordinary hydrogen (${}^1_1\text{H}$), deuterium (${}^2_1\text{H}$), and tritium (${}^3_1\text{H}$) are all isotopes of the element hydrogen. Specific isotopes can be indicated by the element name, and the atomic mass number. For example, carbon-12 is another way of writing ${}^{12}_6\text{C}$.

The chemical properties of an atom are determined by its electron configuration. Because the numbers of electrons and protons are equal in a neutral atom, the chemical properties are essentially determined by Z . All the isotopes of a particular element have the same number of protons and electrons. So, these isotopes have almost identical chemical properties. However, the physical properties can differ dramatically. In particular, one isotope of an element may be highly radioactive, while another is quite stable. Bombarding materials with electrons, neutrons, or other particles can create radioactive isotopes.

Nuclides with the same neutron number are called isotones (for example, ${}^{14}_6\text{C}$, ${}^{15}_7\text{N}$, ${}^{16}_8\text{O}$, and ${}^{17}_9\text{F}$) Nuclides with the same value of A are called isobars (for example, ${}^{16}_6\text{C}$, ${}^{16}_7\text{N}$, ${}^{16}_8\text{O}$ and ${}^{16}_9\text{F}$)

24.5 Atomic Mass Units

Atoms and nuclei are much, much smaller than everyday objects. So, even though a kilogram may be a convenient unit for expressing the mass of apples or oranges, it is not particularly useful for measuring the mass of a proton or a carbon nucleus. For calculations involving nuclei and subatomic particles, it is often convenient to use a mass unit that is much smaller than the kilogram.

The atomic mass unit (u) is defined as one-twelfth of the mass of the carbon atom $^{12}_6\text{C}$. Now the number of molecules in 1 mole of carbon is 6.02×10^{23} , Avogadro's constant, and since carbon is monoatomic, there are 6.02×10^{23} atoms of carbon. These have a mass of 12 g.

$$\text{mass of one atom of carbon} = \frac{12}{6.02 \times 10^{23}} \text{ g} = \frac{12}{6.02 \times 10^{26}} \text{ kg} = 12u$$

The atomic mass unit (u) is defined as exactly $\frac{1}{12}$ of the mass of the carbon-12 atom.

$$1u = \frac{1}{12} \times \frac{12}{6.02 \times 10^{26}} \text{ kg} = 1.66 \times 10^{-27} \text{ kg}$$

24.6 Nuclear Size and Density

The radius of the nucleus depends on the nucleon number A of the nucleus. Fast-moving electrons have a de Broglie wavelength of about 10^{-15} m. Diffraction of such electrons has been used to determine the radii of isotopes. Experiments have shown that the radius R of a nucleus is given by the equation;

$$R = r_0 A^{\frac{1}{3}} \quad (1)$$

r_0 has an approximate value of 1.2 fm ($1 \text{ fm} = 10^{-15} \text{ m}$).

The simplest nucleus is that of hydrogen - ^1_1H , with $A = 1$. You can therefore think of r_0 as roughly the radius of a proton.

The volume V of a sphere is equal to $V = \frac{4}{3}\pi R^3$ so from equation (1) the volume of a nucleus is proportional to A .

$$\text{i.e. } V = \frac{4}{3}\pi(r_0 A^{\frac{1}{3}})^3 = \frac{4}{3}\pi r_0^3 A$$

Dividing A (the approximate mass in u) by the volume gives us the approximate density and cancels out A .

$$\begin{aligned} \text{density} &= \frac{\text{mass}}{\text{volume}} = \frac{A u}{\frac{4}{3}\pi r_0^3 A} = \frac{1.66 \times 10^{-27}}{\frac{4}{3}\pi r_0^3} \text{ kg m}^{-3} = \frac{1.66 \times 10^{-27}}{\frac{4}{3}\pi \times (1.2 \times 10^{-15})^3} \\ &= 2.3 \times 10^{17} \text{ kg m}^{-3} \end{aligned}$$

Thus, all nuclei have approximately the same density. This fact is of crucial importance in understanding nuclear structure.

The nucleus of an atom is very small, massive, and hence extremely dense. All nuclei have a density of about $10^{17} \text{ kg m}^{-3}$ - about 10^{14} times denser than water. A spoonful of nuclear material would have a mass of about a thousand million tons.

24.7 Einstein's Mass and Energy

Einstein's work on special relativity led him to publish a paper in 1905. This paper suggested that energy and mass were different ways of expressing the same thing – that energy and mass were interchangeable and linked using the equation, $E = mc^2$, where E is energy (J), m is change in mass (kg) and c is the speed of light, $3 \times 10^8 \text{ ms}^{-1}$. By 1932 his ideas were proved experimentally by Cockcroft and Walton.

So, what does this equation mean? If we use a helium nucleus as an example, the helium nucleus contains four nucleons – two protons and two neutrons. The mass of the helium nucleus is very slightly smaller than the mass of its separate nucleons.

As the helium nucleus forms, some mass is converted to energy and released. Calculating the energy released when an alpha particle is formed is straightforward:

- mass of a proton is $1.6726 \times 10^{-27} \text{ kg}$
- mass of a neutron is $1.6749 \times 10^{-27} \text{ kg}$
- mass of two protons and two neutrons is $6.6950 \times 10^{-27} \text{ kg}$
- measured mass of a helium nucleus is $6.6337 \times 10^{-27} \text{ kg}$
- mass difference is $6.13 \times 10^{-29} \text{ kg}$

Using $E = mc^2$, the energy released when a single alpha particle is formed is $5.5 \times 10^{-12} \text{ J}$, or 34MeV. You can see that, in Einstein's own words, 'a very small amount of mass may be converted into a very large amount of energy and vice versa'.

Work must be done to overcome the very strong nuclear forces that bind the nucleons together and pull a helium nucleus apart. The energy put in to do this creates the extra mass.

A nucleus of Z protons and N neutrons has a mass that is less than the mass of the protons and neutrons that make it up. This difference in mass is called the mass defect, where mass defect $\Delta m = Zm_p + Nm_n - M_{\text{nucleus}}$ measured in kg or atomic mass units (u).

Since mass and energy are interchangeable, we can also express mass as energy. Binding energy is the energy that corresponds to the mass defect, and is related to the mass defect using $\text{binding energy} = \text{mass defect} \times c^2$.

Binding energy is the energy that would have to be supplied to the nucleus to separate it back into its constituent protons and neutrons. The binding energy can be expressed in J or in MeV.

24.7.1 Mass-energy Equivalence

The equivalence of mass and energy is part of the theory of relativity that Albert Einstein developed in 1905. This theory correctly predicted that mass and energy are related by the equation

$E = mc^2$, where E is energy, m is mass, and c is the speed of light.

The idea that mass and energy are equivalent was proposed by Albert Einstein In 1905 as part of the theory of relativity with his famous equation $E = mc^2$, where E is energy, m is mass, and c is the speed of light in a vacuum. This equation has two interpretations.

The first is that mass is a form of energy. The interaction of an electron-positron pair illustrates this idea well — the particles completely destroy each other (annihilation) and the entire mass of the particles is transformed into two gamma photons.

The second interpretation is that energy has mass. The change in mass Δm of an object, or a system, is related to the change in its energy ΔE by the equation $\Delta E = \Delta mc^2$. A moving ball has kinetic energy, implying that its mass is greater than its rest mass. The same happens to electrons in particle accelerators. However, because they can have speeds close to the speed of light, their mass could be a hundred times greater than their rest mass. Similarly, a decrease in the energy of a system means the mass of the system must also decrease. For example, the mass of a mug of hot tea decreases as it cools and loses thermal energy. However, the change in mass is negligibly small (the conversion factor c^2 is enormous).

For example:

- A sealed torch that radiates 10W of light for 10h (= 36000s) would lose 0.36MJ of energy (= $10\text{W} \times 36000\text{s}$). Its mass would therefore decrease by $4.0 \times 10^{-12} \text{ kg}$ (= $0.36\text{MJ}/c^2$), an insignificant amount compared with the mass of the torch.
- A car of 1000kg mass that speeds up from a standstill to 30ms^{-1} would gain 450kJ of kinetic energy so its mass when moving at 30ms^{-1} would be $5.0 \times 10^{-12} \text{ kg}$ (= $450\text{kJ}/c^2$) more than when it is at rest.
- An unstable nucleus that releases a 5MeV γ photon would lose $8.0 \times 10^{-13} \text{ J}$ of energy. Its mass would therefore decrease by $8.9 \times 10^{-30} \text{ kg}$ (= $8.0 \times 10^{-13} \text{ J}/c^2$) which is not an insignificant amount compared with the mass of a nucleus.

As we know physicists commonly use the atomic mass unit, u , for calculations involving nuclei and subatomic particles. For nuclear calculations, it is useful to know the energy equivalent for 1 u :

$$\begin{aligned} E = 1u \times c^2 &= (1.660539 \times 10^{-27})(2.997925 \times 10^8)^2 = 1.492418 \times 10^{-10} \text{ J} \\ &= 1.492418 \times 10^{-10} \text{ J} \times \frac{1\text{eV}}{1.602177 \times 10^{-19}\text{J}} = 931.4941 \text{ MeV} \end{aligned}$$

Thus, 1 u is equivalent to about 931.5 MeV.

Nuclear reactions can involve conversions between mass and energy. The law of conservation of energy still applies if the conversions are taken into account. For any closed system, the total of the energy and the energy equivalent of the mass in the system is constant.

Everyday situations

Consider a person with rest mass 70 kg sitting in a stationary car. Now imagine the car travelling at a steady speed of 15ms^{-1} . The person has gained kinetic energy, an increase in energy ΔE . The person will therefore have increased mass. The change in mass Δm can be calculated using the mass—energy equation $\Delta E = \Delta mc^2$.

$$\Delta m = \frac{\Delta E}{c^2} = \frac{\frac{1}{2}mv^2}{c^2} = \frac{\frac{1}{2} \times 70 \times 15^2}{(3.00 \times 10^8)^2} = 8.8 \times 10^{-14} \approx 10^{-13} \text{ kg}$$

This is a minuscule change in mass and is not noticeable.

24.8 Annihilation and creation

When a particle and its corresponding antiparticle meet, they destroy each other in a process known as annihilation. Positrons are the antiparticles of electrons. When they meet, they annihilate each other, and their entire mass is transformed into energy in the form of two identical gamma photons. This is not science fiction. It does happen, and medical physicists have exploited this phenomenon in positron emission tomography (PET). A PET scanner is used to examine the function of organs, including the brain. Consider an electron—positron pair annihilating each other.

- change in mass $\Delta m = 2m_e$ ($m_e = (\text{mass of electron or positron}) = 9.1 \times 10^{-31} \text{ kg}$)
- energy released $\Delta E = \Delta mc^2 = 2m_e c^2$
- minimum energy of two gamma photons $= 2m_e c^2$
- minimum energy of each gamma photon $= m_e c^2$
- minimum energy of each gamma photon $= m_e c^2 = 9.1 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.2 \times 10^{-14} \text{ J}$
- Therefore, the minimum energy of each photon is $8.2 \times 10^{-14} \text{ J}$ or about 0.51 MeV. If the interacting particles also have kinetic energy, then the energy of each photon would be even greater.

The opposite process to annihilation is called pair production. In pair production, a single photon vanishes, and its energy creates a particle and its corresponding antiparticle. In this process, a photon with enough energy can interact with a large nucleus and be converted directly into a particle–antiparticle pair.

- The rest energy of an electron (and therefore a positron) is 0.51 MeV, or $8.2 \times 10^{-14} \text{ J}$.

$$E = mc^2 = 9.1 \times 10^{-31} \times (3.0 \times 10^8)^2 = 8.19 \times 10^{-14} \approx 8.2 \times 10^{-14} \text{ J}$$

In order to create this particle–antiparticle pair, the photon must have enough energy to create both particles, in other words,

$$(2 \times 0.51) = 1.02 \text{ MeV, or } 16.4 \times 10^{-14} \text{ J.}$$

24.9 Forces in the Nucleus

Aside from hydrogen, all nuclei consist of two or more protons and a number of neutrons. Like charges repel each other, so what keeps these nuclei from flying apart?

Example:

Can gravitational force bind two protons in a nucleus together?

Given,

proton mass $m = 1.67 \times 10^{-27} \text{ kg}$ and proton charge $q = 1.60 \times 10^{-19} \text{ C}$.

Required,

Determine if gravitational force can bind two protons in a nucleus together.

Analysis and Solution:

Compare the gravitational and electrostatic forces between two protons in a nucleus.

The magnitude of the gravitational force is $F_g = \frac{Gm_1m_2}{r^2}$..

The magnitude of the electrostatic force is $F_e = \frac{kq_1q_2}{r^2}$..

$$\text{So, } \frac{F_g}{F_e} = \frac{\frac{Gm_1m_2}{r^2}}{\frac{kq_1q_2}{r^2}} = \frac{Gm_1m_2}{kq_1q_2}$$

This ratio shows that the relative strength of the two forces does not depend on the distance between the protons. For the gravitational attraction between the protons to overcome the electrostatic repulsion, the ratio would have to be greater than 1. Substituting the known values into the ratio of the forces gives;

$$\frac{F_g}{F_e} = \frac{(6.67 \times 10^{-11})(1.67 \times 10^{-27})^2}{(8.99 \times 10^9)(1.60 \times 10^{-19})^2} = 8.08 \times 10^{-37}$$

Paraphrase

The gravitational attraction is vastly weaker than the electrostatic repulsion, so gravity cannot be the force that holds a nucleus together.

Since gravity is far too weak, there must be some other force that holds the particles in a nucleus together. Physicists call this force the strong nuclear force, and think that it is a fundamental force of nature, like gravity and the electrostatic force. The strong nuclear force has a very short range. Although it is more powerful than the electrostatic force within a nucleus, the strong nuclear force has a negligible effect on particles that are more than a few femtometres apart. The strong nuclear force acts on both neutrons and protons, but does not affect electrons.

24.10 Binding Energy

As discussed earlier, a strong nuclear force acts between neighbouring nucleons within the nucleus. It has a very short range $\approx 10^{-15} \text{ m}$ or 1 fm. The stability of many nuclei provides evidence for the strong nuclear force.

Protons and neutrons are very tightly bound to each other in a nucleus. To separate them, energy must be supplied to the nucleus. To completely dismantle a nucleus into all of its constituent nucleons work must be done to separate the nucleons and overcome the strong nuclear force acting between them. This work is known as the nuclear binding energy. The binding energy of a nucleus is the energy required to separate all of its protons and neutrons and move them infinitely far apart.

Suppose we could reverse the process and construct a nucleus from a group of individual nucleons. We would expect there to be energy released as the strong nuclear force pulls them together. This would be equal to the nuclear binding energy needed to separate them. This implies that energy is needed to deconstruct a nucleus from nucleons and is given out when we construct a nucleus.

24.11 Mass Defect

Deuterium is an isotope of hydrogen. A nucleus of deuterium consists of one proton and one neutron. Now, imagine separating these two nucleons. All nucleons are bound together by the strong nuclear force, so they can only be separated by doing work to overcome that force. External energy has to be supplied to make this happen. According to Einstein's mass-energy equation, energy and mass are equivalent, therefore, the total mass of the separated nucleons must be greater than the mass of the deuterium nucleus.

Is this really true? We can use a mass spectrometer to determine the mass of particles accurately. In terms of unified atomic mass units u ($1.661 \times 10^{-27} \text{ kg}$), a deuterium nucleus has a mass of $2.013553u$, a proton has a mass of $1.007276u$, and a neutron has a mass of $1.008665 u$. The total mass of the separated proton and neutron is indeed more than the mass of the deuterium nucleus. The difference is $0.002388u$, which is equivalent to an energy of about $3.5 \times 10^{-13} \text{ J}$ or 2.2 MeV . In simple terms, this means that a minimum energy of 2.2 MeV is needed to completely separate the nucleons of a deuterium nucleus. Suppose we could reverse the process and construct a deuterium nucleus from a proton and a neutron. This time, an energy of 2.2 MeV would be released — most likely in the form of a photon.

In the example of the deuterium nucleus above, the difference in mass of $0.002388u$ is known as the mass defect of the deuterium nucleus.

The mass defect of a nucleus is defined as the difference between the mass of the completely separated nucleons and the mass of the nucleus.

In other words, the mass of a nucleus is less than the total mass of its individual protons and neutrons. This difference in mass is called a mass defect.

The energy difference of 2.2 MeV for the deuterium nucleus is known as its binding energy. The binding energy of a nucleus is defined as the minimum energy required to completely separate a nucleus into its constituent protons and neutrons.

To calculate the binding energy of a nucleus, we can use Einstein's mass—energy equation.

$$\text{binding energy of nucleus} = \text{mass defect of nucleus} \times c^2$$

The binding energy is not the same for all nuclei.

A uranium-235 has 92 protons and 143 neutrons, and the external energy required to split this nucleus into its constituent protons and neutrons is much greater than 2.2 MeV — there are many more strong nuclear bonds to be broken.

The following formula uses atomic masses to calculate the mass defect for a nucleus:

$$\Delta m = Zm_{\text{proton}} + Nm_{\text{neutron}} - m_{\text{nuclide}}$$

where Z is the proton number, and N is the neutron number.

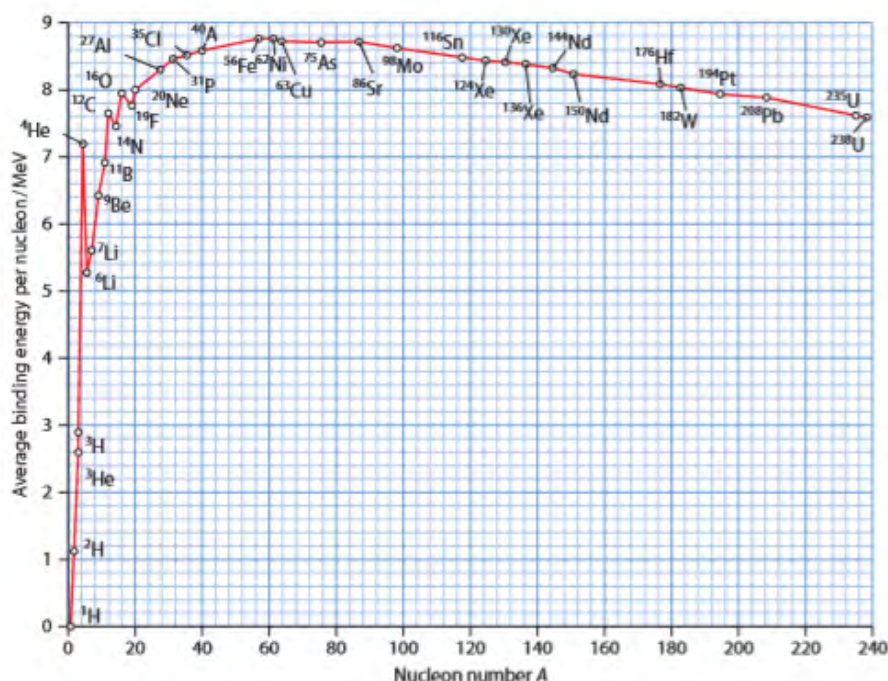
And for binding energy,

$$\text{Binding energy}(BE) = \Delta mc^2 = (Zm_{\text{proton}} + Nm_{\text{neutron}} - m_{\text{nuclide}}) \times c^2$$

24.11.2 The Binding Energy per Nucleon

It is defined as the total binding energy of a nucleus divided by A, the total number of nucleons. We calculated above that the binding energy of ${}^2_1\text{H}$ is 2.2 MeV, so its binding energy per nucleon is $2.2\text{MeV}/2 = 1.1 \text{ MeV}$.

The binding energy per nucleon is the average energy needed to separate a nucleus into its nucleons. A greater binding energy per nucleon means a more tightly bound nucleus in other words a nucleus is more stable if it has a greater BE per nucleon.

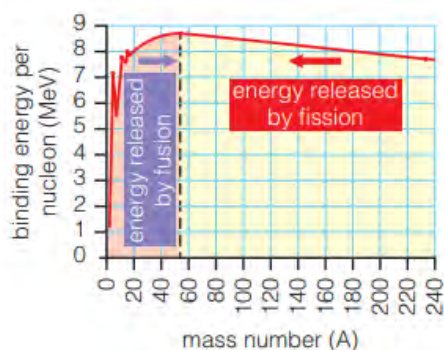


The above figure is a graph of BE per nucleon against nucleon number A. The shape of the graph helps us to understand the processes such as natural radioactive decay, fission, and fusion. The last two processes are covered in greater depth in the next two topics. From the graph you can see that:

- The binding energy per nucleon for hydrogen, ${}^1_1\text{H}$, is zero because there is only one particle in the nucleus.
- The curve rises sharply for low values of A .
- For nuclei with $A < 56$, the BE per nucleon increases as A increases.
- For nuclei with $A > 56$, the BE per nucleon decreases as A increases.
- The nucleus of iron-56 (${}^{56}_{26}\text{Fe}$) has the greatest BE per nucleon — it is the most stable isotope in nature. (and ${}^{62}_{28}\text{Ni}$ as well)
- There are peaks at the position of the nuclei ${}^4_2\text{He}$, ${}^{12}_6\text{C}$ and ${}^{16}_8\text{O}$, which makes these nuclei unusually stable compared to their immediate neighbours.
- The curve drops gently from the peak at $A = 56$ (After $A=62$) and onwards.
- In a fusion process, two lower A -number nuclei join together to produce a higher A number nucleus. The newly formed nucleus has much greater binding energy than the initial nuclei and therefore energy is released. Fusion is the process by which the Sun and other stars produce their energy. Thanks to fusion, we have life on the Earth.
- In a fission process, a high A -number nucleus splits into two lower, A -number nuclei. Energy is released because the two nuclei produced have higher binding energy than the parent nucleus. All fission reactors use this process to produce energy.
- Energy is released in natural radioactive decay. Figure above can be used to show that in cases of spontaneous decay the total binding energy of the parent nucleus is less than the binding energy of the daughter nucleus and the alpha particle. The difference is the energy released in the decay as kinetic energy
- Most nuclei have a binding energy per nucleon between 7 and 9 MeV. The short range of the force implies that a given nucleon can interact with its immediate neighbours only, and not with all of the nucleons in the nucleus. So, for a large nuclei (i.e., roughly $A > 20$) any one nucleon is surrounded by the same number of immediate neighbours and so the energy needed to remove that nucleon from the nucleus is the same. Thus, the short-range nature of the nuclear force explains why the binding energy per nucleon is roughly constant above a certain value of A .

24.12 Nuclear Reactions:

Some nuclei can release energy from nuclear fission or nuclear fusion. Almost all nuclear reactions that occur naturally result in nuclei that are more stable. This increases the binding energy per nucleon compared with the original nuclei. The mass difference between the original nuclei and the nuclei of the products corresponds to the amount of energy released. The figure below shows that, to increase the binding energy per nucleon, lighter elements tend to fuse and heavier elements tend to undergo radioactive decay or fission.



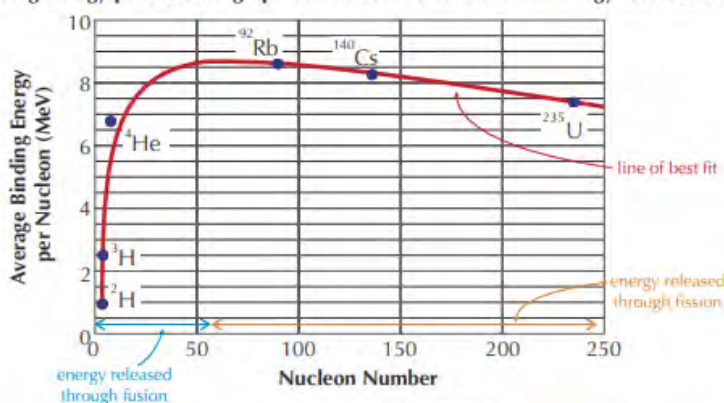
Conservation Laws in Nuclear Processes (reactions)

In addition to conserving momentum, all nuclear processes obey these additional conservation laws:

- **Charge (Proton number):** The net electrical charge cannot change in a decay process. Any change in the electrical charge of the nucleus must be exactly offset by an opposite change elsewhere in the system. For example, if the charge on a nucleus decreases by $+2e$, then a particle with a charge of $+2e$ must be emitted.
- **Nucleon number (Atomic Mass Number):** The total of the nucleon (atomic mass) numbers for the final products must equal the nucleon number of the original nucleus. In other words, the total number of nucleons remains constant.
- **Total mass-energy (mass-energy are interchangeable in nuclear processes)** is conserved.

The Change in Average Binding Energy Gives the Energy Released

The average binding energy per nucleon graph can be used to estimate the energy released from nuclear reactions.



Fusion

- 1) ${}^4\text{He}$ nuclei have 4 nucleons, so the binding energy of ${}^4\text{He} = 4 \times 6.8 = 27.2 \text{ MeV}$.
- 2) The binding energy of ${}^2\text{H}$ and ${}^3\text{H} = (2 \times 1.1) + (3 \times 2.6) = 10.0 \text{ MeV}$.
- 3) So if ${}^2\text{H}$ and ${}^3\text{H}$ nuclei were fused together to form ${}^4\text{He}$ (and a neutron), the energy released would be $27.2 - 10.0 = 17.2 = 17 \text{ MeV}$ (to 2 s.f.).

Fission

- 1) The binding energy of ${}^{235}\text{U} = 235 \times 7.4 = 1739 \text{ MeV}$.
- 2) The binding energy of ${}^{92}\text{Rb}$ and ${}^{140}\text{Cs} = (92 \times 8.8) + (140 \times 8.2) = 1957.6 \text{ MeV}$.
- 3) So if a ${}^{235}\text{U}$ nucleus splits into ${}^{92}\text{Rb}$ and ${}^{140}\text{Cs}$ (plus a few neutrons) during nuclear fission, the energy released would be $= 1957.6 - 1739 = 218.6 = 220 \text{ MeV}$ (to 2 s.f.).

24.13 Nuclear Fission

Nuclear fission is a decay process in which an unstable nucleus splits into two fragments of comparable mass.

If we are able to take a large nucleus and split it into two smaller ones, the binding energy per nucleon will increase as we move from the right-hand side to the centre of the figure of a graph of BE per nucleon against nucleon number A. This means that energy must be given out in the form of the kinetic energy of the fission products.

When a nucleus with $A > 120$ splits into smaller nuclei, they have greater binding energy per nucleon. This fission reaction gives off energy equal to the difference between the binding energy of the original nucleus and the total binding energy of the products.

For both nuclear fission and fusion, the energy released, ΔE , is

$$\Delta E = E_{b_f} - E_{b_i} = (\text{net change in the mass defect}) \times c^2$$

where E_{b_i} is the total binding energy of the original nucleus or nuclei, and E_{b_f} is the total binding energy of the product(s).

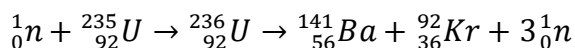
Since the binding energies correspond to the mass defects for the nuclei, the energy released corresponds to the decrease in the total mass defect. This change in the total mass defect equals the change in the total mass. Thus, the energy released corresponds to the mass that the reaction transforms into energy:

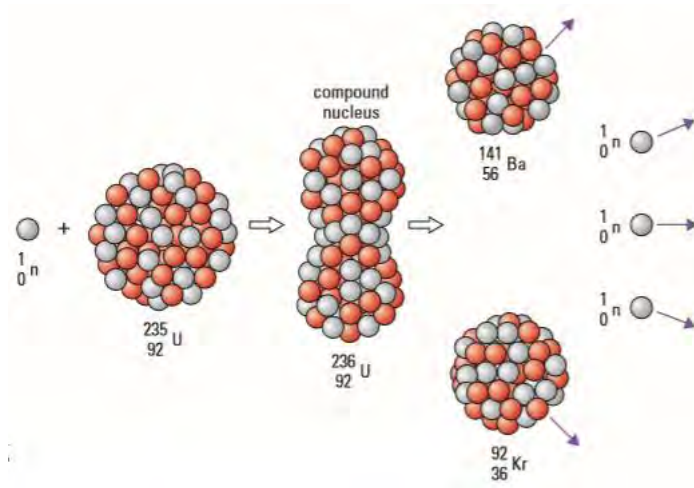
$$\Delta E = (m_f - m_i) \times c^2$$

where m_i is the total mass of the original nucleus or nuclei, and m_f is the total mass of the product(s).

Often, fission results from a free neutron colliding with a large nucleus. The nucleus absorbs the neutron, forming a highly unstable isotope that breaks up almost instantly.

When a neutron is absorbed by a nucleus of uranium-235, uranium momentarily turns into uranium-236. It then splits into lighter nuclei plus neutrons. One possibility is:





The energy released can be calculated as follows:

$$\begin{aligned}
 \text{the total mass of the original nuclei}(m_i) &= m_{{}^{235}_{92}\text{U}} + m_{{}^1_0\text{n}} \\
 &= 235.043\,930\,u + 1.008\,665\,u \\
 &= 236.052\,595\,u
 \end{aligned}$$

$$\begin{aligned}
 \text{total mass of the products}(m_f) &= m_{{}^{141}_{56}\text{Ba}} + m_{{}^{92}_{36}\text{Kr}} + 3m_{{}^1_0\text{n}} \\
 &= 140.914\,412\,u + 91.926\,156\,u +
 \end{aligned}$$

$$3(1.008\,665\,u)$$

$$= 235.866\,563\,u$$

$$\begin{aligned}
 m_i - m_f &= 236.052\,595\,u - 235.866\,563\,u = \\
 &0.186\,032\,u
 \end{aligned}$$

Now, use mass-energy equivalence to calculate the energy released.

1 u is equivalent to 931.5 MeV, so,

$$\Delta E = 173.3\,\text{MeV}$$

Note that the fission process is induced and begins when a neutron collides with a nucleus of uranium-235.

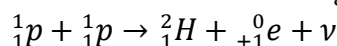
Fission of a uranium nucleus, triggered by neutron bombardment, releases other neutrons that can trigger more fissions, suggesting the possibility of a chain reaction.

The conditions for fusion are just right in the core of our stars like our Sun. The temperature is close to $1.4 \times 10^7 \text{K}$ and the density is $1.5 \times 10^5 \text{kg m}^{-3}$. The enormous density ensures a high number of fusion reactions per second.

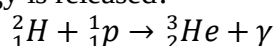
Examples of fusion

Many different types of fusion reactions can take place within the stars - they all release energy. Fusion reactions often occur in cycles or sequences. One such cycle is shown here.

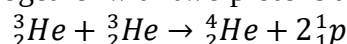
- Two protons fuse together to produce a deuterium nucleus (${}^2_1\text{H}$), a positron, and a neutrino. This reaction produces about 2.2 MeV of energy.



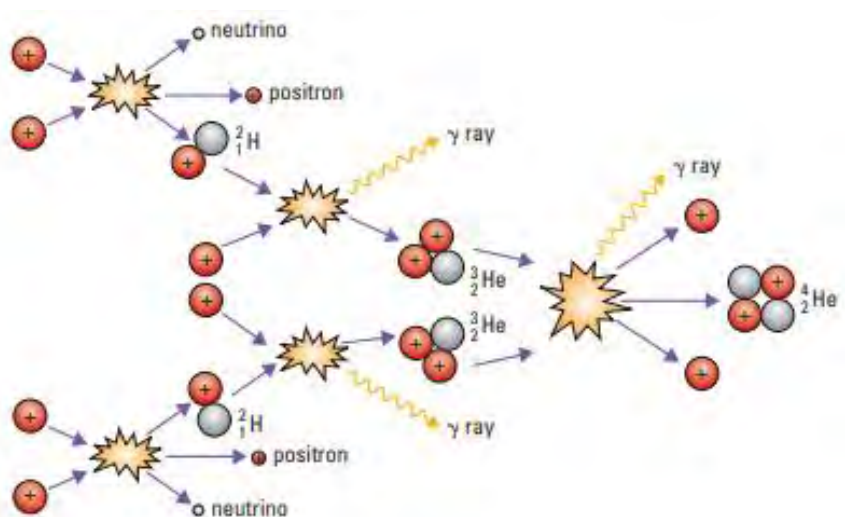
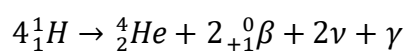
- We can easily explain this energy using the graph of binding energy per nucleon against nucleon number. The two single protons have zero binding energy and the deuterium nucleus has a binding energy of $1.1 \times 2 = 2.2 \text{ MeV}$. The difference in the binding energies, of 2.2 MeV, is the energy released in this fusion reaction.
- The deuterium nucleus from the first reaction fuses with a proton. A helium-3 nucleus is formed and 5.5 MeV of energy is released.



- The helium-3 from the second reaction combines with another helium-3 nucleus. A helium-4 nucleus is formed, together with two protons and 12.9 MeV of energy.



Total,



The whole cycle is repeated again with the two protons. This cycle is known as the proton-proton cycle or the hydrogen-burning cycle, and is one of the main production routes for helium in stars. The proton-proton cycle occurs around 9×10^{37} times each second inside the Sun.

Workedout Example:

1. Find the total energy released in the fusion reactions in the proton–proton cycle.

$$\text{mass of proton} = 1.007\,825\,u, \text{mass of helium} = 4.002\,603\,u$$

Solution:

The net nuclear result of the proton–proton cycle is to fuse four protons to form an alpha particle. Study the reactions above for the proton–proton cycle to be sure you understand how four protons become an alpha particle.

$$\text{the initial mass of the system} = 4(1.007\,825\,u) = 4.031\,300\,u$$

$$\text{the change in mass of the system} = 4.031\,300\,u - 4.002\,603\,u = 0.028\,697\,u$$

$$\text{energy released}(E) = 0.028\,697\,u \times 931.494 \frac{\text{MeV}}{u} = 26.7\,\text{MeV}$$

This energy is shared among the alpha particle and other particles such as positrons, gamma rays, and neutrinos.

2. The Sun radiates about $4 \times 10^{26}\,\text{W}$ and has a mass of $1.99 \times 10^{30}\,\text{kg}$. Astronomers estimate that the Sun can convert only the innermost 10% of its hydrogen into helium. 73% of the mass of the sun is hydrogen. Estimate how long the Sun can continue to shine at its present intensity.

$$\text{Given Power} = 4 \times 10^{26}\,\text{W}$$

$$m_{\text{sun}} = 1.99 \times 10^{30}\,\text{kg}$$

$$\text{Hydrogen available for conversion} = 10\% \text{ of total hydrogen}$$

To find time the Sun will take to convert 10% of its hydrogen into helium (t)

Solution:

The fusion of four hydrogen atoms produces $26.71\,\text{MeV}$. To find the rate at which helium nuclei are produced, divide the Sun's power by the energy released during the formation of each helium nucleus:

$$\begin{aligned} \text{Rate of helium production} &= \frac{\text{Power of sun}}{\text{energy released per helium atom}} \\ &= \frac{4 \times 10^{26}\,\text{W}}{26.71\,\text{MeV/atom}} = \frac{4 \times 10^{26}\,\text{J/s}}{26.71 \frac{\text{MeV}}{\text{atom}} \left(\frac{1.60 \times 10^{-13}\,\text{J}}{1\,\text{MeV}} \right)} = 9.36 \times 10^{37}\,\text{atoms/s} \end{aligned}$$

Since 4 hydrogen atoms are needed for each helium produced, multiply by 4 to find the rate at which the Sun converts hydrogen atoms into helium.

Then, convert this rate to mass per second by multiplying it by the mass of a hydrogen atom:

$$\begin{aligned} \text{Rate of hydrogen conversion} &= 4 (9.36 \times 10^{37}\,\text{atoms/s}) \times (1.67 \times 10^{-27} \frac{\text{kg}}{\text{atoms}}) \\ &= 6.25 \times 10^{11}\,\text{kg/s} \end{aligned}$$

Hydrogen makes up 73% of the mass of the Sun, but only 10% of this hydrogen can be converted into helium.

The lifespan of the Sun approximately equals the amount of hydrogen that can be converted divided by the conversion rate.

$$t = \frac{\text{amount of hydrogen available}}{\text{rate of conversion}} = \frac{1.99 \times 10^{30} \text{ kg} \times 73\% \times 10\%}{6.25 \times 10^{11} \text{ kg/s}}$$

$$= 2.32 \times 10^{17} \text{ or about } 7 \times 10^9 \text{ years}$$

The Sun can continue to produce energy at its present rate for about 7 billion years.

1. Calculate the binding energy of ${}_{92}^{235}\text{U}$.

Given: mass of ${}_{92}^{235}\text{U}$ = 235.004393, mass of proton = 1.007276u and mass of neutron = 1.008665u

Solution:

The uranium nucleus has 2 protons and $(235 - 92) = 143$ neutrons

Therefore total mass of nucleons = $(92 \times 1.007276) + (143 \times 1.008665)$
 $= 236.908487\text{u}$

mass defect for Uranium = $236.908487\text{u} - 235.004393\text{u} = 1.904094\text{u}$

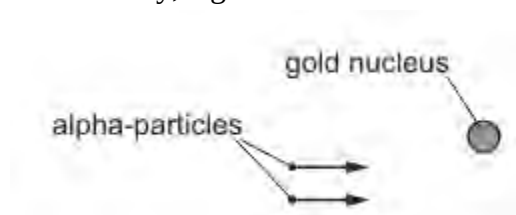
mass defect in kg = $1.904094\text{u} \times 1.66 \times 10^{-27} = 3.16 \times 10^{-27} \text{ kg}$

B.E. = $\Delta m \times c^2 = 3.16 \times 10^{-27} \times (3.0 \times 10^8)^2 = 2.84 \times 10^{-10} \text{ J}$

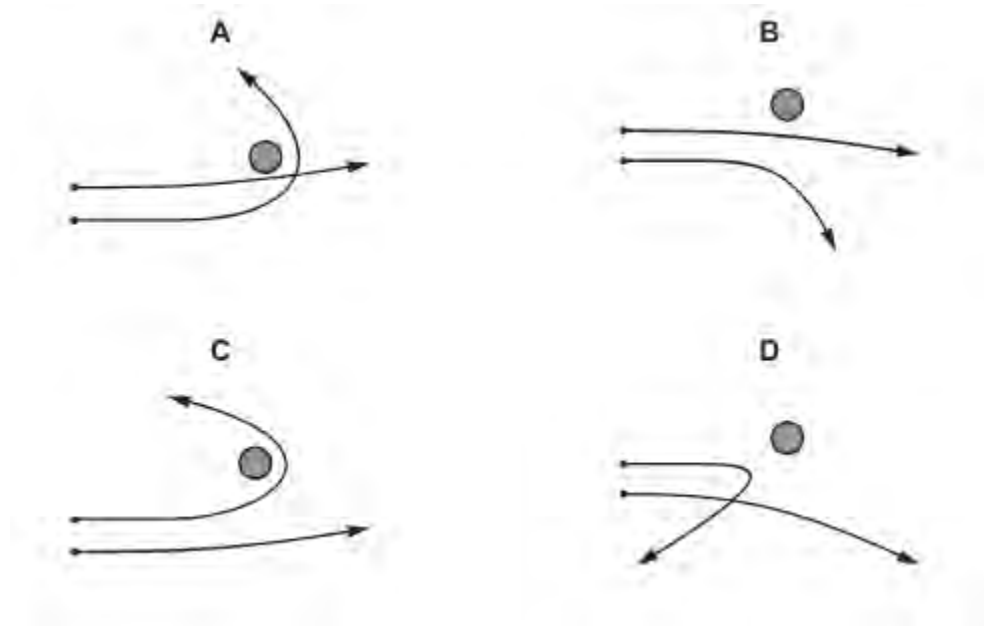
BE in MeV = $\frac{2.84 \times 10^{-10}}{1.60 \times 10^{-13}} = 1777.9 \approx 1780 \text{ MeV}$

Exercise

18. In the early 20th century, experiments were carried out to measure the scattering of alpha particles after striking thin gold foil.
- Which of the following could not be concluded from the results of these experiments?
- A. The nucleus contains most of the mass of the atom.
 - B. The nucleus is made of protons and neutrons.
 - C. The nucleus is charged.
 - D. The nucleus has a much smaller radius than the radius of the atom.
19. Two alpha-particles with the same kinetic energy are moving towards, and are then deflected by, a gold nucleus.



Which diagram could show the paths of the two alpha-particles?



20. In an experiment to investigate the nature of the atom, a very thin gold film was bombarded with α -particles. What pattern of deflection of the α -particles was observed?
- A. A few α -particles were deflected through angles greater than a right angle.
 - B. All α -particles were deflected from their original path.
 - C. Most α -particles were deflected through angles greater than a right angle.
 - D. No α -particle was deflected through an angle greater than a right angle.

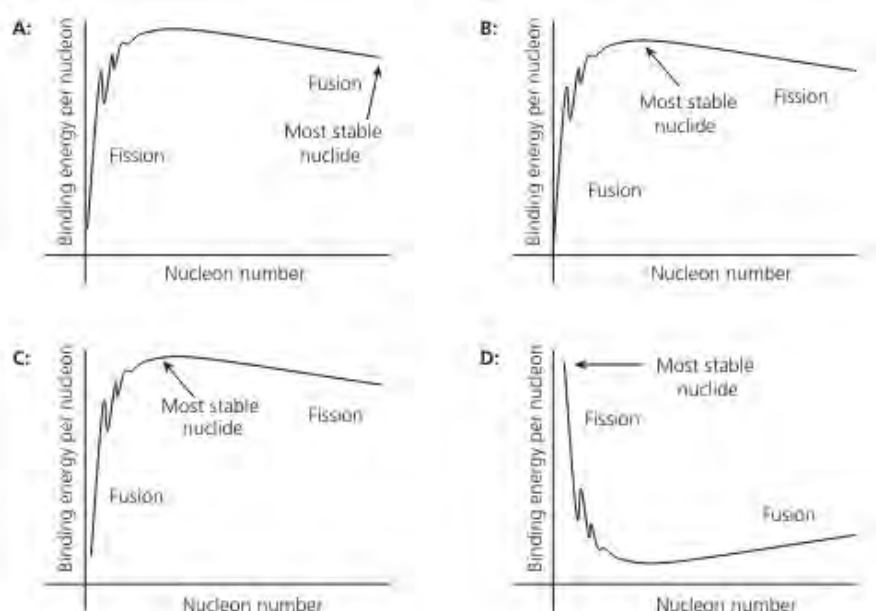
21. The numbers of protons, neutrons and nucleons in three nuclei are shown.

nucleus	number of protons	number of neutrons	number of nucleons
X	15	16	31
Y	15	17	32
Z	16	16	32

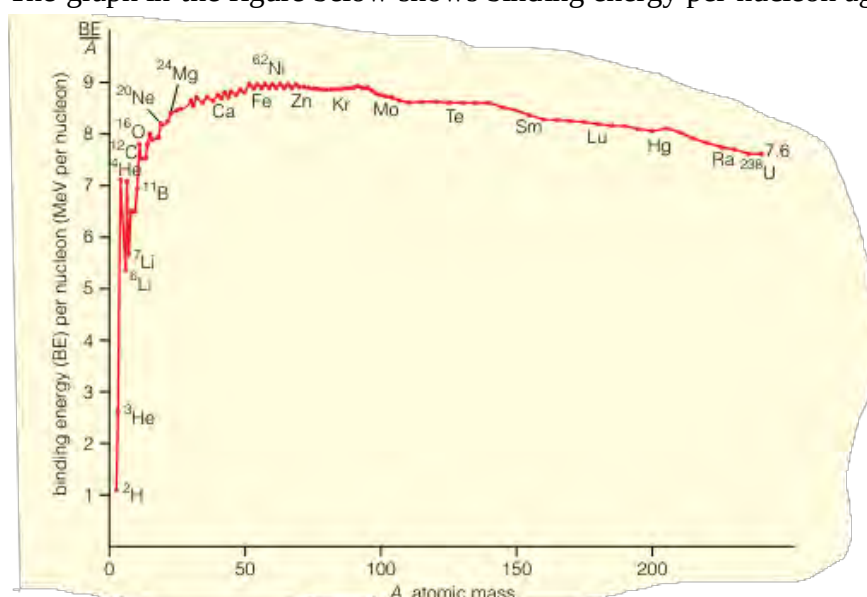
Which nuclei are isotopes of the same element?

- A. X and Y B. X and Z C. Y and Z D. none of them

22. Which one of the graphs of binding energy per nucleon against nucleon number in the figure below is correct?



23. The graph in the figure below shows binding energy per nucleon against atomic mass



Which of the four nuclei shown below is the most stable?

- A. helium (He) B. magnesium (Mg) C. calcium (Ca) D. mercury (Hg)

24. The mass defect for a carbon-12 nucleus is 0.0990u. What is the average binding energy per nucleon

- A. 8.01MeV B. 1.19MeV C. 92.2MeV D. D
7.68MeV

25. The most common isotope of calcium is ${}^{40}_{20}\text{Ca}$.

It has a nuclear mass of 39.9626 u.

- a) Calculate the mass defect of this isotope.
b) Calculate the binding energy in MeV.

26. a) Show that the mass defect in a helium nucleus (alpha particle) is 0.03039 u.

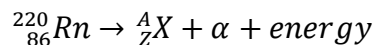
b) Calculate the binding energy per nucleon in MeV.

27. a) Write down an equation to show how helium is formed from hydrogen by fusion.

b) Explain the conditions needed for the reaction to occur.

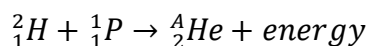
28. Calculate the mass of material that is converted to 5.0×10^{13} J.

29. A typical alpha decay is shown by the decay of radon-220:



- a) Give the values for A and Z for both X and α .
b) Calculate the energy released in MeV and J by 1 nucleus of radon.

30. In this reaction, a proton and a deuterium nucleus fuse to form a helium nucleus, releasing energy:



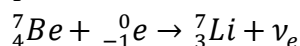
- a) Write down the isotope of helium formed, in isotope notation.
b) Calculate the energy given out by this interaction.

Data: mass of proton = 1.00728u

mass of deuterium = 2.01355u

mass of helium = 3.01493u.

31. Beryllium decays by electron capture to lithium. The equation is:



Calculate the energy change in this event.

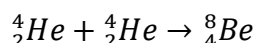
Data: mass of beryllium nucleus = 7.014727u

mass of lithium nucleus = 7.014356u

mass of electron = 0.000549u

mass of electron neutrino is negligible.

32. a. Beryllium-9 (Be) has a proton number of 4. Its nuclear mass is 9.01218 u.
1. Calculate the number of neutrons beryllium has.
 2. Calculate the mass defect of the beryllium nucleus.
- b. Beryllium-8 has a nuclear mass of 8.00531u.
1. Calculate the mass defect of a beryllium-8 nucleus.
 2. Show that the energy from the mass defect in a beryllium-8 nucleus is 54.48MeV.
 3. Calculate the binding energy per nucleon.
- c. The binding energy worked out from the mass defects of each helium nucleus is 28.31MeV. The two alpha particles fuse as shown by the equation:



1. Calculate the total binding energy of the two alpha particles.
2. Calculate the energy change as the reaction proceeds.

33. Use the data for the nuclear masses to answer this question.

proton mass = 1.00782u

neutron mass = 1.00867u

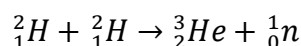
deuterium mass = 2.01355u

tritium mass = 3.01550u

helium-3 mass = 3.01493 u

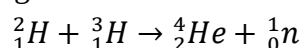
helium-4 mass = 4.00151 u

- a. This reaction involves the fusion of two deuterium nuclei:



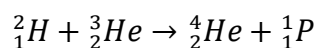
1. Show that the change in mass (mass defect) in this reaction is about 3.5×10^{-3} u.
2. Determine the energy in MeV that is emitted.

- b. Deuterium and tritium fuse to give a helium-4 nucleus and a neutron:



1. Show that the mass defect in this reaction is about 1.9×10^{-2} u.
2. Determine the energy in MeV that is emitted.

- c. Deuterium and helium-3 react to form helium-4 and a proton:



1. Show that the mass defect is about 0.02 u.
2. Determine the energy in MeV

- d. The total energy released in these reactions is about 43MeV. 6.0×10^{23} particles react.

1. Show that the total energy is about 43 MeV.
2. Calculate the total energy released in J.
3. Calculate the total mass that has been turned into energy.

34. a. In a certain fission reaction from U-235, the mass change is 3.1×10^{-28} kg for each nucleus.

1. Determine the energy equivalence to this mass change.
2. Calculate how many nuclei would be needed to give 1.0 J. Give your answer to an appropriate number of significant figures.

35. A nucleus written in isotope notation is ${}^A_ZX$. It has a mass M . If the mass of a proton is m_p and the mass of a neutron is m_n :

1. Write an expression to determine the mass defect Δm of the nucleus.
2. Write an expression to determine the binding energy per nucleon, E .

36. a. Uranium-235 has the following nuclide notation: ${}^{235}_{92}U$

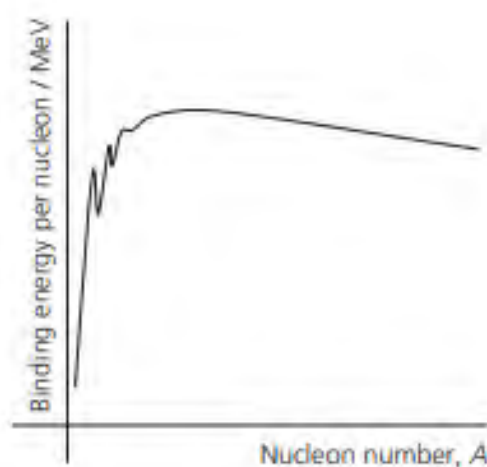
Use this data to calculate the binding energy per nucleon:

mass of U-235 nucleus = 235.044u

mass of proton = 1.00728u

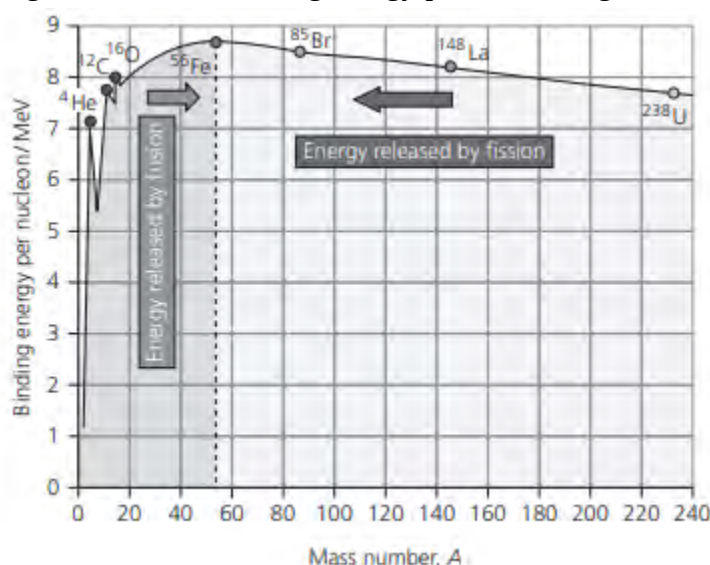
mass of neutron = 1.00867u.

b. The figure below shows the variation of binding energy per nucleon with the nucleon number, A .



1. State where uranium-235 and the fission fragments are on the graph. (No values are needed.)
2. Explain why fission energy is released.

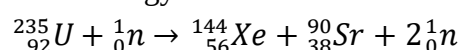
37. The figure shows the binding energy per nucleon against the number of nucleons.



a. Use the figure to determine the binding energy per nucleon for:

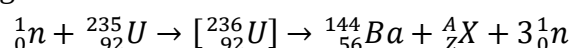
1. U-235
2. Sr-90
3. Xe-144.

b. Use the data to calculate the energy released for the reaction:



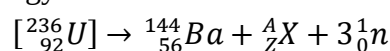
38. When a nucleus of uranium-235 undergoes fission, it mostly splits into two fragments. The fragments can be a range of different elements. The most commonly occurring fragments have nucleon numbers of about 95 and 137.

a. One such reaction gives:



Determine the proton number and the nucleon number of the isotope of the element X.

b. This reaction gives out energy:



The nuclear masses are shown in the table below.

Isotope	Nuclear mass/u
Uranium 236	235.995061
Barium 144	143.892211
X	88.898071
Neutron	1.00867

1. Calculate the change in mass given by the reaction.
2. Show that the energy emitted is about 170MeV.
- c. A nuclear power station has an output power of 1500MW. The power station is about 40% efficient.

1. Show that the amount of fissionable material used in a day is less than 5 kg per day,
assuming that 1 kg of the uranium fuel contains 6.8×10^{13} J.
 2. If the fuel contains 5% of fissionable material, determine the total amount of fuel that is needed each day.
39. The mass defect of a uranium-235 nucleus is 1.864557 u.
- a) Calculate the binding energy, in joules, of a uranium-235 nucleus. ($1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$)
 - b) Calculate the average binding energy (in MeV) per nucleon for uranium-235.
40. The following equation shows a nuclear reaction between two deuterium (${}^2_1\text{H}$) nuclei, to form helium-3:
- $${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n}$$
- a) State what type of nuclear reaction this is.
 - b) The binding energy per nucleon is 0 MeV for a neutron, approximately 1.11 MeV for a ${}^2_1\text{H}$ nucleus, and approximately 2.58 MeV for a ${}^3_2\text{He}$ nucleus. Use these values to estimate the energy released by this reaction.
41. The Sun radiates about $4 \times 10^{26} \text{ W}$ and has a mass of $1.99 \times 10^{30} \text{ kg}$. Astronomers estimate that the Sun can convert only the innermost 10% of its hydrogen into helium. 73% of the mass of the sun is hydrogen. Estimate how long the Sun can continue to shine at its present intensity.
42. One of the fusion reactions taking place in a star much more massive than the Sun, involves a nucleus of calcium undergoing fusion with a helium-4 nucleus to form a nucleus of titanium: ${}^{40}_{20}\text{Ca} + {}^4_2\text{He} \rightarrow {}^{44}_{22}\text{Ti} + \text{energy}$
Determine the energy released in this fusion reaction. Binding energy per nucleon data is shown in the table.

Nucleus	Binding energy per nucleon / MeV
${}^{40}_{20}\text{Ca}$	8.551
${}^4_2\text{He}$	7.073
${}^{44}_{22}\text{Ti}$	8.533

Chapter 25

Energy Levels

According to the nuclear model of an atom, the orbiting electrons cannot occupy any possible orbit around the nucleus. Different orbits correspond to different amounts of energy, or energy levels, and the electrons are restricted to orbits with specific energies. Electrons change energy so that they can jump from one energy level to another, but they can only occupy allowed energy levels. Although you may ask what makes an allowed orbit or allowed energy level, it takes an understanding of the wave nature of electrons and quantum mechanics to attempt to explain this consistently, but for now it is better to accept that this is how nature operates.

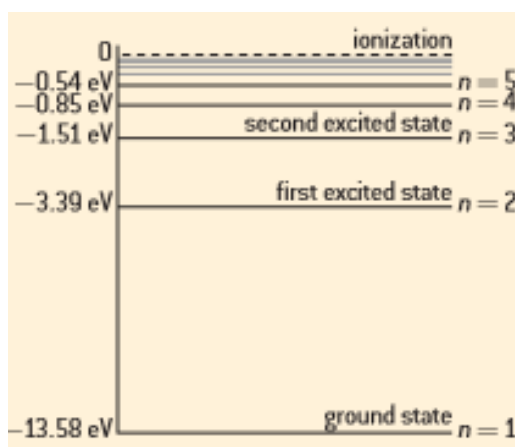
The levels can be represented by horizontal lines, arranged one above the other to form an energy level diagram (or a ladder, of unequally spaced rungs), each line indicating by its position a particular energy value. Every *atom* has a characteristic set of energy levels. While an electron is permitted to pass from one level to another by gaining or losing energy, it is not allowed to have an amount of energy that would put it between two levels. An atom can thus only accept 'parcels' of energy of certain definite sizes, i.e., its energy is **quantized**.

Hydrogen is the simplest of all elements, with the hydrogen atom normally consisting of a proton bound to a single electron by the electromagnetic force (an attractive force between oppositely charged particles). If a hydrogen atom completely loses the electron it becomes a positively-charged hydrogen ion. A common visualization of the atom is the “planetary model” in which the electron orbits the proton mimicking the Earth orbiting the Sun. Despite weaknesses in this model, it still gives a good introduction to energy levels in the atom.

The energy levels for the hydrogen atom are shown in the table below:

Energy level(n)	Energy/eV
1	-13.58
2	-3.39
3	=1.51
4	=0.85
5	-0.54

The energy levels are usually shown diagrammatically as in figure.



Energy Level diagram for hydrogen

This diagram shows the energy level and the energy of an electron that occupies this level. An electron in the ground state of the hydrogen atom must have exactly -13.58 eV of energy; one in the second energy level must have -3.39 eV , etc. An atom with an electron occupying an energy level higher than the ground state is said to be excited; so, an electron in level 2 is in the first excited state, one in level 3 is in the second excited state, etc. The energy levels in atoms are said to be quantized – which means they must have discrete finite values. An electron in a hydrogen atom cannot have -5.92 eV or -10.21 eV or any other value between -13.58 and -3.39 eV ; it must have one of the values in the table.

Band Theory

A fundamental result from basic modern physics is that atoms are characterized by discrete energy levels. What happens to this energy level model when we assemble many atoms together into a solid?

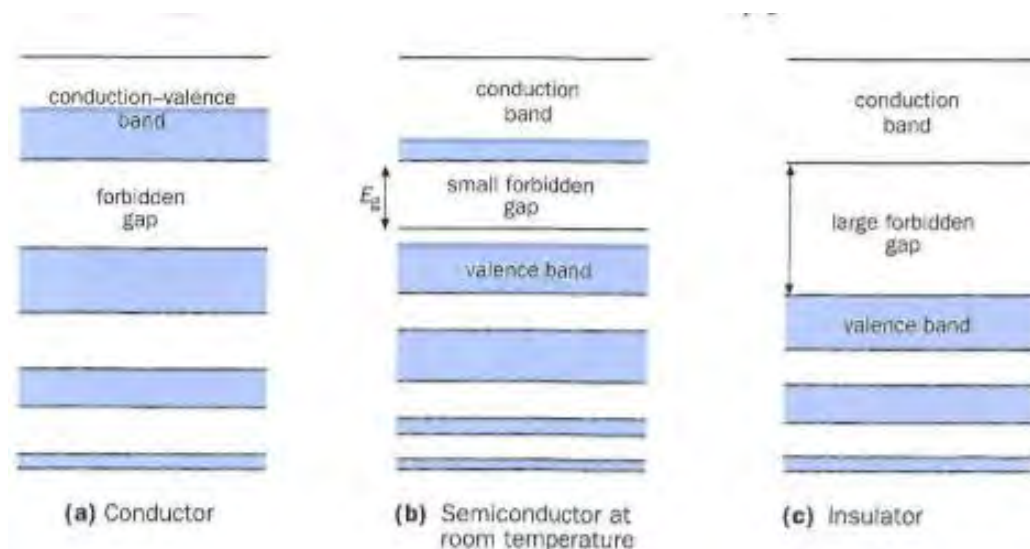
As the atoms are brought together, as in solid, electron waves from different atoms overlap and each single energy level broadens into a band of allowed energy levels which is shared by all the atoms. Energy bands are separated by gaps or forbidden regions where there are no allowed energy levels.

The crucial aspect of a good conductor is that the highest energy band (containing electrons) is only partially filled. When a potential difference is applied across the metal, electrons can respond by accelerating and increasing their energy, since there are plenty of unoccupied states of slightly higher energy available. The characteristic of all good conductors is that the highest energy band is only partially filled, or two bands overlap so that unoccupied states are available.

In a material that is a good insulator, on the other hand, the highest band containing electrons, called the valence band, is completely filled. The next highest energy band, called the conduction band, is separated from the valence band by a "forbidden" **energy gap** (or **band gap**), E_g of typically 5 to 10 eV. So, at room temperature (300 K), where thermal energies (that is, average kinetic energy) are on the order of $\frac{3}{2}kT \approx 0.04 \text{ eV}$, almost no electrons can acquire the 5 eV needed to reach the conduction band. When a

potential difference is applied across the material, no available states are accessible to the electrons, and no current flows. Hence, the material is a good insulator.

The figure below compares the relevant energy bands (a) for conductors, (b) semiconductors, and (c) for the insulators.



The bands for a pure (or intrinsic) semiconductor, such as silicon or germanium, are like those for an insulator, except that the unfilled conduction band is separated from the filled valence band by a much smaller energy gap, E_g , typically on the order of 1 eV. At room temperature, a few electrons can acquire enough thermal energy to reach the conduction band, and so a very small current may flow when a voltage is applied. At higher temperatures, more electrons have enough energy to jump the gap. So, the resistivity of semiconductors can decrease with increasing temperature. But, this is not the whole story of semiconductor conduction. When a potential difference is applied to a semiconductor, the few electrons in the conduction band move toward the positive electrode. Electrons in the valence band try to do the same thing, and a few can because there are a small number of unoccupied states which were left empty by the electrons reaching the conduction band. Such unfilled electron states are called holes. Each electron in the valence band that fills a hole in this way as it moves toward the positive electrode leaves behind its own hole, so the holes migrate toward the negative electrode. As the electrons tend to accumulate at one side of the material, the holes tend to accumulate on the opposite side.

Points to be remembered

In an intrinsic semiconductor at zero kelvin (0K),

- the valence band is fully occupied;
- the conduction band is empty; and
- the forbidden band has a width of about 1 eV.

For an intrinsic semiconductor above zero kelvin (0K),

- some electrons are able to jump the forbidden band to enter the conduction band;
- electrons in the conduction band act as negative charge carriers;
- the vacancies left behind in the valence band by the electrons entering the conduction band are known as holes;

- holes behave as positive charge carriers in the valence band;
- lattice vibrations increase as the temperature rises;
- any effect due to increase in the number of charge carriers outweighs any effect due to increase in lattice vibrations overall, resistance decreases as temperature rises; and
- the number density, the number per unit volume for electrons in the conduction band, is equal to the number density of holes in the valence band.

In a metal;

- the valence band overlaps the conduction band,
- the number of charge carriers does not increase as the forbidden band does not exist, and
- lattice vibrations increase as the temperature rises and hence resistivity increases as temperature increases.

In an insulator;

- the valence band is fully occupied,
- the conduction band is empty,
- the forbidden band is very wide, and
- the insulator does not conduct as free electrons (electrons in conduction band) are not available.

Light-dependent Resistors

Thermal energy is not the only form of energy that can cause electrons in the valence band to move to the conduction band. Semiconductor materials can be produced with a forbidden band such that photons of light promote electrons from the valence band to the conduction band. Increase in light intensity gives a rise to an increase in the number of photons per unit time and an increase in the number of electrons promoted to the conduction band. Thus, the resistance of these materials will decrease as the intensity of light increases - the light-dependent resistor (LDR).

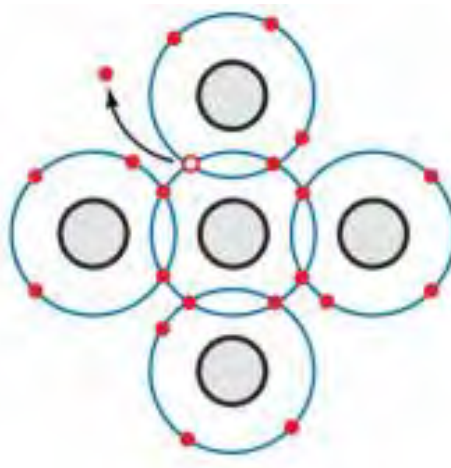
Semiconductors

The best conductors (silver, copper, and gold) have one valence electron, whereas the best insulators have eight valence electrons. A semiconductor is an element with electrical properties between those of a conductor and those of an insulator. As you might expect, the best semiconductors have four valence electrons. Examples: Germanium, Silicon

The Hole

The ambient temperature is the temperature of the surrounding air. When the ambient temperature is above absolute zero (-273°C), the heat energy in this air causes the atoms in a silicon crystal to vibrate. The higher the ambient temperature, the stronger the mechanical vibrations become. When you pick up a warm object, the warmth you feel is the effect of the vibrating atoms.

In a silicon crystal, the vibrations of the atoms can occasionally dislodge an electron from the valence orbit. When this happens, the released electron gains enough energy to go into a larger orbit, as shown in the figure below. In this larger orbit, the electron is free.



But, that is not all. The departure of the electron creates a vacancy in the valence orbit called a hole (See the figure above.). This hole behaves like a positive charge because the loss of the electron produces a positive ion. The hole will attract and capture any electron in the immediate vicinity. The existence of holes is the critical difference between conductors and semiconductors. Holes enable semiconductors to do all kinds of things that are impossible to do with the conductors.

At a room temperature, thermal energy produces only a few holes and free electrons. To increase the number of holes and free electrons, it is necessary to dope the crystal.

Points to remember

A hole and an electron each possess a charge of 1.6×10^{-19} C, but of opposite polarity.

Main Ideas

At any instant, the following is taking place inside a silicon crystal:

1. some free electrons and holes are being created by thermal energy,
2. other free electrons and holes are recombining,
3. some free electrons and holes exist temporarily, awaiting recombination.

Intrinsic Semiconductors:

An intrinsic semiconductor is a pure semiconductor. A silicon crystal is an intrinsic semiconductor if every atom in the crystal is a silicon atom. At room temperature, a silicon crystal acts like an insulator because it has only a few free electrons and holes produced by thermal energy.

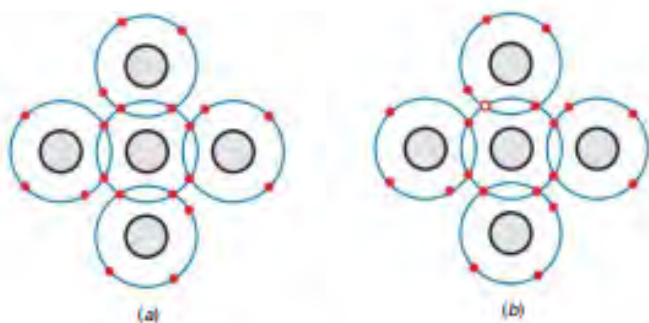
Doping a Semiconductor

One way to increase the conductivity of a semiconductor is by doping. This means adding impurity atoms to an intrinsic crystal to alter its electrical conductivity. A doped semiconductor is called an extrinsic semiconductor.

Increasing the Free Electrons

How does a manufacturer dope a silicon crystal? The first step is to melt a pure silicon crystal. This breaks the covalent bonds and changes the silicon from a solid to a liquid. To increase the number of free electrons, pentavalent atoms are added to the molten silicon. Pentavalent atoms

have five electrons in the valence orbit. Examples of pentavalent atoms include arsenic, antimony, and phosphorus. Because these materials will donate an extra electron to the silicon crystal, they are often referred to as donor impurities.



The figures above show how the doped silicon crystal appears after it cools down and re-forms its solid crystal structure. A pentavalent atom is in the center, surrounded by four silicon atoms. As before, the neighboring atoms share an electron with the central atom. But this time, there is an extra electron left over. Remember that each pentavalent atom has five valence electrons. Since only eight electrons can fit into the valence orbit, the extra electron remains in a larger orbit. In other words, it is a free electron.

Each pentavalent or donor atom in a silicon crystal produces one free electron. This is how a manufacturer controls the conductivity of a doped semiconductor. The more impurity that is added, the greater the conductivity. In this way, a semiconductor may be lightly or heavily doped. A lightly doped semiconductor has a high resistance, whereas a heavily doped semiconductor has a low resistance.

Increasing the Number of Holes

How can we dope a pure silicon crystal to get an excess of holes? By using a trivalent impurity, one whose atoms have only three valence electrons. Examples include aluminum, boron, and gallium.

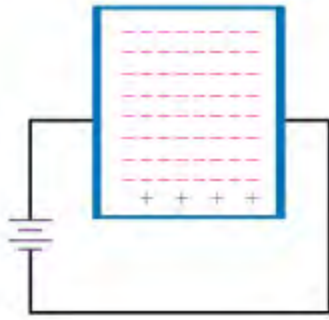
Figure b shows a trivalent atom in the center. It is surrounded by four silicon atoms, each sharing one of its valence electrons. Since the trivalent atom originally had only three valence electrons and each neighbor shares one electron, only seven electrons are in the valence orbit. This means that a hole exists in the valence orbit of each trivalent atom. A trivalent atom is also called an acceptor atom because each hole it contributes can accept a free electron during recombination.

Two Types of Extrinsic Semiconductors

A semiconductor can be doped to have an excess of free electrons or an excess of holes. Because of this, there are two types of doped semiconductors.

N-Type Semiconductor

Silicon that has been doped with a pentavalent impurity is called an n-type semiconductor, where the n stands for negative. The figure below shows an n-type semiconductor. Since the free electrons outnumber the holes in an n-type semiconductor, the free electrons are called the majority carriers and the holes are called the minority carriers.

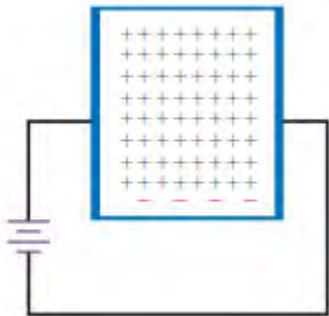


Because of the applied voltage, the free electrons move to the left and the holes move to the right. When a hole arrives at the right end of the crystal, one of the free electrons from the external circuit enters the semiconductor and recombines with the hole.

The free electrons shown in the figure above flow to the left end of the crystal, where they enter the wire and flow on to the positive terminal of the battery.

P-Type Semiconductor

Silicon that has been doped with a trivalent impurity is called a p-type semiconductor, where the p stands for positive. The figure below shows a p-type semiconductor. Since holes outnumber free electrons, the holes are referred to as the majority carriers and the free electrons are known as the minority carriers.



Because of the applied voltage, the free electrons move to the left and the holes move to the right. In the figure above, the holes arriving at the right end of the crystal will recombine with free electrons from the external circuit. There is also a flow of minority carriers in the figure. The free electrons inside the semiconductor flow from right to left. Because there are so few minority carriers, they have almost no effect in this circuit.

Exercise

- Barrier potential difference is the potential difference across:
 - Terminals of a cell
 - Depletion layer
 - Capacitor plates
 - Ends of a conductor
- Transformer is a device to:
 - Convert mechanical energy to electrical energy.
 - Convert AC to DC
 - Convert DC to AC
 - Transform AC voltage
- AC is converted to DC by:
 - Transformer
 - Rectifier
 - Diode
 - Triode
- When pure germanium is doped with trivalent impurity like aluminum; the conduction is due to:
 - electrons
 - Holes
 - Protons
 - Positrons
- If germanium is doped with donor impurity, then donor atoms should be
 - tetravalent
 - pentavalent
 - trivalent
 - none of these
- A sample on n-type silicon;
 - contains an excess of free electrons and therefore it is negatively charged
 - contains an excess of free electrons and it is electrically neutral
 - predominantly contains trivalent impurities
 - contains only tetravalent impurities.
- When arsenic is added as an impurity to silicon, the resulting material is
 - n-type conductor
 - n-type semiconductor
 - p-type conductor
 - p-type semiconductor
- Three semi-conductors are arranged in the increasing order of their energy gap as follows. The correct arrangement is:
 - Tellurium, Germanium, Silicon
 - Tellurium, Silicon, Germanium
 - Silicon, Germanium, Tellurium
 - Silicon, Tellurium, Germanium
- A p-type semiconductor is
 - a silicon crystal doped with arsenic impurity
 - a silicon crystal doped with antimony impurity.
 - a germanium crystal doped with boron impurity
 - a germanium crystal doped with phosphorus impurity
- The current through a p-n junction is negligible when the junction is
 - forward biased
 - reverse biased
 - positive biased
 - negative biased
- Silicon is a semi-conductor. On adding a small quantity of arsenic in it, its conductivity
 - increases
 - decreases
 - remains the same
 - becomes zero
- Indium impurity is mixed in a silicon specimen. If on the average, one indium atom per 10^8 silicon atoms is doped and the number density of atom in the silicon specimen is 5×10^{28} atoms/ m^3 then the number and nature of atoms in silicon per cubic meter will be
 - 5×10^{20} acceptor
 - 5×10^{20} donor
 - 5×10^{36} acceptor
 - 5×10^{36} donor

Answers

1. b	2. d	3. b	4. b	5. b	6. b	7. b	8. a	9. c
10.b	11.a	12.a						

13. Base on the band theory of solids how which yours distinguish between conductor and semiconductor?
14. How is p- type semiconductor formed? Explain.
15. When p and N type materials are interface there exists a depletion layer at the interface. Why?
16. How does conductivity of semiconductor vary with temperature? Explain.
17. The conductivity of an intrinsic semiconductor is low, why? (University physics)
18. Why is a semi conductor virtually an insulator at room, temperature? (University physics)
19. What do you meant by doping?
20. What do you meant by forbidden energy gap?
21. What is extrinsic semiconductor? Explain the formation of potential barrier and depletion layer in p-n junction.

Chapter 26

Solids

26.1 Particles and Antiparticles

The positron is basically a positive electron. That is, many of its properties are the same as for the electron, such as mass, but it has the opposite charge. Other quantum numbers are also reversed for antiparticles. The positron is said to be the antiparticle to the electron. After the positron was discovered in 1932, it was predicted that other particles also ought to have antiparticles. In 1955 the antiparticle to the proton was found, the antiproton ($\bar{p}$), which carries a negative charge. (The bar over the p is used to indicate antiparticle.) Soon after, the antineutron ($\bar{n}$) was found. All particles have antiparticles. But, a few, like the photon and the π_0 , do not have distinct antiparticles—we say that they are their own antiparticles!

Antiparticles are produced in nuclear reactions when sufficient energy is available, and they do not live very long in the presence of matter. For example, a positron is stable when by itself; but if it encounters an electron, the two annihilate each other. The energy of their vanished mass, plus any kinetic energy they possessed, is converted into the energy of γ rays or of other particles. Annihilation also occurs for all other particle—antiparticle pairs.

There are possibly only two classes of ‘particles’ in the universe – Fermions and Bosons.

All elementary particles (Quarks, Leptons, Gauge Bosons, etc.) will fall under either of these two classes.

26.2 Fundamental Particles

All matter around us is made of elementary particles, the building blocks of matter.

Protons, neutrons and electrons were thought of as fundamental particles, which could not be subdivided further. However, in the middle decades of the 20th century, physicists discovered many other particles that did not fit this pattern. They gave them names such as pions, kaons, muons and so on, using up most of the letters of the Greek alphabet.

These new particles were found in two ways:

- by looking at cosmic rays, which are particles that arrive at the Earth from outer space
- by looking at the particles produced by high-energy collisions in particle accelerators

More than half a century of painstaking experimental and theoretical work has resulted in what is believed today to be the complete list of the fundamental particles of nature.

A particle is called elementary/fundamental if it is not made out of any smaller component particles.

These particles occur in three basic types; quarks, leptons and exchange particles . The quarks, the leptons and the exchange particles (bosons) are the fundamental particles.

Production of Elementary Particles:

Fundamental particles are not permanent entities. They can be created and destroyed. Different elementary particles are produced by different methods.

For instance, electrons are easiest to produce. Simply, heat a piece of metal sufficiently and electrons come boiling off. To obtain protons, hydrogen gas can be ionized.

Besides these, there are three main sources to observe and create elementary particles:

1. Cosmic rays.
2. Particle accelerators.
3. Nuclear reactors.

Fermions-Bosons:

Not only elementary particles but also composite particles like Baryons (Eg: Protons, Neutrons, etc.) will also fall under this basic classification of all particles into fermions and bosons.

All fermions have half-integer multiple spins (i.e. $1/2$, $3/2$, $5/2$...). Fermions are subject to the Pauli Exclusion Principle which states that no particle can exist in the same state in the same place at the same time. Fermions obey Fermi–Dirac statistics. Fermions are usually associated with matter while Bosons are the force carriers.

Exemples of Fermions:

Leptons (Electrons, Neutrinos, etc), Quarks (Up, Down, etc.), Baryons (Protons, Neutrons, etc.)

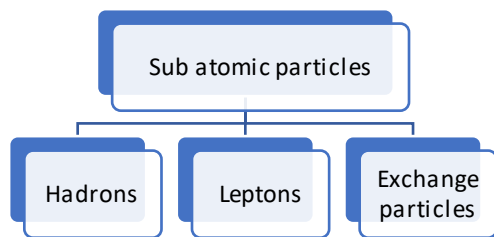
All bosons have either zero spin or an even integer spin. Bosons are gregarious. Bosons may occupy the same quantum state as other bosons. Bosons obey Bose–Einstein statistics. Bosons are the force carriers.

Examples of bosons include fundamental particles such as photons, gluons, and W and Z bosons (the four force-carrying gauge bosons), the Higgs boson; and composite particles (e.g. mesons).

Classification of Particles:

The subatomic particles that make up our universe can be divided into three main categories:

- hadrons — these particles ‘feel’ a strong nuclear interaction (force)
- leptons — these particles ‘feel’ a weak nuclear interaction
- exchange particles



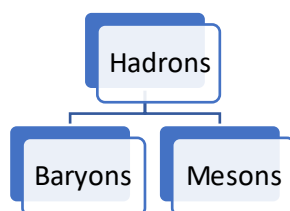
These three categories are further divided as follows:

Hadrons are composed of two or three quarks:

Baryons — protons, neutrons (each composed of three quarks)

Mesons — pions, kaons (each composed of two quarks)

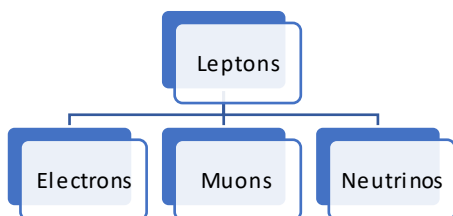
(Every particle has its own antiparticle.)

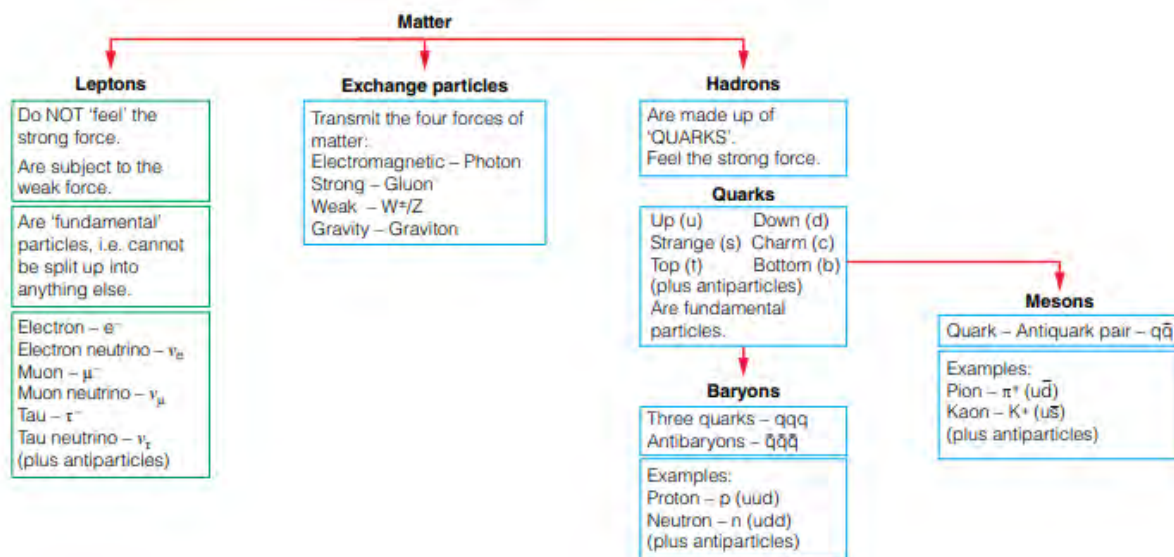


Leptons — electrons, muons, neutrinos — subject to the weak interaction.

(Every particle has its own antiparticle.)

Leptons do not contain quarks. They themselves are considered to be fundamental particles.



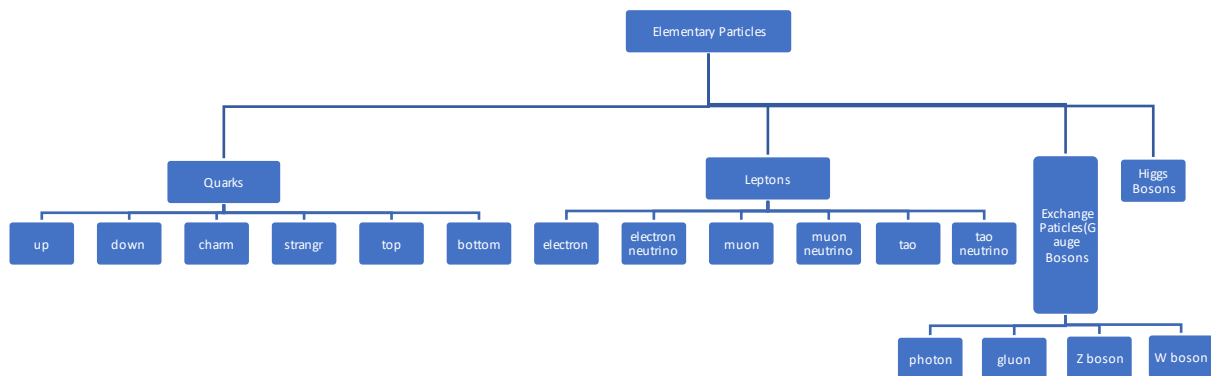


26.3 Quarks Quarks were first proposed by two physicists working independently: Murray Gell-Mann and Georg Zweig .

There are six different types or flavours of quarks. The six flavours are the up (u), charm (c) and top (t) quarks with electric charge $\frac{2}{3}e$, and the down (d), strange (s) and bottom (b) quarks with electric charge $-\frac{1}{3}e$.

The top and bottom quarks are alternatively called truth and beauty.

There is solid experimental evidence for the existence of all six flavours of quarks. In addition, we have the anti-particles of each of these. These have the same mass but all other properties are opposite, for example, electric charge. Anti-particles are denoted with a bar on top of the symbol for the name. We have already met two anti-particles: the anti-neutrino in beta minus decay, and the positron in beta plus decay. Quarks are never found as free particles. They are always combined together. Particles composed of the combination of quarks are called hadrons. Hadrons are particles that experience both the weak and strong nuclear force. Quarks combine in just two ways to form hadrons.



A hadron is a particle made out of quarks.

- When three quarks combine, they form a baryon. (When three antiquarks combine they form an anti-baryon.)
- When a quark combines with an anti-quark, they form a meson.
- However, experiments in 2003 produced strong evidence that the recently discovered theta particle (θ^+) consists of five quarks: two up quarks, two down quarks, and an antistrange quark.

The proton is a baryon made out of two u quarks and one d quark, uud. The neutron is another baryon made out of two d quarks and one u quark, ddu.

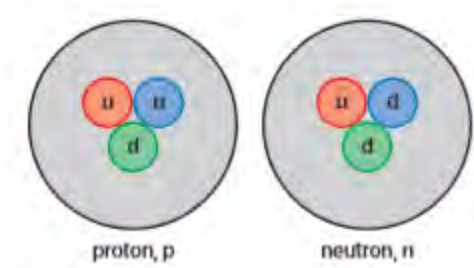


Fig: a representation of the proton and neutron in terms of quarks.

The electric charge of the proton is thus predicted to be:

$$Q_p = +\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = e$$

and that of the neutron is predicted to be:

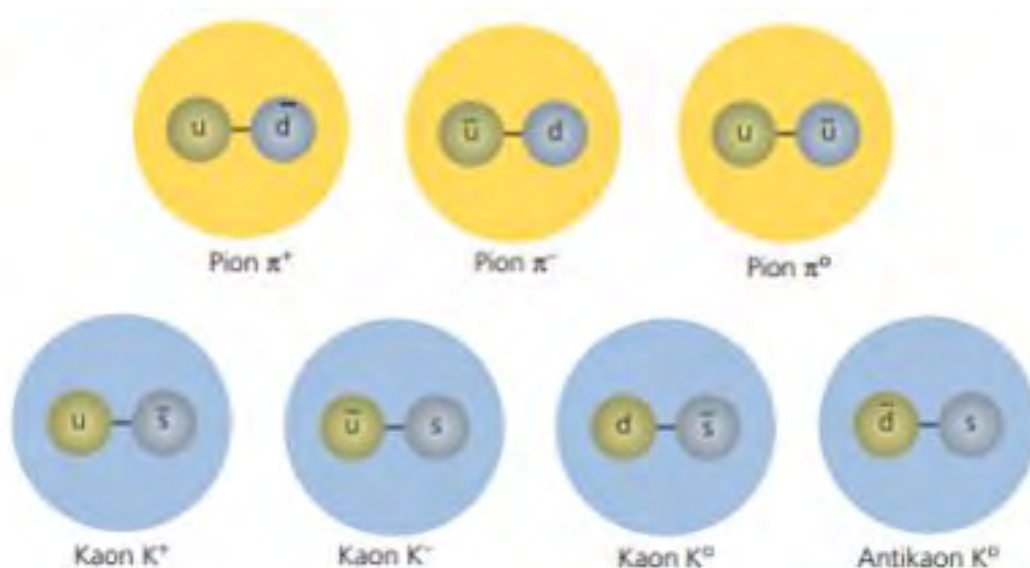
$$Q_n = -\frac{1}{3}e - \frac{1}{3}e + \frac{2}{3}e = 0$$

These are, of course, the correct values.

Pions are examples of mesons. The positively charged pion (π^+ meson) is made up as follows:

$$\pi^+ = (u\bar{d})$$

The bar over 'd' shows this is an anti-particle. Thus, the positive pion is made out of a u quark and the anti-particle of the d quark (the d antiquark).



Apart from electric charge, quarks have another property called baryon number, B. Each quark is assigned a baryon number of $+\frac{1}{3}$ and each antiquark a baryon number of $-\frac{1}{3}$. To find the baryon number of the hadron that is formed by quarks, just add the baryon numbers of the quarks in the hadron. For example:

$$u\bar{c}t \text{ baryon number} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1 \text{ (a baryon)}$$

$$u\bar{d} \text{ baryon number} = +\frac{1}{3} - \frac{1}{3} = 0 \text{ (a meson)}$$

Since all baryons are made from three quarks, all baryons have baryon number +1. All anti-baryons have baryon number -1, and all mesons have baryon number 0.

Quarks interact with the strong nuclear interaction, the weak nuclear interaction, and the electromagnetic interaction.

In all reactions, electric charge and baryon number are conserved, i.e., they have the same value before and after the reaction.

Properties of Quarks

Quark flavour	Charge, Q
u (up)	$+\frac{2}{3}e$
d (down)	$-\frac{1}{3}e$
s (strange)	$-\frac{1}{3}e$
c (charm)	$+\frac{2}{3}e$
b (bottom or beauty)	$-\frac{1}{3}e$
t (top or truth)	$+\frac{2}{3}e$

Workedout Example

1. Find the charge for the following quark combinations:

(a) udd (neutron)

$$Q = \left(+\frac{2}{3}\right)e + \left(-\frac{1}{3}\right)e + \left(-\frac{1}{3}\right)e = 0$$

(b) uud (proton)

$$Q = \left(+\frac{2}{3}\right)e + \left(+\frac{2}{3}\right)e + \left(-\frac{1}{3}\right)e = (+1)e$$

(c) $u\bar{u}$ (π^0 pion)

$$Q = \left(+\frac{2}{3}\right)e + \left(-\frac{2}{3}\right)e = 0$$

(d) uss (Σ^0 sigma)

$$\left(+\frac{2}{3}\right)e + \left(-\frac{1}{3}\right)e + \left(-\frac{1}{3}\right)e = 0$$

(e) sdd (Σ^- sigma)

$$\left(-\frac{1}{3}\right)e + \left(-\frac{1}{3}\right)e + \left(-\frac{1}{3}\right)e = (-1)e$$

(f) $b\bar{u}$ (B^- meson)

$$Q = \left(-\frac{1}{3}\right)e + \left(-\frac{2}{3}\right)e = (-1)e$$

2. One possible outcome when two protons interact is:

$$p + p \rightarrow p + n + \pi^+.$$

Analyse this reaction in terms of the charge and deduce the quark structure of the π^+ .

Solution:

Left-hand side of equation $p + p$ has charge $(+e) + (+e) = +2e$.

$\therefore$ Right-hand side $p + n + \pi^+$ has zero charge from n so must have $+2e$ from p and π^+ .

As p has charge $+e$, π^+ must also have charge $+e$ for charge to be conserved.

Writing the known quarks in brackets: $p + p \rightarrow p + n + \pi^+$ becomes:

$$p(uud) + p(uud) \rightarrow p(uud) + n(udd) + \pi^+(q\bar{q})$$

as π^+ is a meson and must consist of a quark and an antiquark.

$\therefore$ The meson must be a $u\bar{d}$ meson, of charge $\left(+\frac{2}{3}e\right) + \left(+\frac{1}{3}e\right) = +e$, as needed.

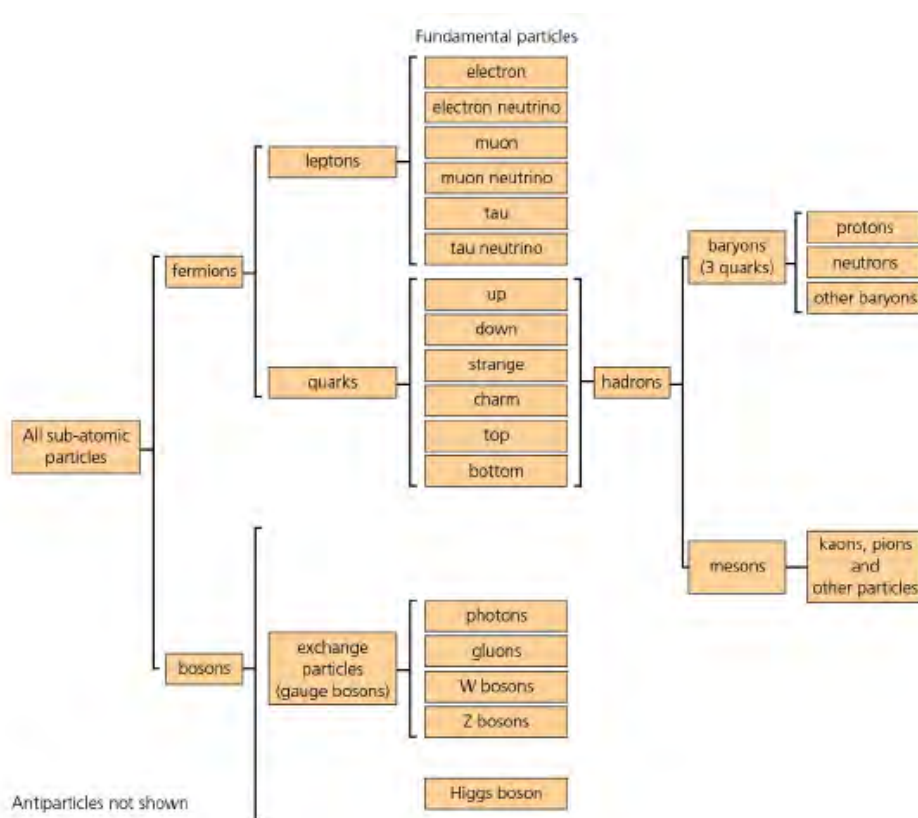
The quarks are thus: $uud + uud \rightarrow uud + udd + u\bar{d}$

26.4 Leptons

Leptons are particles that can travel on their own, meaning that they are not trapped inside larger particles.

Particles that are acted on by the weak nuclear force not by the strong nuclear force are called leptons. There are six types of lepton. These are the electron and its neutrino, the muon and its neutrino, and the tau and its neutrino. They are denoted by e^- , ν_e , μ^- , ν_μ and τ^- , ν_τ . The muon is heavier than the electron, and the tau is heavier than the muon. There is now conclusive evidence that neutrinos have a very small non-zero mass. There is solid experimental evidence for the existence of all six leptons. In addition, we have the anti-particles of these. All leptons interact with the weak nuclear interaction. Those that have electric charge ($\bar{e}$, $\bar{\mu}$, and $\bar{\tau}$) also interact with the electromagnetic interaction.

Leptons (and only leptons) are assigned a new quantum number called family lepton number. There is an electron, muon and tau lepton number. The family lepton number is conserved in all interactions.



26.5 Cosmology

Cosmology is the study of the universe – how it began, how it developed, and what will happen to it in the future. It has always been the nature of many individuals, societies, and cultures to wonder what lies beyond the Earth. The fact that the Sun and the stars appear to revolve around

the Earth led early civilizations, understandably but wrongly, to believe that a stationary Earth was the centre of everything. This belief was often fundamental to their religions. Indeed, even today some people still believe from their everyday observations, or their religious beliefs, that the Sun orbits around the Earth rather than the other way around.

26.6 Models of the Universe

26.6.1 The Newtonian Model of the Universe

In the Newtonian model of the universe, the Earth, the Sun and the planets were just tiny parts of an infinitely large and unchanging (static) universe that had always been the way as it is, and would always be the same. In this model, the universe, on the large scale, is more or less the same everywhere. In other words, it is uniform with stars approximately evenly distributed. Newton reasoned that unless all of these assumptions (sometimes called postulates) were valid, then gravitational forces would be unbalanced, resulting in the movement of stars (which were thought to be stationary at that time). But, there is a big problem with this Newtonian model of the universe. If the universe is infinite and contains an infinite number of stars, there should be no such thing as a dark sky at night, because light from the stars should be arriving from all directions at all times. (This is known as Olbers's paradox, named after one of the leading astronomers of the nineteenth century, Heinrich Wilhelm Olbers. A paradox is an apparently true statement that seems to contradict itself. 'I always lie' is a widely quoted paradoxical statement.) It was clear that either the reasoning given above and/or the Newtonian model of the universe needed changing or rejecting. Since the mid to late 1960s, the Big Bang model of the universe has been widely accepted by astronomers, and has solved Olbers's paradox.

26.6.2 The Big Bang Model

The Big Bang model is the current theory about how the universe began at one precise moment in time, 13.8 billion years ago. The Hot Big Bang model is a theory that describes the origin and expansion of the universe, and is supported by extensive experimental evidence. Before looking more closely at this theory, we will first consider the evidence for an expanding universe.

26.7 The Expanding Universe: Redshift and Hubble's Law

The universe as a whole: is it static, or does it change? One of the most important scientific results of the twentieth century was that distant galaxies are racing away from us, and that the farther they are from us, the faster they are moving away. How astronomers arrived at this astonishing idea, and what it means for - the past history of the universe as well as its future, will be discussed here.

The idea that the universe is expanding was first put forth by Edwin Hubble in 1929. This idea was based on distance measurements of galaxies, and the determination of their velocities by the Doppler shift of spectra lines in the light received from them. As we are familiar with how the frequency and wavelength of sound are altered if the source is moving toward or away from an observer. If the source moves toward us, the frequency is higher, and the wavelength is shorter. If the source moves away from us, the frequency is lower, and the wavelength is longer.

The amount of redshift, z , allows us to calculate how fast it is moving. This can be done by using measurements of either frequency or wavelength changes:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$$

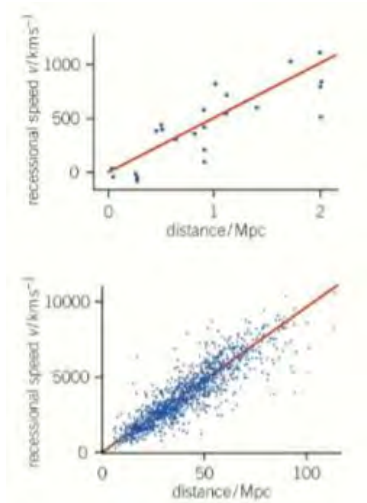
26.8 Hubble's Law

During the late 1920s Hubble analysed the Doppler shift in the absorption spectra of many distant galaxies. Using the data available then he made two key observations:

1. He confirmed earlier observations that the light from the vast majority of galaxies was red-shifted, that is, they had a relative velocity away from the Earth.
2. He found that in general, the further away the galaxy was the greater the observed red shift, and the faster the galaxy was moving.

Using these observations, he formulated what is now called Hubble's law: the recessional speed v of a galaxy is almost directly proportional to its distance d from the Earth, that is, $v \propto d$.

On a graph of recessional speed against distance for all galaxies, the plotted data should produce a straight line through the origin. As we can see in the figure, the spread in the original data points suggests that Hubble's law is valid, but there is a large uncertainty in the value for the gradient of the best-fit line.



The Hubble Constant

The gradient of the graphs in the figure is a constant of proportionality now called the Hubble constant H_0 . From Hubble's law it follows that:

$$v = H_0 d$$

The SI unit for the Hubble constant can be determined by dividing ms^{-1} by m (the SI units). This gives the unit s^{-1} . However, as you see in the figure, cosmologists prefer to express speed in kms^{-1} , and distance in Mpc , giving an alternative unit of $\text{kms}^{-1} \text{Mpc}^{-1}$, which must be

equivalent to s^{-1} . In 2013, the most reliable data gave $67.80 \pm 0.77 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the Hubble constant, or about $2.2 \times 10^{-18} s^{-1}$.

26.9 The Expanding Universe

Hubble's law is the key evidence for the Big Bang theory and the model of the expanding universe following the Big Bang. This model is the most widely accepted explanation of the observation that the light from nearly all the galaxies we can see is red-shifted. It states that the fabric of space, and time, is expanding in all directions. It is not simply the galaxies moving away from each other, but the actual space itself expanding. As a result, any point, in any part of the universe, is moving away from every other point in the universe, and the further the points are apart the faster their relative motion will be away from each other. From our position on the Earth we can explain why light from more distant galaxies is more red-shifted, indicating that they are moving faster than those nearer to us.

26.10 The Cosmological Principle

The cosmological principle is the assumption that, when viewed on a large enough scale, the universe is homogeneous and isotropic, and the laws of physics are universal.

- i) The laws of physics can be applied across the universe. This is a bold assumption. It means that the theories and models tested here on the Earth can be applied to everything within the universe over all time and space.
- ii) 'Homogeneous' means that matter is distributed uniformly across the universe. For a very large volume, the density of the universe is uniform. This means that the same type of structures (galaxies) are seen everywhere.
- iii) 'Isotropic' means that the universe looks the same in all directions to every observer. It follows that there is no centre or edge to the universe.

In essence, the cosmological principle means that the universe would look the same wherever you are.

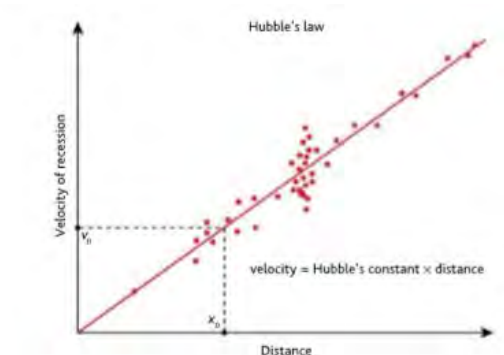
26.11 The Big Bang

Hubble's conclusion that the galaxies are moving further apart provides compelling evidence that they were once much closer together. According to the Standard Model, about 13.7 billion years ago the universe occupied a space smaller than the size of an atom. At that instant the entire universe exploded in a Big Bang, undergoing an immense expansion in which both time and space came into being. Starting off at a temperature of 10^{32} K , the universe rapidly cooled so that one second after the Big Bang it had fallen to 10^{10} K . In the time since the Big Bang, the universe has continued to cool to 2.7 K . In this time there has been an expansion of the fabric of space – the universe has not been expanding into a vacuum that was already there. As the galaxies move apart the space already between them becomes stretched. This is what we mean by “expansion” and why we believe the redshift occurs. The space through which the electromagnetic radiation travels is expanding and it stretches out the wavelength of the light. The further away the source of the light, the greater space becomes stretched, resulting in a more stretched-out wavelength and increasing the cosmological redshift – not to be confused with local redshift due to the Doppler Effect.

Cosmologists have spent a great deal of time considering what happened between the time that the Big Bang occurred and the present day. One of the key questions is “how did the universe appear in the distant past?” Hubble’s law certainly suggests that the galaxies were closer together than they are now and, logically, it follows that there must have been a point in time when they were all in the same place – that time being the Big Bang. It is possible to estimate the age of the universe using Hubble’s law.

How old is the Universe?

As all distant objects show a red shift, they are all moving away from us. This implies that the universe as a whole is expanding. If we imagine time running backwards from the present, then the universe would contract until a moment where everything was in the same place. This would be the time of the Big Bang, when everything first exploded outwards from that single point. Thus, if we can find the Hubble constant, it will tell us how quickly the Universe is expanding. From this we can work out when it all started (fig.).



For an object to travel a distance d_0 from the beginning of time, at a speed of v_0 , the time taken, T_0 , can be calculated from the basic equation for speed:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$v_0 = \frac{d_0}{T_0}$$

$$T_0 = \frac{d_0}{v_0}$$

If we consider the gradient of the Hubble graph, $H_0 = \frac{v_0}{d_0}$

$$T_0 = \frac{1}{H_0}$$

Note that in this calculation you should use the same units for the distance and for the length component of the units for recession velocity. Usually, the, Hubble constant is quoted in units of $\text{kms}^{-1} \text{Mpc}^{-1}$

$$H_0 = 71 \text{kms}^{-1} \text{Mpc}^{-1} = 71000 \text{ms}^{-1} \text{Mpc}^{-1}$$

$$1 \text{pc} = 3.08 \times 10^{16} \text{m}$$

$$1 \text{Mpc} = 3.08 \times 10^{22} \text{m}$$

$$H_0 = \frac{71000}{3.08 \times 10^{22}} = 2.31 \times 10^{-18} s^{-1}$$

$$So, T_0 = \frac{1}{H_0} = \frac{1}{2.31 \times 10^{-18}} = 4.33 \times 10^{17} s$$

This value for T_0 gives the age of the Universe as 13.7 billion years.

Big Distance Units

Astronomical distances are very large. Our nearest neighbour is the Moon, and yet that is nearly 400000000m away. The Sun is 150 000 000000m away, and the distance to the orbit of Pluto is 5 750 000000 000m. Measuring across space quickly generates very large values. Using standard form notation helps with this, but astronomers have defined a number of alternative distance units to cut down the magnitudes of the numbers involved. You must have heard of the light year. This is the distance that light can travel in one year, which is about 10^{16} m. We also use the astronomical unit (AU), which is the radius of the Earth's orbit around the Sun, $1AU = 1.5 \times 10^{11}$ m. We will also consider distances in parsecs.

Distances are often specified in terms of parallax angle given in second of arc: 1second (1") is $\frac{1}{60}$ of a minute (') of arc, which is $\frac{1}{60}$ of a degree, so $1'' = \frac{1}{3600}$ of a degree. Then the distance is specified in parsecs (meaning parallax angle in second of arc) where the *parsec (pc)* is defined as $\frac{1}{\phi}$ with ϕ seconds

If $\phi = (6 \times 10^{-5})^0 = 0.22''$ of arc then the *distance* $= \frac{1}{0.22''} = 4.5 pc$

$$1parsec(pc) = 3.08 \times 10^{16}m = 3.26 lightyear(ly)$$

26.12 Black Holes

We have seen that neutron stars have enormous densities and small radii. This means that the gravitational potential at the surface of such a star will also be enormous. The escape velocity from the surface of a planet or star is related to the gravitational potential at its surface. So, what would happen if the potential was such that the escape velocity was equal to the speed of light? It would in fact mean that no electromagnetic radiation could leave the surface of the star; it would have become a black hole.

The correct way in which to think of the formation of a black hole is in terms of the General Theory of Relativity. This theory predicts that space is warped by the presence of mass. Hence the path of light travelling close to large masses will be curved. Near to a black hole the space is so severely warped that the path of any light leaving the surface will be bent back in on itself. The General Theory also predicts that time slows in a gravitational field. At a point close to a black hole, where the escape velocity just equals the speed of light, time will cease. This point is known as the event horizon of the black hole. When a dying star contracts within its event horizon the entire mass of the star will shrink to a mathematical point at which its density will be infinite. Such a point is known as a singularity.

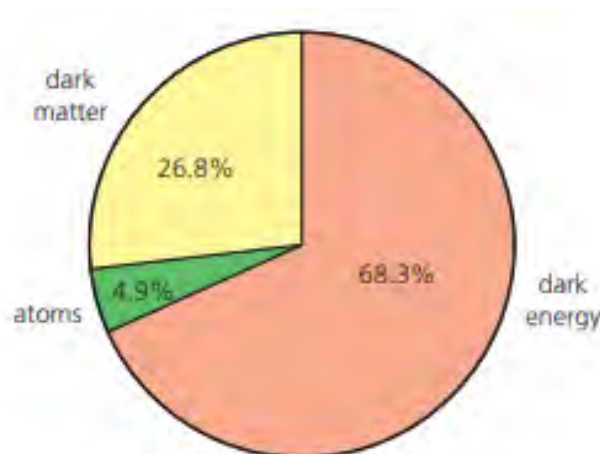
A black hole therefore consists of an event horizon and a singularity. So, do black holes exist? We cannot see a black hole, however, its existence might be inferred from the effect that its

gravitational field would have on its surroundings. The intense gravitational field of the black hole draws material from the supergiant and as this material spirals into the black hole it reaches a temperature at which it emits X-rays. Other potential candidates for black holes have been found and some theorists think that there is a black hole at the centre of all galaxies including our own Milky Way.

26.13 Dark Matter

Dark matter is the name given to the proposed matter that must be present in the universe, but which has never been detected directly because it neither emits nor absorbs radiation.

Dark matter is believed to be about five times more plentiful in the universe than observable matter (atoms), but dark energy is thought to be the dominant form of mass–energy in the universe (see Figure).



Many theories have been proposed about the nature of dark matter, including:

MACHOs – *massive astronomical compact halo objects*. These could be old stars or small ‘failed’ stars, very large planets or even black holes, any or all of which simply do not emit enough radiation for us to detect them. It is considered unlikely that there could be a sufficient number of such objects to explain dark matter fully.

WIMPs – *weakly interacting massive particles*. (Note that here ‘massive’ simply means ‘with mass’; it does not imply large individual masses.) There could be very large numbers of particles that we do not know about yet simply because they are very difficult to detect. Including...

Neutrinos – the masses of neutrinos are presumed to be very small, but they are still not known with any certainty. Neutrinos are present in the universe in vast numbers so they could contribute significant mass.

The 5% Universe

At the time of writing, attempts to measure distances in our universe has unearthed (an unfortunate pun) two problems:

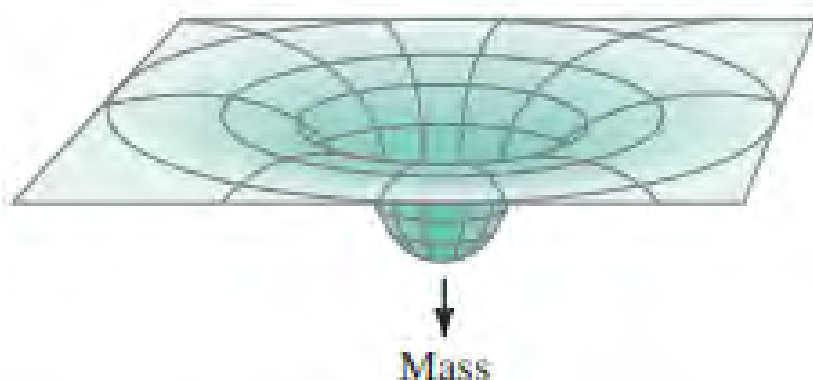
1. Nearby galaxies do not have enough visible gravitationally attracting (hadronic) matter to keep the outermost stars moving in circles around the galaxy's centre.
2. Distant galaxies appear to be moving away from the Solar system at an accelerating rate, rather than slowing down as predicted by Newton's (attractive) law of gravitation.

The first 'problem' suggests that about 24% of the mass of nearby galaxies consists of ... we do not know what. It is called dark matter. The second 'problem' suggests that about 71% of our universe consists of a mysterious 'antigravity' material known as dark energy. As 24% plus 71% add up to 95%, this means that, at the moment, we know nothing about 95% of our universe. A further issue that is concerning astrophysicists and cosmologists is that, possibly, our universe is only one of many multiverses, but any knowledge of other universes is denied to us.

Whatever dark matter consists of, it exerts a gravitational pull on ordinary matter (hadronic matter like you). Because of this, the total mass of gravitationally attracting matter within the orbit of a star as it moves around the edge of a galaxy can be calculated. There are a number of suggestions about what dark matter might consist of, but so far, no scientific theory to enable such suggestions to be tested. The position about dark energy is even more mysterious: theories relating to what happened in the very early universe and to Einstein's geometrical theory of gravitation are proposed, but the acceleration remains unexplained.

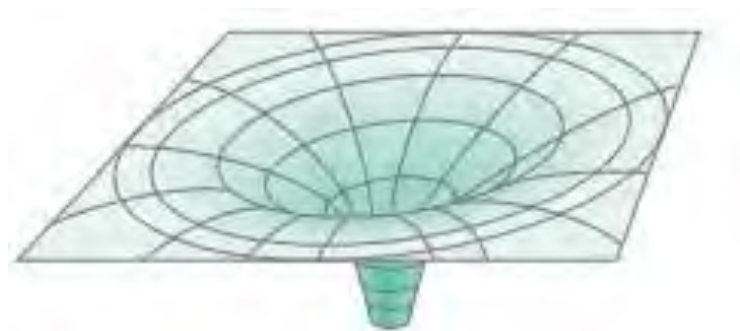
Black holes are massive, extremely dense objects from which matter and radiation cannot escape. Space-time inside the event horizon is so warped that any path taken by light inside the Schwarzschild radius will curve farther into the black hole.

According to Einstein's theory of general relativity (sometimes abbreviated GR), space-time is curved near massive objects. We might think of space as being like a thin rubber sheet: if a heavy weight is placed on the sheet, it sags as shown in the figure.).



The weight corresponds to a huge mass that causes space (space itself!) to curve. Thus, in the context of general relativity we do not speak of the "force" of gravity acting on objects. Instead we say that objects and light rays move as they do because space-time is curved. An object starting at rest or moving slowly near the great mass of Fig. above would follow a geodesic (the equivalent of a straight line in plane geometry) toward that great mass.

The extreme curvature of space-time shown in Fig. could be produced by a black hole.



A black hole has such strong gravity that even light cannot escape from it. To become a black hole, an object of mass M must undergo gravitational collapse, contracting by gravitational self-attraction to within a radius called the Schwarzschild radius,

$$R = \frac{2GM}{c^2}$$

where G is the gravitational constant, and c is the speed of light. If an object collapses within this radius, it is predicted by general relativity to collapse to a point at $r=0$ forming an infinitely dense singularity. This prediction is uncertain, however, because in this realm we need to combine quantum mechanics with gravity, a unification of theories not yet achieved.

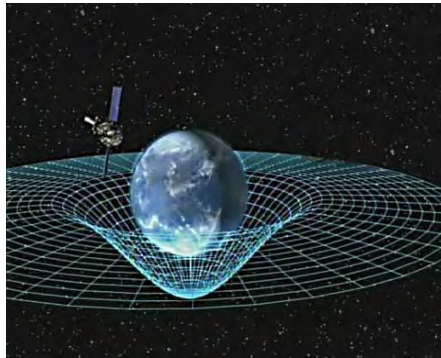
The Schwarzschild radius also represents the event horizon of a black hole. By event horizon, we mean the surface beyond which no emitted signals can ever reach us, and thus inform us of events that happen beyond that surface. As a star collapses toward a black hole, the light it emits is pulled harder and harder by gravity, but we can still see it. Once the matter passes within the event horizon, the emitted light cannot escape but is pulled back in by gravity (=of space-time).

All we can know about a black hole is its mass, its angular momentum (rotating black holes), and its electric charge. No other information, no details of its structure or the kind of matter it was formed of, can be known because no information can escape.

How might we observe black holes?

We cannot see them because no light can escape from them. They would be black objects against a black sky. But, they exert a gravitational force on nearby objects, and also on light rays (or photons) that pass nearby. The black hole is believed to be at the center of our Galaxy ($(M \approx 4 \times 10^6 M_{\text{sun}})$), and it was discovered by examining the motion of matter in its vicinity. Another technique is to examine the stars that appear to move as if they are one member of a binary system (two stars rotating about their common center of mass), but without a visible companion. If the unseen star is a black hole, it might be expected to pull off gaseous material from its visible companion. As this matter approaches the black hole, it becomes highly accelerated, and emits X-rays of a characteristic type before plunging inside the event horizon. Such X-rays, plus a sufficiently high mass estimate from the rotational motion, can provide evidence for a black hole. One of the many candidates for a black hole is, in the binary-star system, Cygnus X-1. It is widely believed that the center of most galaxies is occupied by a black hole with the mass of a typical star, like our Sun.

26.14 What are Gravitational Waves?



Two-dimensional illustration of how mass in the Universe distorts space-time. [Credit: NASA]

Gravitational waves are 'ripples' in space-time caused by some of the most violent and energetic processes in the universe. Albert Einstein predicted the existence of gravitational waves in 1916 in his general theory of relativity. Einstein's mathematics showed that massive accelerating objects (such as neutron stars or black holes orbiting each other) would disrupt space-time in such a way that 'waves' of undulating space-time would propagate in all directions away from the source. These cosmic ripples would travel at the speed of light, carrying the information about their origins, as well as, the clues to the nature of gravity itself.

Exercise

- Which of the following is not a fundamental particle?
 - electron
 - proton
 - neutrino
 - quark
- Which combination of *up* (*u*) and *down* (*d*) quarks forms a proton?
 - u u u*
 - u u d*
 - u d d*
 - d d d*

- Baryons and mesons are types of particles that consist of quarks. Which row of the table below shows a possible quark composition for a baryon, and for a meson?

	Baryon	Meson
A	$c\bar{u}$	$u\bar{c}d$
B	ucd	cu
C	$c\bar{u}$	ucd
D	ucd	$c\bar{u}$

- The group of particles known as hadrons is composed of quarks. There are two sub-groups of hadrons: mesons and baryons.
 - What is the name of the property that defines a hadron?
 - State the quark structure of a meson.
 - State the quark structure of a baryon.
 - The antiproton is the antiparticle of the proton. State one way that the proton and the antiproton are similar, and one way that they are different.
 - What are the following values of an antiproton:
 - its charge
 - its quark structure
- Particles can be leptons, hadrons, or exchange particles. Here is a list of particles:
 electron proton neutron muon pion
 - From the list state the name of one lepton and one hadron.
 - State one difference between leptons and hadrons.
 - State the difference in structure between baryons and mesons.
 - From the list, state the names of one baryon and one meson.
- The table below shows some basic information about three hadrons.

Sub-atomic particle	Quark structure	Baryon or meson	Charge
	$u\bar{d}$		
	udd		
<i>Kaon K⁺</i>			

- Copy and complete the table.

- b) All sub-atomic particles shown in the table have a corresponding antiparticle. State one example of a baryon and its antibaryon **not** shown in the table and state their quark structures
- c) The electron and the positron are an example of a lepton particle-antiparticle pair. State **one** property of positron that is the same as an electron, and **one** that is different.

7. A sodium atom contains 11 electrons, 11 protons, and 12 neutrons. Which row in the table contains the correct numbers of hadrons, baryons, mesons, and leptons in a sodium atom?

	Hadrons	Baryons	Mesons	Leptons
A	12	11	12	11
B	23	23	0	11
C	23	0	23	23
D	12	11	0	23

8. What is the charge on the following particles?
- electron
 - muon-neutrino
 - up quark
 - positron
 - down quark
 - strange quark.
9. State one similarity and one difference between a 'particle' and its 'antiparticle' (for example, an electron and a positron).
10. Why are quarks believed to be 'fundamental particles'?
11. Use the following list to answer.

proton pion muon photon neutron

- Which particle is a lepton?
muon, ...?
 - Which particles are hadrons?
proton, neutron
 - Which particles are fundamental particles?
photon, muon
 - Which particle is a meson?
pion, ...?
 - Which particle is an exchange particle?
photon.
12. A quark is a fundamental particle but a neutron is not. Explain the meaning of this statement.
13. A proton and a neutron each contain three quarks, either up or down quarks.
- Copy and complete the table to show the charge on a proton, a neutron and the quarks that they contain.

Particle	Charge	Quarks
proton	$+1(e)$	<i>uud</i>
neutron	<i>0</i>	<i>udd</i>

- b. Using the information from your table, suggest why some quarks must have a positive charge and some quarks a negative charge.

14. The Hubble constant is $2.3 \times 10^{-18} \text{s}^{-1}$.

(a)

- (i) A galaxy is $1.6 \times 10^8 \text{ly}$ from Earth. Show that its recessional speed as measured from Earth is about $3.5 \times 10^6 \text{ms}^{-1}$.

- (ii) A line in the hydrogen spectrum when measured on the Earth has a wavelength of 486nm . Calculate, in nm , the wavelength of the same hydrogen line when observed in the galaxy's emission spectrum. [2]

- (b) Outline how observations of spectra from distant galaxies provide evidence that the universe is expanding. [1]

15.

- (a) State Hubble's law.

- (b) Explain the principles used by observers on the Earth to determine the speeds of distant galaxies.

- (c) Describe how observations about distant galaxies provide evidence for:

- (i) the expanding universe

- (ii) the Big Bang theory